University of California, Davis
Department of Economics
Macroeconomics

Date: September 4, 2008
Time: 5 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Directions: Answer all questions.

Short Answer Questions - Keep your answers short and concise. (Each question is worth 5 points.)

1. In the Lucas tree model of asset prices, demonstrate that the price of equity is a monotonically increasing function of dividends if the dividend process is *i.i.d.* Does this necessarily hold when the dividend process exhibits positive autocorrelation? (Recall that in the Lucas model, the *level* of the dividend is stationary.)

2. Habit persistence in consumption has been shown to help resolve the equity premium puzzle. How does this modification of preferences improve the performance of the consumption-based capital asset pricing model.

3. For the Solow model with capital accumulation, population growth, and exogenous technical progress, derive the Golden Rule . Be explicit about the objective function, the optimal allocation of capital, and how that allocation is supported as an equilibrium. To keep things simple, assume that the government can determine the economy's saving rate.

4. In the context of endogenous growth theory, what is a 'scale effect.' Give an example. Is this a positive or negative feature of a model?

Longer Answer Questions (Each question is worth 20 points.)

5. Suppose that a representative consumer receives an endowment of a non-storable consumption good. The endowment evolves exogenously according to

$$\Delta \ln C_t = \mu + \rho \Delta \ln C_{t-1} + \sigma \varepsilon_t, \tag{1}$$

where Δ is the difference operator and ε_t is an *iid* $N(0,1)$ random variable. The consumer's preferences are

$$E_t \sum_{j=0}^{\infty} \beta^j \ln C_{t+j}. \tag{2}$$

(a) Solve recursively for the value of the endowment process. (I.e., write the Bellman equation and solve for the value function.)

(b) What is the marginal value of an increase in the mean growth rate of the endowment?

(c) What is the marginal value of a reduction in volatility, as measured by a decline in σ?

(d) Provide intuition about the relative magnitudes of the marginal values in (b) and (c).

6. Consider a variation of the Sidrauski monetary model with a constant population. Specifically, assume that the representative agent's maximize lifetime utility is given by:

$$\sum_{t=0}^{\infty} \beta^t \left[U(c_t) + V\left(\frac{M_t}{P_t}\right)\right]$$

where $U(\cdot)$ and $V(\cdot)$ are concave, twice-differentiable functions, $c(t)$ denotes consumption and $M(t)$ is money chosen in period t. Each period, agents use beginning of period nominal balances, the revenue from sales of output and a lump-sum monetary transfer to purchase consumption, investment and new money. In contrast to the Sidrauski model, both capital and money are used as inputs into the production process. Letting y_t denote output, the production function is given by:

$$y_t = \left(1 - z\left(\frac{M_t}{P_t}\right)\right) f(k_t)$$

where $z'(\cdot) < 0, z''(\cdot) > 0, z(0) = 1, \lim_{M_t/P_t \to \infty} z(M_t/P_t) = 0$. The function $f(k_t)$ has standard properties. The money supply in this economy is growing at the constant rate $\mu > 0$ and capital depreciates at the constant rate of $\delta < 1$.

(a) Derive and interpret the necessary conditions associated with the agent's maximization problem.

(b) Define a steady-state equilibrium in this economy. Contrast the effects of money growth in this setting with those obtained in the original Sidrauski model. (NOTE: It is NOT necessary to solve explicitly for $dc/d\mu, dk/d\mu$, etc. An intuitive argument is sufficient.)

7. Consider a representative agent economy in which household preferences are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln c_t + A \ln(1 - h_t)\right]$$

Each period, households supply labor h_t, and capital, k_t, to firms; the factor prices are denoted w_t and r_t,respectively. In addition, they choose consumption and investment every period. The depreciation rate of capital is 100%. Firms in the economy have technology given by:

$$y_t = z_t k_t^{\alpha} h_t^{1-\alpha}$$

where z_t denotes an *i.i.d.* technology shock with $E(z_t) = 1$. Firms make input choices in order to maximize profits. Given this environment, do the following:

(a) Define a recursive competitive equilibrium.

(b) Solve for the equilibrium functions which determine consumption, capital, and labor.

(c) Suppose a new asset that entitles the owner to the stream of future consumption is introduced into this economy. That is, this asset is identical to equity whose dividend is consumption. Let q_t denote the price of this asset; solve for the equilibrium price of equity.

(d) Since capital chosen in period t, i.e. k_{t+1}, can be used to generate the future path of expected consumption, it would seem logical that $k_{t+1} = q_t$. Prove that this is not the case. Why?

(e) Suppose one used Hall's method to test the implications of the permanent income hypothesis within this economy; would the stochastic implications of the life-cycle hypothesis be supported? (Note: The analysis is simplified if all variables are in logs.)