University of California, Davis
Department of Economics
Macroeconomics

Date: September 8, 2005
Time: 4 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Directions: Answer all questions; the weights for each question are given in parentheses.

Short Answer Questions

1. (25% - parts (a)-(d) are equally weighted.) Keep your answers short and concise.

   (a) Two modifications to the basic real business cycle model that have been incorporated are *household production* and *labor adjustment costs.* What is the rationale for these modifications? Have they been successful?

   (b) In a simple RBC model, what is the correlation between interest rates and the marginal productivity of capital?

   (c) In the Solow growth model, show how an increase in the savings rate influences the level and growth rate of output per worker.

   (d) In a linear-quadratic control problem, how does an increase in uncertainty affect decisions? E.g., does it promote 'saving for a rainy day?'

Longer Answer Questions

2. (25 %) Consider a version of the Ramsey model in which population is constant and there is no exogenous technical progress. Households have preferences of the form

$$U = \int_0^\infty e^{-\rho t} U(C_t) dt,$$

where ρ is the discount rate, C_t is consumption, and $U(C_t) = C_t^{1-\theta}/(1-\theta)$. Firms produce output from labor and capital using a Cobb-Douglas production function, $Y_t = K_t^\alpha (AL)^{1-\alpha}$, where A and L are both constant. Capital depreciates at a constant rate $\delta > 0$.

   (a) What is the social planner's optimization problem and first-order conditions? How does it relate to a competitive equilibrium for this economy?

   (b) Draw a phase diagram and characterize the steady-state and golden rule allocations.

(c) Can the steady-state level of capital exceed the golden rule capital stock? Why or why not?

(d) Does the result in (c) necessarily carry over to models with overlapping generations? Why or why not?

3. (25 %) Imagine an infinitely-lived representative consumer who can borrow or lend freely at a constant gross interest rate R, subject only to a constraint that rules out infinite debt. She has an exogenous stochastic stream of labor income y_t and must choose how much to consume and save each period. Her preferences are time separable, and period utility is quadratic. Collecting these assumptions, we can write her decision problem as

$$\max_{\{c_t\}} E_0 \left[-\frac{1}{2} \sum_{t=0}^{\infty} \beta^t (c_t - \bar{c})^2 \right],$$

subject to

$$b_{t+1} = R(b_t + c_t - y_t),$$

where β is the subjective discount factor and b_t is the amount borrowed ($b < 0$ if she is a lender). To keep things simple, let's also suppose that $R = \beta^{-1}$.

(a) What is the first-order condition for optimal consumption? What does it imply about the univariate time series representation for consumption?

(b) Now assume that income is a first-order autoregressive process,

$$y_t = \phi y_{t-1} + \varepsilon_t,$$

where ε_t has mean zero and is identically and independently distributed across time. Use the budget constraint along with the first-order condition to solve for the decision rule for consumption.

(c) How does the autoregressive parameter ϕ influence the variance for Δc_t? What is the economic intuition behind this result?

4. (25 %) Consider an overlapping generations version of the Lucas tree model. Specifically, an exchange economy is characterized by a random total endowment of x_t each period. It is assumed that x_t is identically and independently distributed over time. Each period a new generation (that lives for two periods) is born. There is no population growth so you can think of the economy at any point in time consisting of a representative young and representative old person. At birth, each agent receives a fraction (ηx_t) of the endowment (with

$0 < \eta < 1$); this income is used to finance current consumption and saving. There is no income when old so consumption in the final period of life is financed from the returns to savings. There are two assets: a one period bond, b_t, that costs p_t units of consumption in period t and returns one unit of of consumption in the following period and equity. Equity purchased in period t yields a dividend of $(1-\eta)\,x_{t+1}$ next period (and, of course, the equity can be sold as well). Let q_t denote the price of equity and assume that the total shares in the economy (denoted $\bar{z}$) are normalized to 1. Agents make choices in order to maximize lifetime expected utility given by:

$$U(c0_t) + \beta E\left[U(c1_{t+1})\right]$$

Given this environment, do the following:

(a) Write down the agents' maximization problem.

(b) Derive and interpret the associated necessary conditions.

(c) Define a recursive competitive equilibrium in this economy.

(d) Prove that, in equilibrium, equity prices are procyclical while interest rates are countercyclical.

(e) Given an *intuitive* proof for the sign of the equity premium in this economy.