University of California, Davis
Department of Economics
Macroeconomics

Date: June 28, 2004
Time: Four Hours
Reading Time: 20 minutes

Preliminary Examination for the Ph. D. Degree

Directions: Answer all questions. Note that, while you have four hours for the exam, ***the test is designed to be finished in 3 hours.***

1. **Briefly** discuss the following statements (keep your answers short and concise):
   (a) The consumption-based capital asset pricing model is inconsistent with high volatility of stock prices.
   (b) In real business cycles, the MPK is highly procyclical. This implies that interest rates (i.e. real) will be as well.
   (c) In the standard Overlapping Generations Model with no technological growth the competitive equilibrium is dynamically inefficient when the growth rate of population is large.
   (d) The Solow model assumes that all countries converge to their own balanced growth path. Therefore, in the long run, differences in income per capita across countries should be fully explained by differences in their saving rates.

2. Consider the standard set up of the Ramsey model modified to include capital depreciation, taxation of investment and lump sum transfers in the way described below. The representative household maximizes her lifetime utility:

$$\max_{c(t)} \int_0^\infty u(c(t))e^{-(\rho-n)t}dt \quad , \qquad \rho > n > 0$$

Subject to the dynamic constraint:

$$\dot{a}(t) = w(t) + (1-\tau)r(t)a(t) - na(t) - c(t) + T(t)$$

And to the transversality condition:

$$\lim_{t\to\infty}\left(a(t)e^{-\int_0^t r(s)ds} \right) = 0$$

$c(t)$ is consumption per capita, $a(t)$ are assets per capita, *n* is the growth rate of the population, ρ is the inter-temporal discount rate, $w(t)$ denotes wage and $r(t)$ denotes the return to capital. The tax rate on the return to assets is $\tau > 0$ while $T(t)$ is a lump-sum transfer per capita from the government to the families.

The production function for the representative firm is $Y(t)=K(t)^{\alpha}L(t)^{1-\alpha}$. Y(t) is output, K(t) is physical capital and L(t) is labor. Total supply of labor is equal to total population. The production function does not exhibit any productivity growth. The representative firm rents capital and labor from the households and maximizes profits, taking *w(t)* and *r(t)* as given.

Capital depreciates at rate $\delta>0$ so that the firm has to replace depreciated capital before paying the production factors.

a) Write down the maximization problem of the firm and solve it. Say whether the firm's problem is a dynamic problem or not and explain why. Find the equilibrium interest rate and wage rate as a function of capital per worker k=K/L. Show that the total net product of the economy is fully exhausted by the payments to the production factors.

b) Write down the maximization problem for the representative household. Find the Euler equation that defines the optimal consumption using the Hamiltonian technique.

c) In equilibrium the family assets are equal to the physical capital in the economy, so that $a(t)=k(t), \dot{a}(t)=\dot{k}(t)$. Moreover the government has a balances budget so that its tax income is equal to the transfers to households. Using these conditions and the equilibrium values of *w(t)* and *r(t)* write down the dynamic constraint as a function of *k(t), c(t)* and parameters only.

d) Using the Euler equations and the dynamic constraint draw the phase diagram for this economy on the k(t), c(t) axis. Indicate the dynamics with arrows and mark clearly the saddlepath.

e) Calculate the steady state values of capital per person k*, and consumption per person c*.

f) Describe on a new phase diagram the effect of an unexpected increase in τ at time t_0. Show what happens over time to c and k.

g) Describe on a new phase diagram the effect of an anticipated increase in τ. Show what happens over time to c and k when the government announces at t_0 that an increase in τ will take effect at t_1.

3. Consider a standard search model in which workers look for jobs and firms post vacancies and look for workers. In continuous time the rate at which vacant jobs are filled is $q(\theta)$(equal to number of matches/Vacancies), with $\partial q/\partial \theta < 0$ and $\theta =$(Vacancies/Unemployed) is the measure of market tightness. The rate at which unemployed find jobs is $\theta q(\theta)$ and $\partial \theta q(\theta)/\partial \theta > 0$. Let λ denote the rate at which a match is broken and a worker becomes unemployed.

   (a) Standardize the labor force to 1 and define, in steady state, the relation between *u*, unemployment rate and θ. Show that for constant λ unemployment in steady state is always a negative function of market tightness.

   (b) The cost of a vacant job per unit of time is *c,* the interest rate is r, the productivity of a match (i.e. total product to be split between worker and firm) is *a* and wage is *w*. Define recursively, using the Bellman Equation, the value of a vacancy, V, and of a filled vacancy J.

   (c) Imposing free posting of vacancies V=0, derive the "job creation equation" in which *w*, the wage, depends on productivity, market tightness and other parameters.

   (d) Assume unemployment benefits equal to z<a. Use the Bellman equation to derive the value of being unemployed U, and of being employed W.

(e) Workers and firms split the surplus of a match so that (W-U)=(β/(1−β))(J-V) where β indicates the relative bargaining power of the workers. Use this condition and those under d) and c) to find the "wage-setting equation" as function of z, β, a, c and θ.

(f) Draw on a diagram with w (wage) and θ (Market tightness) on the axis, the job creation equation and the wage setting equation.

(g) Show on the diagram (or using the equations) how an increase in productivity a changes wages, market tightness and unemployment in equilibrium (i.e. say if they increase of decrease). Do the same (on a new diagram) for an increase in β, the bargaining power of workers.

4. Consider a representative agent economy in which household preferences are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln c_t + A \ln(1 - h_t)\right]$$

Each period, households supply labor, h_t, and capital, k_t, to firms; the factor prices are denoted, r_t and w_t respectively. In addition, they choose consumption and investment every period. The depreciation rate of capital is 100%.

Firms in the economy have technology:

$$y_t = z_t k_t^{\alpha} h_t^{1-\alpha}$$

where z_t denotes an *i.i.d.* technology shock with $E[z_t] = 1$. Firms make input choices in order to maximize profits. Given this environment, do the following:

a. Define a recursive competitive equilibrium.
b. Solve for equilibrium consumption, capital, and labor.
c. Suppose a new asset that entitles the owner to the stream of future consumption was introduced into this economy. That is, this asset is identical to equity whose dividend is consumption. Let q_t denote the price of this asset; solve for the equilibrium price of equity.
d. Since capital chosen in period t, i.e. k_{t+1}, can be used to generate the future path of expected consumption, it would seem logical that $k_{t+1} = q_t$. Prove that this is **not** the case. Why?
e. Suppose one used Hall's method to test the implications of the permanent income hypothesis within this economy; would the stochastic implications of the life-cycle hypothesis be supported? (Note: The analysis is simplified if all variables are in logs.)