University of California, Davis
Department of Economics
Macroeconomics

Date: July 3, 2006
Time: 4 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Directions: Answer all questions. Part 1 collectively counts for 25 percent of the grade. Each of the questions in Part 2 count for 25 percent of the grade.

Part 1: Short Answer Questions - Keep your answers short and concise.

1. The solution to a linear-quadratic Gaussian dynamic program exhibits 'certainty equivalence.' State precisely what that means.

2. What is dynamic efficiency? Can the equilibrium of the Ramsey-Cass-Koopmans model be dynamically inefficient? Why or why not?

3. In a simple RBC model with log preferences and 100% depreciation, consumption is typically a constant fraction of income. This result is invariant to whether the technology shocks are temporary (i.i.d.) or persistence (positively autocorrelated). Is this a contradiction to the permanent income hypothesis?

4. Suppose that consumption growth follows the process:

$$\frac{c_{t+1}}{c_t} = \bar{c}_\Delta \exp\left\{\varepsilon_{c,t+1} - \sigma_c^2/2\right\}$$

where $\varepsilon_{c,t}$ is assumed to be a normally distributed $i.i.d.$innovation with mean of zero. The return on riskless bonds must satisfy the standard intertemporal Euler equation:

$$1 = \left(1 + \bar{r}^b\right) \beta E\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\right]$$

Using the properties of lognormally distributed random variables, this implies that the riskless interest rate is given by (do not derive this):

$$\bar{r}^b = -\ln\beta + \gamma \ln\bar{c}_\Delta - \gamma\left(1+\gamma\right)\frac{\sigma_c^2}{2}$$

Explain the significance of each term in this expression.

Part 2: Longer Answer Questions

6. Imagine a representative consumer who lives forever and who can borrow or lend freely at a constant gross interest rate R, subject only to a constraint that rules out infinite debt. She has an exogenous stochastic stream of labor income y_t and must choose how much to consume and save each period. Her preferences are time separable, and period utility is quadratic. Collecting these assumptions, we can write her decision problem as

$$\max_{\{c_t\}} E_0 \left[-\frac{1}{2} \sum\nolimits_{t=0}^{\infty} \beta^t (c_t - \bar{c})^2 \right],$$

subject to

$$b_{t+1} = R(b_t + c_t - y_t),$$

where β is the subjective discount factor and b_t is the amount borrowed ($b < 0$ if she is a lender). To keep things simple, suppose that $R = \beta^{-1}$.

(a) Write the Bellman equation for this problem.

(b) What is the functional form of the value function? How can you prove that? (You don't have to do the proof; that would take too much time. Just outline the key steps.)

(c) What is the first-order condition for optimal consumption? What does it imply about the univariate time series representation for consumption?

(d) Now assume that income is a first-order autoregressive process,

$$y_t = \phi y_{t-1} + \varepsilon_t,$$

where $\varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$. Use the budget constraint along with the first-order condition to solve for the decision rule for consumption.

(e) How does the autoregressive parameter ϕ influence the innovation variance for consumption? What is the economic intuition behind this result?

7. Consider the following optimal growth model. Agents' preferences are given by

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \ln c_t \right]$$

while output is a concave function of beginning of period capital and a technology shock. That is,

$$y_t = z_t k_t^{\alpha}$$

It is assumed that z_t is *i.i.d.* The law of motion for the capital stock is

$$k_{t+1} = k_t^{1-\delta} i_t^{\delta}$$

Note that the parameter δ incorporates both depreciation and costs of adjustment that affect investment, i_t. Solve for the policy functions describing optimal consumption and investment. Show that if $\delta = 1$, the solution is identical to that studied in class. (Note: When manipulating the associated necessary conditions, it is useful to express the variables as ratios when possible. For instance, the $MPK = \alpha z_t k_t^{\alpha-1} = \alpha \frac{y_t}{k_t}$. And the resource constraint is $\frac{y_t}{c_t} = 1 + \frac{i_t}{c_t}$.)

8. Consider the following variation of the Sidrauski model. Assume that preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t \left[U \left(c_t, \frac{M_{t-1}}{P_t} \right) \right]$$

Note that utility from real balances depends upon money chosen in the previous period. Output is produced using the techology $y_t = f(k_t)$ where $f' > 0$, $f'' < 0$. In addition to money and capital, agents also trade one-period nominal bonds that cost \$1 at time t and return \$$(1 + n_t)$ in period $t + 1$. The money supply is assumed to grow at the rate μ per period so that the law of motion of the money stock is $\bar{M}_t = (1 + \mu) \bar{M}_{t-1}$. New money is introduced via lump-sum transfers.

(a) Express the maximization problem as a dynamic programming problem; derive and interpret the associated first-order conditions.

(b) Define a steady-state equilibrium in this economy.

(c) Using steady-state analysis, calculate and interpret: $\frac{dk}{d\mu}, \frac{dm}{d\mu}, \frac{dn}{d\mu}$. (Where m is steady-state level of real balances.)