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Department of Economics
Macroeconomics

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Time: 5 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Directions: Answer all questions. Feel free to impose additional structure on the problems below, but please state your assumptions clearly.

Short Answer Questions. *Keep your answers short and concise. (Each question is worth 7 points.)*

1. Consider the following equation

$$U = b + e^{-\delta}(e^{-p}U + (1 - e^{-p})V)$$

   (a) If b equals unemployment benefits, and $1 - e^{-p}$ is the probability of finding a job, what is the interpretation of U?

   (b) In the equation above, one time-period is equal to one unit of time. Reformulate the equation such that one time-period equals Δ units of time. How does the probability of finding a job change as Δ becomes smaller? Why is this?

   (c) What is the continuous-time formulation of the above equation?

2. Consider a Solow growth model in which the aggregate production function is of the form: $Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha}$ where Y denotes output, K is the capital stock, L is labor and A is labor-augmenting technology. Assume that L and A grow exogenously at the rates n and g respectively. Demonstrate that, in the steady-state, the equilibrium in this economy is consistent with Kaldor's stylized facts of growth.

3. In order for the equilibrium in a standard Ramsey-Cass-Koopmans economy to exhibit balanced growth (which, in turn, is consistent with Kaldor's stylized facts), it is necessary to place restrictions on agents' preferences. Provide an intuitive argument for the nature of these restrictions. That is, you do not need to formally derive the restrictions; instead, discuss their motivation.

Longer Answer Questions. *(Each question is worth 20 points.)*

4. Consider the following (very simple) consumption-savings problem with habits

$$V(a_0, c_{-1}) = \max_{\{a_{t+1}, c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t - \gamma c_{t-1}) \tag{1}$$

$$\textit{subject to } c_t + a_{t+1} = w + (1+r)a_t, \quad t = 0, 1, \ldots \tag{2}$$

with a_0 and c_{-1} given, and $\gamma \in [0, 1)$.

(a) Derive the Bellman equation corresponding to the optimization problem above (i.e. prove "Theorem 1").

(b) Prove that this is a contraction mapping with modulus β.

(c) Use the envelope theorem to derive the derivative of the value function with respect to (both) state variables (no proof).

(d) Derive the Euler equation.

5. Let z be a random variable that takes on values in $Z = \{0, 1\}$. Here, 1 denotes employment and 0 unemployment. Consider an arbitrary process of consumption $\{c_t(z^t)\}_{t=0}^{\infty}$ where $c_t : Z^{t+1} \to \mathbb{R}_+$. For this question, the probability of an unemployed individual finding a job is endogenous and depends on her search effort. Search effort is simply the binary choice between being a searcher, or not being a searcher. If the agent is a searcher, she sets variable x to 1, and the probability of finding a job equals $\bar{p}$; that is $P(z_{t+1} = 1 | z_t = 0) = \bar{p}$. If the agent decides not to search, she sets x to 0, and the probability of finding a job is zero; that is $P(z_{t+1} = 1 | z_t = 0) = 0$. Let us, for simplicity, assume that once a job is found, it lasts for perpetuity and is unaffected by the agent's choice of x; that is $P(z_{t+1} = 1 | z_t = 1) = 1$.

We can summarize these assumptions concisely by

$$\lambda(z^{t+1}) = p(x)\lambda(z^t), \text{ if } z_t = 0, \text{ and } \lambda(z^{t+1}) = \lambda(z^t), \text{ if } z_t = 1$$

where $p(x) = \bar{p}$ if $x = 1$ and zero if $x = 0$. $\lambda(z^{t+1})$ denotes as usual *the probability of history z^{t+1} occurring.*

Lastly, an agent's preferences are given by

$$\sum_{t=0}^{\infty} \sum_{z^t \in Z^{t+1}} \beta^t \{u(c_t(z^t)) - x_t(z^t)\} \lambda(z^t)$$

where $x_t(z^t)$ is an agent's choice of search effort in period t, after having observed history z^t. Notice that the agent gains disutility 1 of being a searcher.

(a) After any history z^t, define a *continuation value* in this economy. Denote this $\vec{V}(z^t)$.

(b) Show that if $z_t = 0$, a *necessary* condition for an agent to be a searcher (i.e. to set $x_t(z^t) = 1$) is given by

$$\vec{V}((z^t, 1)) - \vec{V}((z^t, 0)) \geq \frac{1}{\beta \bar{p}}$$

(c) If $z_t = 1$, what is the optimal choice of $x_t(z^t)$?

(d) Now, let us assume there is a government providing unemployment insurance and taxing labor income. More precisely, the government will collect *all* labor income once the agent is employed, and always provide some sort of benefits equal to $c_t(z^t)$. The government does so in order to maximize her own (expected present value) revenue, and such that the agent is given some minimum (expected present value) utility. However, the government must also recognize that the agent will freely decide whether to search or not. To make the problem simple(r), we assume that *the government* always prefers the agent to be a searcher when unemployed (i.e. to set $x_t(z^t) = 1$ whenever $z_t = 0$, but not otherwise). The government's optimization problem is then given by

$$J(V, z_0) = \max_{\{c_t(z^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{z^t \in Z^{t+1}} \beta^t \{z_t w - c_t(z^t)\} \lambda(z^t) \tag{3}$$

$$\textit{subject to} \quad \vec{V}((z^t, 1)) - \vec{V}((z^t, 0)) \geq \frac{1}{\beta \bar{p}}, \quad \forall z_t = 0, \ t = 0, 1, \ldots \tag{4}$$

$$V = \sum_{t=0}^{\infty} \sum_{z^t \in Z^{t+1}} \beta^t \{u(c_t(z^t)) - (1 - z_t)\} \lambda(z^t) \tag{5}$$

Provide the Bellman equation associated with the optimization problem above (no need for a proof). Hint: A good starting point is to derive the Bellman equation for $z_0 = 1$, and then for $z_0 = 0$.

6. In Kydland and Prescott's original RBC model, they made the assumption that investment does not produce capital immediately; i.e. the economy exhibited "time to build". Consider a representative-agent (i.e. no population growth), non-stochastic version of their economy and assume that agents have preferences given by:

$$\sum_{t=0}^{\infty} \beta^t \left(\ln c_t + \theta \ln \left(1 - h_t\right) \right)$$

where c_t is consumption and h_t is time spent in work activity. Aggregate output is produced using a standard Cobb-Douglas production function:

$$y_t = k_t^{\alpha} h_t^{1-\alpha}$$

where y_t is output and k_t denotes capital. (Note that there is no technological progress in the economy.) In each period, agents choose consumption, work effort and investment in order to maximize lifetime utility. In this economy, an investment project started at time t does not produce capital until period $t + 2$. The costs associated with this project are spread out over the two-period horizon. Let s_{it} denote an investment project that is finished after i periods ($i = 1, 2$) where households pay the fraction ω_i of the total costs. Then total investment expenditures are given by:

$$i_t = \omega_1 s_{1t} + \omega_2 s_{2t}$$

and, of course, $\omega_1 + \omega_2 = 1$.The law of motion for the capital stock is given by:

$$k_{t+1} = k_t \left(1 - \delta\right) + s_{1t}$$

Given this environment, do the following:

(a) Express the associated social planner problem for this economy as a dynamic programming problem. Be explicit in identifying the states and control variables in each period (along with the laws of motion for the state variables). (Note: it is easiest to write the law of motion for capital as an additional constraint with an associated Lagrange multiplier.)

(b) Derive and interpret the necessary conditions associated with an optimum.

(c) Solve for the steady-state output-capital ratio, the investment-capital ratio, and the ratio of time spent in work activity to time spent in leisure as a function of the exogenous parameters.

7. Consider a simple stochastic, representative agent cash-in-advance economy in which the level of the endowment, x_t, is an *i.i.d.* process while the aggregate money stock is constant. In this economy, agents face two markets. In the beginning of the period, agents visit the asset market where money, one-period nominal bonds and one-period real bonds are traded. They next visit the goods market where money (acquired in the asset market) is used to finance consumption. At the beginning of each period agents receive the endowment (so all uncertainty is resolved) and they sell this in the goods period. The money from these sales along with any unspent money is carried over into the next period. Hence agents face two constraints associated with each market: in the asset market, beginning of period wealth is used to purchase money, nominal bonds and real bonds; in the goods market, money is used to purchase consumption goods (the CIA constraint). Nominal bonds cost \$1 at time t and return $\$\left(1+n_t\right)$ in the following period's asset market. Real bonds cost one unit of consumption in period t and return $\left(1+r_t\right)$ units of consumption in the asset market next period. Agents make their consumption and portfolio decisions in order to maximize:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t U\left(c_t\right)\right]$$

where the utility function has standard properties. Given this description, do the following:

(a) Express the individual's maximization problem as a dynamic programming problem. (Note: At the individual level, do not assume that the CIA constraint is binding.)

(b) Define a recursive competitive equilibrium in this economy. Assume that the cash-in-advance constraint is always binding.

(c) Characterize and explain the behavior of nominal and real interest rates in this economy.

(d) The Fisher relationship states that the expected real return on nominal bonds is equal to the real interest rate. Is that true in this economy? Explain.