University of California, Davis — Date: September 1, 2011
Department of Economics — Time: 5 hours
Macroeconomics — Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Directions: Answer all questions. Feel free to impose additional structure on the problems below, but please state your assumptions clearly.

Part A: Questions on ECN 200D (Rendahl)

1. (8 points) Consider the following transition matrix

$$T = \begin{array}{c} \\ e \\ u \end{array} \begin{array}{c} \begin{array}{cc} e & u \end{array} \\ \begin{pmatrix} p & (1-p) \\ (1-s) & s \end{pmatrix} \end{array}$$

where e denotes employment and u unemployment. The first row gives the probabilities of transitioning to state e and u in the subsequent period conditional on being employed in the current period. The second row gives the equivalent probabilities conditional on being unemployed in the current period.

   (a) What is the duration of *employment*?

   (b) Suppose there is a measure one of individuals, and that the shocks are entirely idiosyncratic. What is the unemployment rate in the economy? How is it related to p and s, and why?

   (c) What is the *unconditional probability* of employment?

2. (8 points) One of the most important themes of the course was to show that functional equations such as

$$v(x) = \max_{x' \in \Gamma(x)} \{F(x, x') + \beta v(x')\} \tag{1}$$

have a unique solution (or a unique fixed point).

Explain the following concepts, and explain why we needed them in order to understand this.

   (a) Cauchy sequence.

   (b) Complete metric space.

(c) The theorem of the maximum.

(d) Contraction mapping theorem.

3. (17 points) In a stationary equilibrium of the Aiyagari-economy, an individual's Bellman equation is given by,

$$
\begin{aligned}
v(a, \theta) &= \max_{c, a'} \{u(c) + \beta \sum_{\theta' \in \{1,0\}} v(a', \theta') \lambda(\theta', \theta)\} \\
\text{s.t.} \qquad c + a' &= (1 + r - \delta)a + \theta w \\
a' &\geq \underline{a}
\end{aligned}
$$

where $\theta = 1$ equals employment and $\theta = 0$ unemployment. $\lambda(\theta', \theta)$ denotes, as usual, the probability of transitioning to state θ' in the subsequent period conditional on being in state θ in the current period. The policy function which solves the Bellman equation above is denoted $g(a, \theta)$.

The law of motion for the distribution of capital holdings is given by

$$
\psi_{t+1}(a', \theta') = \sum_{\theta \in \{0,1\}} \sum_{a : a' = g(a, \theta)} \psi_t(a, \theta) \lambda(\theta', \theta)
$$

And a representative firm's problem is given by

$$
\max_{K, N} \{F(K, N) - wN - rK\} \tag{2}
$$

where K is the amount of capital rented in the capital markets, and N the measure of employed individuals.

(a) Using the information from above, define a *stationary competitive* equilibrium in the Aiyagari economy.

(b) In contrast to the Aiyagari economy, the "Hugget economy" abstracts from both firms and capital, and considers w as a purely exogenous and perishable endowment. Thus, the only equilibrium price is therefore the interest rate, which ensures that aggregate (or net-) savings are zero. Modify the equations above to incorporate the ideas underlying a Hugget economy. Define, again, a *stationary competitive* equilibrium in the Hugget economy.

4. (17 points) Consider an infinitely lived individual born at time zero. The agent has momentary preferences $u(c)$, where c denotes contemporary consumption.

Additionally, the agent discounts the future at rate β. In each period the agent is offered a job which pays the wage w. Wages, however, are stochastic and is a function of the state s_t; $w(s_t)$. The state variable s_t can take on any value in the finite and countable set S, and its law of motion is described by the conditional probability $\lambda(s_{t+1}, s_t)$. In each period, the agent may accept or decline a job-offer. If she accepts, she will consume $w(s_t)$, and if she declines, she will consume b – where b denotes government provided unemployment benefits. She has no ability to save nor borrow.

(a) What is the agent's time-zero (or sequence-) problem? (Hint: it is useful to describe the agent's choice as a history dependent binary variable $h_t(s^t)$ which takes on the value 1 if she accepts, and 0 if she rejects.)

(b) What is the Bellman equation associated with the sequence problem above?

(c) How does your answer to (a) and (b) change if the agent receive disutility δ whenever she is working (i.e. δ is subtracted from her momentary utility whenever she is working)?

Part B: Questions on ECN 200E (Geromichalos)

5. (15 points) Consider the standard growth model in discrete time. There is a large number of identical households (normalized to 1). Each household wants to maximize life-time discounted utility

$$U(\{c_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t u(c_t).$$

Each household has an initial capital stock x_0 at time 0, and one unit of productive time in each period, that can be devoted to work. Final output is produced using capital and labor services,

$$y_t = F(k_t, n_t),$$

where F is a CRS production function. This technology is owned by firms whose number will be determined in equilibrium. Output can be consumed (c_t) or invested (i_t). We assume that households own the capital stock (so they make the investment decision) and rent out capital services to the firms. The depreciation rate of the capital stock (x_t) is denoted by δ.[1] Finally, we assume

[1] The capital stock depreciates no matter whether it is rented out to a firm or not.

that households own the firms, i.e. they are claimants to the firms' profits. The functions u and F have the usual nice properties.[2]

(a) First consider an Arrow-Debreu world. Describe the households' and firms' problems and carefully define an AD equilibrium. How many firms operate in this equilibrium?

(b) Write down the problem of the household recursively.[3] Be sure to carefully define the state variables and distinguish between aggregate and individual states. Define a recursive competitive equilibrium (RCE).

For the rest of this question focus again on an Arrow-Debreu setting.

(c) In this economy, why is it a good idea to describe the AD equilibrium capital stock allocation by solving the (easier) Social Planner's Problem?

(d) From now on assume the following functional forms: $F(k_t, n_t) = Ak_t n_t$, $u(c_t) = c_t^{1-\sigma}/(1-\sigma)$. Fully characterize (i.e. find a closed form solution for) the equilibrium allocation of the capital stock. (Hint: Write the Planner's problem recursively. Then, in the Euler equation, guess and verify a "policy rule" of the form $k_{t+1} = \gamma k_t$, where γ is an unknown to be determined.) What happens in this economy in the long run?

6. (17.5 points) Consider the Mortensen-Pissarides model in continuous time. Labor force is normalized to 1. Unemployed workers, with measure $u \leq 1$, search for jobs. At the same time, firms with vacancies, with measure v, search for unemployed workers. The matching technology, which brings unemployed workers and vacant firms together, is described by the function $m(u, v)$. We assume that m is increasing in both arguments and homogeneous of degree 1 (i.e. it exhibits constant returns to scale). It is convenient to define the market tightness $\theta \equiv v/u$.

A large measure of firms decide whether to enter the labor market with exactly one vacancy. When a firm meets an unemployed worker a job is formed. The output of a job is p per unit of time. However, while the vacancy is unfilled, firms have to pay a search cost, given by pc, per unit of time (so this cost is proportional to productivity). In an active match (job), the firm pays the worker

[2] You will not explicitly need them, so there is no need to be more precise.

[3] Here firms face a static problem. I am not asking you to explicitly spell it out, but it will be critical for a correct definition of the RCE.

a wage w per unit of time, which is determined through Nash bargaining when the two parties first match. Let β represent the worker's bargaining power.

The destruction rate of existing jobs is exogenous and given by the Poison rate λ. Once a shock arrives, the firm closes the job down. When this happens, the worker goes back to the pool of unemployment, and the firm exits the labor market. Unemployed workers get a benefit of z per unit of time.

Focus on steady state equilibria of this model. Let the discount rate of agents be given by r.

(a) Express the arrival rate of workers to a vacant firm as a function of θ. Refer to this term as $q(\theta)$. Can you determine the sign of $q'(\theta)$?

(b) Let $\eta(\theta)$ be the elasticity of $q(\theta)$ with respect to θ. Can you say anything about the range in which $\eta(\theta)$ belongs?

(c) Express the arrival rate of jobs to workers as a function of θ. How does it relate to $q(\theta)$? How does it change as θ goes up?

(d) Describe the Beveridge curve of this economy. More precisely, write down a formula that relates the steady state unemployment rate with the market tightness θ. Graph this relationship on the u, θ space.

(e) Let V be the present-dicounted value of a vacant job and J be the present-dicounted value of a filled job. Write down the Bellman equations that these two values satisfy. Explain intuitively.

(f) If there is free entry of firms into the labor market, what does J satisfy in equilibrium? Use your answer, together with your findings in part (e), to derive the job creation (JC) condition.[4] Graph the JC condition in the w, θ space.

(g) Let U be the present-dicounted value of the income stream of an unemployed worker, and W the analogous expression for an employed worker. Write down the Bellman equations that U and W satisfy. Explain intuitively.

(h) Consider a typical match of a vacant firm and an unemployed worker. Write down the Bargaining game solved by these two parties.

[4] **Hint:** The JC condition should be a formula that relates w and θ and shows the wage firms are willing to pay as a function of market tightness.

After solving the bargaining problem, one can show that the equilibrium wage satisfies $w = z + \beta(p + pc\theta - z)$. Take this result as *given*.

(i) Explain the intuition behind the wage curve given above. Also, graph this relationship in the w, θ space.

(j) Which objects should a steady state equilibrium of the model define? Use your asnwers to parts (d), (f), and (i) in order to claim that the equilibrium is unique. You can do so either graphically or algebraically.

7. (12.5 points) Consider a standard "Lucas trees" economy. There is a large number of identical households (normalized to 1) who wish to maximize expected life-time utility, given by

$$E_0 \sum_{t=0}^{\infty} \beta^t ln(c_t).$$

There is only one non-storable commodity that agents consume, call it coconuts. There are also infinitely lived objects, that we call trees, which yield coconuts. There is no production in this world. Agents can buy their shares at some price that they take as given. Let the supply of shares of the trees be normalized to 1. The holder of one share of the tree in period t is a claimant to the fruit d_t. We assume that d_t follows a Markov process. For any t, $d_t \in D \equiv \{d_1, ..., d_N\}$. Let $\Gamma_{ij} = Pr(d_{t+1} = d_j | d_t = d_i)$. Assume that d_0 is given.

a) Characterize the AD equilibrium price of one coconut in period t after a certain history realization.

b) Set up the problem of the agent in a recursive form, and include the following asset: a claim, to be bought in period $t-1$ after history $\hat{h}_{t-1}$, that pays one coconut in t if state j occurs. What is the equilibrium price of this asset?

c) What is the equilibrium price of a bond, bought in period $t-1$ after history $\hat{h}_{t-1}$, which will deliver one coconut (with certainty) in period t?

d) What is the equilibrium price of an option, bought in period $t-1$ after history $\hat{h}_{t-1}$, that allows you to sell shares of the tree in period t, at the predetermined price x?

8. (5 points)[5] Consider again a "Lucas trees" economy as in Question 7. There are only two differences. First, the fruit process is deterministic: the tree yields

[5] **Suggestion:** Answering this questions should take you 2 minutes. The answer has been discussed in class, but even if you do not remember it, you should be able to guess it, since it is very intuitive.

d units of the fruit in each period with certainty. The second difference is the following: except from claims to the tree, there is another asset called fiat money. This is an intrinsically useless object (for example pieces of paper) that gives no fruit or utility to agents. What is special about money is that its supply is not exogenous, like the real asset. The supply of money in period t is given by M_t, and it is controlled by a monetary authority. This authority chooses the growth rate of money supply, μ, so that $M_{t+1} = (1 + \mu)M_t$. As in the case of claims to the tree, agents can purchase any amount of the new asset (fiat money) in a perfectly competitive market.

(a) What is the competitive equilibrium price of one share of the Lucas tree?

(b) If $\mu = 0$ (fixed supply of money), what is the equilibrium price of fiat money? Why?

(c) What is the range of monetary policies (i.e. of values of μ), that induce agents to hold fiat money?

Hence, in this question you do not need to show any work. Full credit will be given just for stating the correct result. In fact, I encourage you to just spell out the intuitive answer, rather than writing down any value functions.