University of California, Davis
Department of Economics
Macroeconomics

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Time: 4 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Directions: Answer all questions. Part 1 collectively counts for 25 percent of the grade. Each of the questions in Part 2 count for 25 percent of the grade.

Part 1: Short Answer Questions - Keep your answers short and concise.

1. In Lucas's consumption CAPM, returns on traded assets satisfy the following Euler equation,
$$\beta E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} R_{j,t+1} = 1, \tag{1}$$
where β is the subjective discount factor, α is the coefficient of relative risk aversion, C_t is consumption, and $R_{j,t+1}$ is the gross real return on security j. Briefly discuss how the equity premium and risk-free rate puzzles follow from this equation.

2. In the Solow model, the golden rule allocation of capital satisfies
$$f'(k_{GR}) = n + g + \delta,$$
where $k \equiv K/AL$, $f(k)$ is the production function, n is the population growth rate, g is the rate of exogenous technical progress, and δ is the depreciation rate. How can we use data to determine whether the US economy has more or less capital than the golden rule allocation? What do the data tell us?

3. Within the context of a real business cycle model, assume that agents's utility is separable in consumption and leisure. Explain how the stylized facts that consumption and the real wage are growing over time while labor is relatively constant place restrictions on the form of the utility function.

Part 2: Longer Answer Questions

4. Consider a finite-horizon version of Hall's (1978) consumption model. A representative consumer lives for T periods and can borrow or lend freely at a constant gross interest rate R. She has an exogenous stochastic stream of labor income y_t and must choose how much to consume and save each period. Her preferences are time separable, and period utility is quadratic. Collecting these assumptions, we can write her decision problem as

$$\max_{\{c_t\}} E_0 \;\Box\; -\frac{1}{2}\sum\nolimits_{t=0}^{T} \beta^t (c_t - \bar{c})^2 \Big], \tag{2}$$

subject to

$$\begin{aligned} A_{t+1} &= R(A_t + y_t - c_t), \\ A_{T+1} &\geqslant 0, \end{aligned} \tag{3}$$

where β is the subjective discount factor and A_t is financial wealth ($A_t < 0$ means she is a debtor). To keep things simple, suppose that $R = \beta^{-1}$.

For period $T - j$, $j = 0, 1, ..., T$, the decision rule can be expressed as

$$c_{T-j} = \gamma_{T-j} \left[A_{T-j} + E_{T-j} \sum\nolimits_{h=0}^{j} R^{-h} y_{T-j+h} \right], \tag{4}$$

where γ_{T-j} is the marginal propensity to consume out of wealth (financial wealth plus human capital). This decision rule can be found by backward induction.

(a) Start in the terminal period T. What is the maximization problem and decision rule for that period? What is the intuition behind this result?

(b) Now step back to period $T - 1$. Write down the maximization problem, first-order condition, and decision rule for date $T - 1$.

Hints: use the decision rule for T. Also, your answer will involve a term $R/(1+R)$. It's convenient to express this as

$$\left(\frac{R}{1+R}\right)\left(\frac{R^{-1}}{R^{-1}}\right) = \frac{1}{1+R^{-1}}. \tag{5}$$

(c) Continuing the backward recursion, derive a formula for γ_{T-j}.

(d) To what does γ_{T-j} converge as T and j grow large? Interpret that limiting value.

5. In a continuous time version of the Solow model, the law of motion for capital is

$$\dot{k} = sk^{\alpha} - (n + g + \delta)k,$$

where $k \equiv K/AL$, s is the savings rate, α is the capital share, n is the population growth rate, g is the rate of exogenous technical progress, and δ is the depreciation rate.

(a) Draw the phase diagram and show that the steady state is globally stable.

(b) Derive an approximation to the rate of convergence.

(c) How does the rate of convergence depend on the capital share? Briefly relate that to the empirical literature on the speed of convergence.

6. Consider the following simple real business cycle model in which output is a linear function of beginning-of-period capital (labor is not an input)

$$y_t = Ak_t$$

and there is no depreciation of capital. Unlike a typical RBC framework, there are no technology shocks in this economy. Uncertainty is instead due to taste shocks, ν_t, that affect household's utility function. This is given by the quadratic utility function:

$$U(c_t) = c_t - \theta (c_t - \nu_t)^2$$

(It is assumed that c_t is in the range so that marginal utility is always positive.) Agents maximize

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U(c_t) \right]$$

It is assumed that the ν_t are mean zero, *i.i.d.* shocks and that $\beta^{-1} = (1 + A)$. Given this environment, answer the following:

(a) Intuitively describe how taste shocks affect agents' consumption demand?

(b) Setup the households' maximization problem as a dynamic programming problem and derive the associated necessary conditions.

(c) Given the linear framework in this economy, it is reasonable to conjecture that the consumption policy function will be a linear function of the state variables. Use this conjecture in the necessary conditions and solve for the coefficients of the policy function. (Hint: use the method of undetermined coefficients. That is, the expression derived when using the conjecture must hold for all values of the state variables. This imposes a set of restrictions that the coefficients must satisfy.) Using the resource constraint, derive the policy function for capital.

(d) What is the correlation between investment and consumption in this model? What is the implication for the role of taste shocks as an important impulse mechanism for the business cycle?