

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Directions: Answer all questions. Feel free to impose additional structure on the problems below, but please state your assumptions clearly.

Short Answer Questions. Keep your answers short and concise. (Each question is worth 7 points.)

1. Consider the following (real valued) sequence $\{x_t\}_{t=0}^{\infty}$ defined by: $x_0 = 1, x_1 = 1+a, x_2 = 1 + a + a^2, x_n = 1 + a + a^2 + \dots + a^n$ and so on. Prove that if $|a| < 1$, then $\{x_t\}_{t=0}^{\infty}$ has a unique limit point. (Hint: Define x_{n+1} recursively. You are then allowed to evoke the contraction mapping theorem.)
2. A large share of macroeconomic models build on the Ramsey growth model augmented with a labor-leisure choice

$$V(k_0) = \max_{\{c_t, k_{t+1}, \ell_t\}_{t=0}^{\infty}} \sum \beta^t u(c_t, \ell_t) \quad (1)$$

$$s.t \quad c_t + k_{t+1} = f(k_t, \ell_t) \quad (2)$$

In class, however, we only considered problems of the type

$$V(x_0) = \max_{\{x_{t+1}\}_{t=0}^{\infty}} \sum \beta^t F(x_t, x_{t+1}) \quad (3)$$

$$s.t \quad x_{t+1} \in \Gamma(x_t) \quad (4)$$

Define $x_t, F(\cdot, \cdot)$ and $\Gamma(\cdot)$ such that these two problems coincide exactly.

3. True or False: An implication of the Hansen indivisible labor RBC model is that consumption and labor productivity will have the same time series properties.

Longer Answer Questions (Each question is worth 20 points.)

4. Consider an exchange economy populated by identical agents that trade equity shares, z_t , defined as title to the endowment process. (That is, this is the same asset priced in the Lucas tree model.) Denote the price of equity as q_t . Agents also trade one-period bonds which cost p_t units of consumption in period t and return 1 unit of consumption in the following period. In addition to these assets, a one-period forward contract on bonds is traded. In this contract, agents agree at time t to pay ϕ_t units of consumption in period $t + 1$ for the promise of one unit of consumption to be received in period $t + 2$. The endowment, x_t , is stochastic and varies over the interval (\underline{x}, \bar{x}) ; furthermore, x_t is assumed to be independently and identically distributed. Given this environment, agents choose a sequence of consumption and assets in order to maximize

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U(c_t) \right]$$

- (a) Formulate the agent's problem as a dynamic programming problem. Be explicit in identifying the state and control variables.
 - (b) Derive and interpret the necessary conditions which characterize the solution to this maximization problem.
 - (c) Define a recursive competitive equilibrium in this economy.
 - (d) Prove that equilibrium bond and equity prices are positively correlated with the endowment while the price of the forward contract is constant. Explain these results.
5. Consider an infinitely lived individual born in time zero, endowed with a cake of size x_0 . The cake is storable (without depreciation) and infinitely divisible. The agent derives contemporary utility from (cake) consumption through $u(c_t)$, and has as her ultimate objective to maximize her infinitely discounted sum of utility streams. Her discount rate is, as usual, given by β .
- (a) Formulate the mathematical problem corresponding to the above description (i.e formulate the Sequence Problem).
 - (b) Provide a recursive formulation and show that its solution also solves the Sequence Problem in (i) (i.e. Theorem 2).
 - (c) Let $u(c) = \ln(c)$. Derive both the value function, $v(x)$, and the policy function, $x' = g(x)$.

6. Consider the following sequence problem

$$V(a_0, w_0) = \max_{\{c_t\}_{t=0}^{\infty}} \sum \beta^t u(c_t) \quad (5)$$

$$s.t \quad a_0 + \sum_{t=0}^{\infty} q^t w_t = \sum_{t=0}^{\infty} q^t c_t \quad (6)$$

$$w_{t+1} = (1 - \delta)\bar{w} + \delta w_t \quad (7)$$

(a) For which value of δ does (7) converge to a unique fixed point? What is the value of this fixed point?

(b) Define

$$a_t = \sum_{s=0}^{\infty} q^s (c_{t+s} - w_{t+s})$$

Use this definition to show that the problem above has a recursive representation. No formal proof is necessary, but explain the logic underlying Theorem 1 in this context. What interest rate do the “bonds” a_t pay?

(c) In a deterministic economy, such as the one above, it appears as if a one period bond is sufficient to allocate resources efficiently. Is this also true in a stochastic economy? Why or why not?

7. Consider a representative agent, exchange economy similar to that studied by Mehra and Prescott. Specifically, it is assumed that the endowment, x_t , grows stochastically as given by $x_{t+1} = \lambda_{t+1}x_t$ where the growth rate, λ_t , is assumed to be independently and identically distributed. Agents maximize lifetime expected utility:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t (c_t - hc_{t-1})^{1-\gamma} \right]$$

where $\beta \in (0, 1)$ and $0 \leq h < 1$. In this economy, agents trade one-period bonds that cost p_t units of consumption in period t and return one unit of consumption in period $t + 1$. Given this environment, do the following:

(a) Define a recursive competitive equilibrium and derive the necessary condition associated with optimal bond purchases.

(b) Suppose that $h = 0$. Characterize the behavior of equilibrium bond prices? How does a value of $h > 0$ affect the volatility of bond prices? Discuss the implications that these results have for the assumption of habit persistence to help resolve some asset pricing puzzles.