Answer Key Part A: Questions on ECN 200D (Rendahl)

1. (5 points) Suppose that an agent will be employed in the next period with probability p. Symmetrically, the agent will remain unemployed with probability $(1-p)$. Suppose that an agent is unemployed in period 0. For how many periods can the agent expect to remain in unemployment (that is, what is her expected unemployment duration)?

   Answer Her unemployment duration is given by $1/p$.

2. (10 points) Consider the following Ramsey growth model augmented with a labor-leisure choice, and habits.

$$v^* = \max_{\{c_t, k_{t+1}, \ell_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, c_{t-1}, \ell_t) \tag{1}$$

$$\text{s.t} \qquad c_t + k_{t+1} = f(k_t, \ell_t) \tag{2}$$

$$k_0, c_{-1}, \ell_0 \quad \text{given} \tag{3}$$

   In class, however, we often considered problems of the type

$$v^* = \max_{\{x_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t F(x_t, x_{t+1}) \tag{4}$$

$$\text{s.t} \qquad x_{t+1} \in \Gamma(x_t) \tag{5}$$

$$x_0 \quad \text{given} \tag{6}$$

   (a) Define x_t, $F(\cdot,\cdot)$ and $\Gamma(\cdot)$ such that these two problems coincide exactly.

   Answer $x_t = (k_t, c_{t-1}, \ell_t)$. $F(x_t, x_{t+1}) = u(c_t, c_{t-1}, \ell_t)$, $\Gamma(x_t) = \{(c_t, k_{t+1}, \ell_{t+1}) : c_t + k_{t+1} = f(k_t, \ell_t)\}$

   (b) What are the first order conditions to the problem in (1)-(3)?

   Answer The first order condition with respect to c_t is

$$\beta^t u_1(c_t, c_{t-1}, \ell_t) + \beta^{t+1} u_2(c_{t+1}, c_t, \ell_{t+1}) - \lambda_t = 0$$

   where $u_n(\cdot)$ denotes the first derivative of u with respect to the nth argument. λ_t denotes the Lagrange multiplier on the constraint in period t.

   The corresponding condition for k_{t+1} and ℓ_{t+1} are, respectively,

$$-\lambda_t + \lambda_{t+1} f_1(k_{t+1}, \ell_{t+1}) = 0$$

$$\beta^{t+1} u_3(c_{t+1}, c_t, \ell_{t+1}) + \lambda_{t+1} f_2(k_{t+1}, \ell_{t+1}) = 0$$

(c) What is the Bellman equation corresponding to the problem in (1)-(3)? (no proof needed)

Answer The Bellman equation is given by,

$$v(k, c_{-1}, \ell) = \max_{c,k',\ell'}\{u(c, c_{-1}, \ell) + \beta v(k', c, \ell')\}$$
$$\text{s.t.} \quad c + k' = f(k, \ell)$$

3. (20 points) Consider the following economy. A continuum of individuals are born in period 0 with no resources. With *idiosyncratic* probability p, an individual will be employed in period 1, and with the complementary probability $(1-p)$ she will be unemployed. However, the agents have the opportunity of writing contracts with each other, promising the payment of some resources contingent on which state occurs. As they consume nothing in period 0, each individual's problem is given by

$$\max_{c_1,c_2,b_1,b_2} \{pu(c_1) + (1-p)u(c_2)\}$$
$$\text{s.t.} \quad 0 = p_1 b_1 + p_2 b_2$$
$$c_1 = b_1 + w$$
$$c_2 = b_2$$

where c_1 and c_2 denotes consumption at the employed and the unemployed state, respectively. b_1 denotes the quantity purchased of the asset (or contract) which pays b_1 units of the consumption good if the agent turns out to be employed. Similarly, b_2 denotes the quantity purchased of the asset which pays b_1 units of the consumption good if the agent turns out to be unemployed.

(a) Is this a complete- or an incomplete markets economy?

Answer This is a complete markets economy, as there is one market for each good.

(b) What is the market clearing condition? And what are the market clearing prices?

Answer Market clearing means that supply equals demand. More precisely, that for each unit of resources that someone receives, some other individual pays that exact amount. The total amount of recourses received by the unemployed is $(1-p)b_2$. The total amount of recourses paid by the employed is $-pb_1$ (and vice versa). Therefore, the market clearing condition is $pb_1 + (1-p)b_2 = 0$. Equilibrium prices satisfies: $\frac{p_1}{p_2} = \frac{p}{1-p}$.

(c) What is aggregate (or average) consumption? Does it equal aggregate resources?

Answer The aggregate amount of consumption for the unemployed is $(1-p)b_2$. The aggregate amount of consumption for the employed is $p(w+b_1)$. Aggregate consumption is therefore pw, which is also equal to aggregate resources.

4. (20 points) Consider the following sequence problem,

$$v^* = \max_{\{c_t, s_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{7}$$

$$\text{s.t} \quad w_t = c_t + s_t \tag{8}$$

$$a_{t+1} = a_t(1+r) + s_t \tag{9}$$

$$w_t = \delta \sum_{i=0}^{t-1} \gamma^{t-i} w_i \tag{10}$$

$$a_0, w_0 \quad \text{given} \tag{11}$$

(a) What conditions do you need on δ and γ in order to ensure that the sequence $\{w_t\}_{t=0}^{\infty}$ is bounded?

Answer We can write w_t as,

$$w_t = \delta(\gamma^t w_0 + \gamma^{t-1} w_1 + \gamma^{t-2} w_2 + \ldots + \gamma^2 w_{t-2} + \gamma w_{t-1})$$

or as

$$w_t = \delta\gamma(\gamma^{t-1} w_0 + \gamma^{t-2} w_1 + \gamma^{t-3} w_2 + \ldots + \gamma w_{t-2}) + \delta\gamma w_{t-1}$$

and w_{t-1} as

$$w_{t-1} = \delta(\gamma^{t-1} w_0 + \gamma^{t-2} w_1 + \gamma^{t-3} w_2 + \ldots + \gamma w_{t-2})$$

Therefore,

$$w_t = \gamma w_{t-1} + \delta\gamma w_{t-1} = (1+\delta)\gamma w_{t-1}$$

As long as $w_0 \neq 0$, we need that $|(1+\delta)\gamma| \leq 1$.

(b) Use the logic of Theorem 1 and derive the Bellman equation associated with the sequence problem. There is no need for a formal proof, but make sure you state the logical steps clearly (i.e. "prove" it verbally).

Answer The Bellman equation is given by,

$$v(a,w) = \max_{c,s}\{u(c) + \beta v(a',w')\}$$

$$\begin{aligned}\text{s.t.} \quad & w = c + s \\ & a' = a(1+r) + s \\ & w' = (1+\delta)\gamma w\end{aligned}$$

(c) What are the first order conditions? Apply the envelope theorem.

Answer Substituting constraint such that a' is the only choice variable, the first order condition is

$$u'(c) = \beta v'(a',w')$$

The envelope condition is

$$v'(a,w) = u'(c)(1+r)$$

University of California, Davis
Macroeconomics Prelim Exam, Part B
Suggested Answer Key
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Question 1 *(20 points)*
Consider the standard growth model in discrete time. There is a large number of identical households (normalized to 1). Each household wants to maximize life-time discounted utility

$$U(\{c_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t u(c_t).$$

Each household has an initial capital stock x_0 at time 0, and one unit of productive time in each period, that can be devoted to work. Final output is produced using capital and labor services,

$$y_t = F(k_t, n_t),$$

where F is a CRS production function. This technology is owned by firms whose number will be determined in equilibrium. Output can be consumed (c_t) or invested (i_t). We assume that households own the capital stock (so they make the investment decision) and rent out capital services to the firms. The depreciation rate of the capital stock (x_t) is denoted by δ.[1] Finally, we assume that households own the firms, i.e. they are claimants to the firms' profits. The functions u and F have the usual nice properties.[2]

a) First consider an Arrow-Debreu world. Describe the households' and firms' problems and carefully define an AD equilibrium. How many firms operate in this equilibrium?

b) Now focus on an alternative environment with spot (sequential) markets. Describe the households' and firms' problems and carefully define a sequential markets equilibrium (SME).

c) Write down the problem of the household recursively.[3] Be sure to carefully define the state variables and distinguish between aggregate and individual states. Define a recursive competitive equilibrium (RCE).

[1] The capital stock depreciates no matter whether it is rented out to a firm or not.

[2] You will not explicitly need them, so there is no need to be more precise.

[3] Since the firms face a static problem, your answer in part (b) also describes the problem they solve here, so do not spend any time on them.

For the rest of this question focus again on an Arrow-Debreu setting.

d) In this economy, why is it a good idea to describe the AD equilibrium capital stock allocation by solving the (easier) Social Planner's Problem?

e) From now on assume the following functional forms: $F(k_t, n_t) = k_t^a n_t^{1-a}$, $a \in (0,1)$, $u(c_t) = ln(c_t)$. Also assume $\delta = 1$. Fully characterize (i.e. find a closed form solution for) the equilibrium allocation of the capital stock. (Hint: Use your answer in part (d) and focus on the Planner's problem. In the Euler equation, guess and verify a "policy rule" of the form $k_{t+1} = \theta k_t^a$, where θ is an unknown to be determined.) What happens as $t \to \infty$?

f) As $t \to \infty$, what happens to the AD equilibrium prices of the consumption good and the capital services?

Question 1 Suggested Answer:

Parts a, b, and c are taken word by word from the lecture notes, so I do not repeat them here to save some space. So let's move on to part d.

d) The Planner's problem is indeed much easier, because one only needs to describe the allocations, and not the prices of all commodities (which are infinite sequences). What allows us to use this technique here, is the fact that in this environment both Welfare Theorem hold. So we know that the competitive allocation and the Planner's allocation will coincide. After characterizing the Planner's allocation, we can construct the whole competitive equilibrium, like we did in class.

e) Using any technique you want, you can derive the Euler equation,

$$u'(f(k_t) - k_{t+1}) = \beta u'(f(k_{t+1}) - k_{t+2}) f'(k_{t+1}).$$

After using the specific functional forms ($u(c) = ln(c)$, $f(k) = k^a$), and imposing our guess, i.e. that $k_{t+1} = \theta k_t^a$ and, therefore, $k_{t+2} = \theta k_{t+1}^a$, one can easily verify that $\theta = a\beta$. This simply means that each period agents should invest a part equal to $a\beta$ of the output and eat the remaining $1 - a\beta$.

This economy has a steady state. No matter where the economy starts (initial capital stock), since $k_{t+1} = a\beta k_t^a$, the economy will always converge to $k^* = (a\beta)^{\frac{1}{1-a}}$.

f) From the definition of competitive equilibrium (part a or your lecture notes), we know that if one knows the Planner's allocation, the whole competitive equilibrium can be recovered fairly easily. For example, we know that the price of output, in the AD setting in period t, can be expressed as

$$p_t = \beta \frac{u'(c_{t+1})}{u'(c_t)},$$

where, as we already explained, $c_t = (1 - a\beta)k_t$. Therefore,

$$p_t = \frac{\beta}{(a\beta)^a} k_t^{a-a^2}.$$

Clearly, as $t \to \infty$, $p_t \to 0$, which simply means that the consumption good in very remote periods is worth nothing (because of discounting).

Now consider the price of capital rental, r_t. Form the AD settup we know that

$$r_t = F_k(k_t, n_t) = F_k(k_t, 1) = ak_t^{a-1}.$$

Hence, as $t \to \infty$, we have $r_t \to a(a\beta)^{\frac{a-1}{1-a}} = 1/\beta$.

Question 2 (*15 points*)
Consider a standard "Lucas trees" economy. There is a large number of identical households (normalized to 1) who wish to maximize expected life-time utility, given by

$$E_0 \sum_{t=0}^{\infty} \beta^t ln(c_t).$$

There is only one non-storable commodity that agents consume, call it coconuts. There are also infinitely lived objects, that we call trees, which yield coconuts. There is no production in this world. Agents can buy their shares at some price that they take as given. Let the supply of shares of the trees be normalized to 1. The holder of one share of the tree in period t is a claimant to the fruit d_t. We assume that d_t follows a Markov process. For any t, $d_t \in D \equiv \{d_1, ..., d_N\}$. Let $\Gamma_{ij} = Pr(d_{t+1} = d_j | d_t = d_i)$. Assume that d_0 is given.

a) Characterize the AD equilibrium price of one coconut in period t after a certain history realization. What is the price of the whole tree in period $t = 0$?

b) Set up the problem of the agent in a recursive form, and include the following asset: a claim, to be bought in period $t-1$ after history $\hat{h}_{t-1}$, that pays one coconut in t if state j occurs. What is the equilibrium price of this asset?

c) Express the price of the asset described in part (b) as a function of the AD prices of coconuts you found in part (a).

d) What is the equilibrium price of a bond, bought in period $t-1$ after history $\hat{h}_{t-1}$, which will deliver one coconut (with certainty) in period t?

e) What is the equilibrium price of an option, bought in period $t-1$ after history $\hat{h}_{t-1}$, that allows you to sell shares of the tree in period t, at the predetermined price x?

f) What is the equilibrium price of the following asset: an option, bought in period $t-1$ after history $\hat{h}_{t-1}$, that allows you to buy shares of the tree in period $t+1$, at the predetermined price y, if the state of the world in $t+1$ is j, where j is an odd number.

Question 2 Suggested Answer

a) Setting up the problem in AD language, we know that the agent wishes to solve

$$\max_{c_t(h_t)} \sum_{t=0}^{\infty} \sum_{h_t \in H_t} \beta^t \pi(h_t) u(c_t(h_t))$$

$$s.t.\ \sum_{t=0}^{\infty}\sum_{h_t\in H_t} p_t(h_t)c_t(h_t) = \sum_{t=0}^{\infty}\sum_{h_t\in H_t} p_t(h_t)d_t(h_t).$$

We can set up the Langrangian and obtain the FOC's with respect to $c_t(h_t)$ and c_0. This yields the following conditions:

$$u'(c_0) = \lambda p_0, \tag{1}$$

$$u'(c_t(h_t))\beta^t\pi(h_t) = \lambda p_t(h_t). \tag{2}$$

Combining (1) and (2), normalizing $p_0 = 1$, and using the fact that in equilibrium $c_t = d_t$, we obtain

$$p_t(h_t) = \beta^t \frac{\pi(h_t)d_0}{d_t(h_t)}. \tag{3}$$

It is now easy to find that the price of the whole tree is just the sum of the values of the fruit over all periods and histories. In other words,

$$P_{tree} = \sum_{t=0}^{\infty}\sum_{h_t\in H_t} p_t(h_t)d_t(h_t).$$

Exploit (3) and after some algebra one can find an even nicer expression:

$$P_{tree} = \sum_{t=0}^{\infty}\sum_{h_t\in H_t} \beta^t \frac{\pi(h_t)d_0}{d_t} d_t(h_t) = \frac{d_0}{1-\beta}.$$

b) Here we want to price the assets that in class we named "the state contingent claim". Since the fruit follows a Markov process, we can easily set up the problem in a recursive form, which is much more convenient than the AD setting. The agent's problem is the following

$$V_i(s,a) = \max_{s',a'} \left\{ ln(c) + \beta \sum_{j=1}^{N} \Gamma_{ij} V_j(s',a') \right\},$$

$$s.t.\ c + \sum_{j=1}^{N} q_j a'_j + \psi_i s' = (\psi_i + d_i)s + a_i$$

Notice that the realization of the fruit today (i) is an aggregate state variable. Also, notice that only the state i contingent claim bought in period $t-1$, pays the agent today (given that state i was realized). On the contrary, in period t the agent needs to buy all possible state contingent claims for tomorrow (there are N of these assets). The competitive price of these assets is denoted by q_j, and this is precisely what we want to determine in this exercise.

Taking the FOC and the envelope condition (see lecture notes for more detail), and imposing the market clearing condition $c = d$ in all possible states, one can obtain that

$$q_j = \beta \Gamma_{ij} \frac{d_i}{d_j},$$

where it is understood that i is the period t state, and j is the period $t+1$ state (conditional on the fact that $d_t = d_i$).

c) This question is taken from your homework. As we saw there, we can express the price of this asset in terms of the consumption good of the AD world in the following way:

$$q_j = \frac{p_t(\hat{h}_{t-1}, d_t = d_j)}{p_{t-1}(\hat{h}_{t-1})},$$

where $\hat{h}_{t-1}$ is a specific history realization up to period $t-1$.

d) This asset will deliver 1 unit of consumption in period t regardless of the state. Hence, one can replicate it by buying a little bit of the N different state contingent claims. We know the prices of these claims (part b), and so all we need to do is use the arbitrage reasoning, developed in class, in order to find the equilibrium price of the bond. Since the bond pays 1 in all states, we can replicate its return by buying 1 unit of all state contingent claims: then no matter what happens tomorrow I have 1 unit of coconut with certainty. What is left for the no-arbitrage condition to hold, is to also equate the cost of the two "investment plans". Clearly, this requires

$$P_{bond} = \sum_{j=1}^{N} q_j(\hat{h}_{t-1}, d_t = d_j) = \beta d_i \sum_{j=1}^{N} \frac{\Gamma_{ij}}{d_j}.$$

Just a clarification regarding notation: $q_j(\hat{h}_{t-1}, d_t = d_j)$ is the price of the claim bought in $t-1$, that will deliver 1 unit of consumption in period t, if state j occurs, and given that the state in $t-1$ was i.

e) We need to use our arbitrage reasoning again. This question is very similar to examples we saw in class and in the homework. The difference here is that you will sell (not buy) these claims. One can conclude that

$$P_{option_1} = \sum_{j=1}^{N} q_j(\hat{h}_{t-1}, d_t = d_j) \max\{x - \psi_j, 0\}$$
$$= \beta d_i \sum_{j=1}^{N} \frac{\Gamma_{ij}}{d_j} \max\{x - \psi_j, 0\}.$$

The terms $q_j(\hat{h}_{t-1}, d_t = d_j)$ are the prices of the state contingent claims described above, and the terms ψ_j are the equilibrium prices of the tree stocks described in part b. These are all known, in the sense that if I knew the Γ matrix, I could find a closed form solution for all the q's and ψ's.[4]

f) Once again we use the same arbitrage reasoning. Here the option allows you to buy shares, not tomorrow, but in two periods. But the logic is exactly the same. One can show that

$$P_{option_2} = \sum_{j=1}^{N} q_j(\hat{h}_{t-1}, d_t = d_j) \sum_{k=odd} q_k(\hat{h}_{t-1}, d_t = d_j, d_{t+1} = d_k) \max\{\psi_k - y, 0\}.$$

[4] Here we use an important fact that we learnt in class: when the fruit follows a Markov process, the prices of the shares of the tree are time invariant, and they only depend on the current period's realization. This is why in the formulas above we write ψ_j, ψ_k without referring to the specific period (i.e. to the t's).

Question 3 (*15 points*)

Consider the following static version of the Mortensen-Pissarides model. Labor force is normalized to 1. There is a large number of firms who can enter the market and search for a worker. Firms who engage in search first have to pay a fixed cost k. If a measure v of firms enters the market, a CRS matching function $m(1, v)$ gives us the total measure of matches in the economy. Within each match, the firm and worker bargain for the wage, w, with β denoting the bargaining power of the worker. If they agree, they can move on to production, which will deliver output equal to y. If they disagree both parties get nothing. Assume that $k/y < 1 - \beta$. Also, throughout this question focus on a specific matching technology, such that the arrival rate of workers to firms (or simply the probability with which a firm finds a worker) is given by $a_F(b) = 1 - e^{-b}$, where $b \equiv 1/v$ is the market tightness.

a) What is the probability of a match for a typical worker, $a_W(b)$?

b) Describe algebraically and graphically the equilibrium value of b in this economy, and show it always exists and it is unique.

c) What value of b would a benevolent Social Planner choose for this economy? Describe it algebraically and graphically. Does it coincide with the equilibrium value of b you described in part (b)?

From now on assume that unemployed workers obtain an unemployment insurance (UI) equal to z.[5] In order to raise funds and pay the UI to all unemployed workers, the government considers two plans. In the first, dubbed "Taxation System 1", the government imposes a lump-sum tax to be payed by all firms who enter the labor market. In the second, dubbed "Taxation System 2", the government imposes a lump-sum tax to be payed only by firms who get matched with a worker and are, therefore, productive. Regardless of which taxation system is adopted, the government publicly announces the level of z before the "matching game" described above is played.

d) Under Taxation System 1, what is the equilibrium value of b? Does an equilibrium always exist? How does the equilibrium value of b compare to the one in part (b)?

e) Repeat the analysis of part (d) under Taxation System 2. Compare the two tax regimes in terms of efficiency.

f) Which taxation system and what level of z should the government choose in orer to maximize efficiency in this economy?

[5] Assume that workers who match, but do not come to an agreement with the firm over a wage, also get this UI.

Question 3 Suggested Answer

a) From the CRS assumption, we know that $a_W(b) = a_F(b)/b = (1-e^{-b})/b$.

b) This part is exactly the same as in Midterm 1. For more details and for a graphical representation of equilibrium, please see the Midterm answer key. The equilibrium satisfies zero expected profit (due to free entry). This means that

$$(1-e^b)(1-\beta)y = k. \tag{4}$$

The fact that $k/y < 1-\beta$ guarantees existence. Also, the fact that $1-e^{-b}$ is strictly increasing in b guarantees uniqueness. A higher k leads to a higher equilibrium b. Since here $b = 1/v$, this is equivalent to fewer firms entering the market when the entree fee goes up.

c) The planner wants to maximize expected matches (employment) times output (y), minus entree fees paid by the firms (these can also be thought as recruitment/job search costs). In other words the Planner chooses b in order to maximize

$$\frac{1-e^{-b}}{b}y - \frac{k}{b}. \tag{5}$$

The FOC after some manipulations yields

$$(1-e^{-b}-be^{-b})y = k. \tag{6}$$

Comparing (4) and (6), there is no reason to believe that the equilibrium value of b will coincide with the solution chosen by the Planner. This will only happen if the so-called Hosios condition is met, but there are no intrinsic forces that would guarantee such a coincidence. In general, if β is relatively big, workers gain too much surplus, and this will lead to an inefficiently low entry of firms, which will lead to excess unemployment. But if β is relatively small, equilibrium unemployment might be too low.

d) Under taxation system 1, all entrant firms pay taxes in order to raise funds and give all unemployed a benefit equal to z. The equilibrium condition will be

$$(1-e^b)(1-\beta)(y-z) - \left(1 - \frac{1-e^{-b}}{b}\right)\frac{z}{\frac{1}{b}} = k. \tag{7}$$

This can be interpreted as follows: given market tightness b, a firm finds a worker with probability $1 - e^{-b}$. In this case, the firm gets a part $1 - \beta$ of the total surplus of the match $(y - z)$. Moreover, there is a mass of $1 - \frac{1-e^{-b}}{b}$ of unemployed workers. All these workers will receive benefit z, and this cost will be divided among all firms that entered. The mass of the latter is given by v, or equivalently, $1/b$.

An equilibrium does not necessarily exist. It is easy to show that the term on the left hand side of (7) is increasing for low values of b, it reaches a global maximum and after that it is decreasing (it reaches $-\infty$ when $b \to \infty$). Whether an equilibrium exists, depends on the value of z. In general, the smaller z is, the more likely it is for the equilibrium to exist. If z is very big it might be the case that no firm wants to enter the market. All these facts can be shown more formally, but this was not required in order to get full credit.

Finally, we need to compare part d to part b. If an equilibrium in d exists, it will ALWAYS involve a higher b and, therefore, a lower v (less firms) compared to part b. This is because the expression on the left hand side of (7) lies below the expression on the left hand side of (4), for all b.

e) Now consider the second txation system. Only firms who got matched pay the tax. The equilibrium condition will be

$$(1 - e^{b})(1 - \beta)(y - z) - (1 - e^{-b})\frac{\left(1 - \frac{1-e^{-b}}{b}\right) z}{\frac{1-e^{-b}}{b}} = k. \tag{8}$$

This can be interpreted as follows: given market tightness b, a firm finds a worker with probability $1 - e^{-b}$. In this case, the firm gets a part $1 - \beta$ of the total surplus of the match $(y - z)$. There is a mass of $1 - \frac{1-e^{-b}}{b}$ of unemployed workers who will receive benefit z. The firm will pay a tax in order to raise this amount with probability $1 - e^{-b}$. Also, the tax will be divided among all firms that got matched. The mass of these firms is given by $(1 - e^{-b})/b$. After a few simplifications, it turns out that (8) can be re-written as

$$(1 - e^{b})(1 - \beta)(y - z) - \left(1 - \frac{1 - e^{-b}}{b}\right) zb = k.$$

But the last expression is identical to (7)! Hence the two taxation systems coincide. Everything we said in part e also applies here.

f) Clearly, for any given announcement z by the government, there is no effect in efficiency by whether system 1 or 2 is used. All that matters is what

z is equal to. So the government has to choose z wisely in order to make the equilibrium b described in part (e), coincide with the value of b chosen by the Planner (described in part c). Of course, one can describe this in more detail, but the algebra is tedious, and in the exam I was just asking for the intuition.