Math Material for Economics 100

I. One Variable Calculus

I.1 R^1

- → the Set of all Real Numbers
- → includes Rational Numbers and Irrational Numbers
- → Graphical Representation: Number Line

I.2 Function: a rule that assigns a number to each number in R^1

x is the independent variable
y is the dependent variable

$y = f(x)$ [y equals f of x]

Ex I.2.1: Transforming Centimeters to Inches

$y = (1/2.54)x$, where y is "inches" and x is "centimeters"

I.3 Functional Forms

I.3.1 Monomials and Polynomials

→ Monomials and Polynomials will often be the kinds of functions we will be working with in this course
→ Monomials:
$y=f(x) = ax^k$ → the general form of a monomial

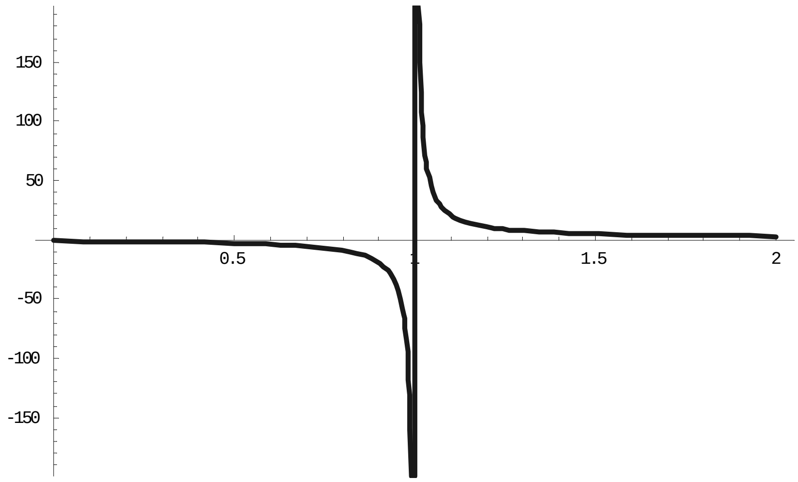
k is a constant, and is called the degree of the polynomial
a is a constant, called the coefficient

→ A Polynomial is a series of monomials added or subtracted together
Ex: $h(x) = -x^7 +3x^4$;
→ the degree of the polynomial is the highest exponent in the function, in this example it is 7

I.3.2 Rational Functions

Rational Functions have the form $h(x) = \frac{f(x)}{g(x)}$, defined where g(x) is not zero.

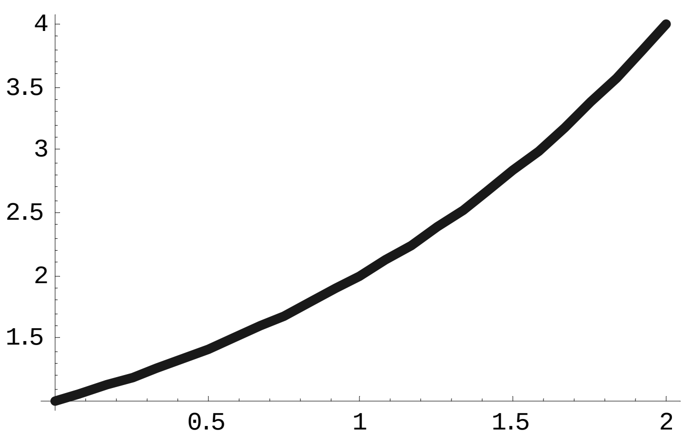
Ex: $y = \frac{f(x)}{g(x)} = \frac{x+1}{x-1}$



I.3.3 Exponential Functions

Exponential Functions are of the form: $y = f(x) = c^x$, where c is a constant, and x is the variable.

Ex: $y=f(x)=2^x$



I.4 Increasing and Decreasing Functions

Def: Increasing (Decreasing) functions means that if $x_1 > (<) x_2$, then $f(x_1) > f(x_2)$.

I.5 Domain of a Function

→Subset of R^1 where the function is defined.

Ex: $y = \sqrt{x-7}$ is defined on $x \geq 7$ because the square root of a negative number is not possible.

→ For economic applications, *usually* the domain is for $x \geq 0$, where x is some economic variable like cars produced, and so on.

I.6 Maximum and Minimum

→ x_0 is a relative/local max [min] if at (x_0, f(x_o)), f(x) changes from an increasing [decreasing] to a decreasing [increasing] function

→ If the graph of f(x) is never above [below] (x_0, f(x_o)), then x_o is the is the absolute max [min] of the function

II. Linear Functions

General Form: $y = f(x) = mx + b$

Where: m is the slope of the function, b is a constant.

The slope is $m = \frac{y_1 - y_0}{x_1 - x_0}$, where the subscripts represent two points on the line. From the general form, we have: $m = \frac{y - b}{x - 0}$.

Interpretation: m is the slope of the function, or rate of change.

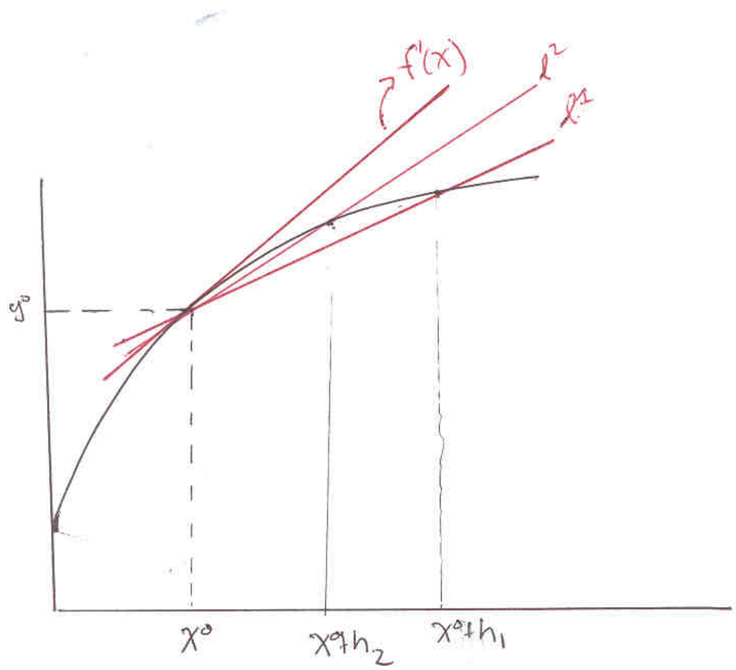
Ex: *Cost Function*: $C = f(q) = Fq$. Here, F is the slope and b is zero.

III. Slope of Non-Linear Functions

Def: Derivative: (x_0, f(x_o)) is a point on the graph y=f(x)

$$f'(x_0) = \frac{dy}{dx}(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

We can interpret the first derivative as the slope of a non- linear function on a point (x_0, f(x_o)).



Ex: For linear Functions:

$f'(x) = \frac{dy}{dx}(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{m(x+h) - mx}{h} = \frac{mh}{h} = m$, so the derivative of a linear function is equal to its slope.

III.1 Rules for Computing Derivatives

a. $(f \pm g)'(x) = f'(x) \pm g'(x)$

b. $kf'(x) = k(f'(x))$

c. $(f * g)'(x) = f'(x)g(x) + f(x)g'(x)$

d. $\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

e. $((f(x))^n)' = n(f(x))^{n-1} f'(x)$

f. Monomial Rule: $(x^k)' = kx^{k-1}$

III.2 Second Derivatives and the Shape of the Function

We write f''(x) as the second derivative. We get the second derivative by finding the derivative of the first derivative.

Ex:

$f(x)=3x^3+2x^2$

First Derivative: $f'(x)=9x^2+4x$

Second Derivative: $f''(x)=18x+4$

As the first derivative tells us the slope of the function is positive or negative, the *second derivative* tells us whether the slope is increasing of decreasing.

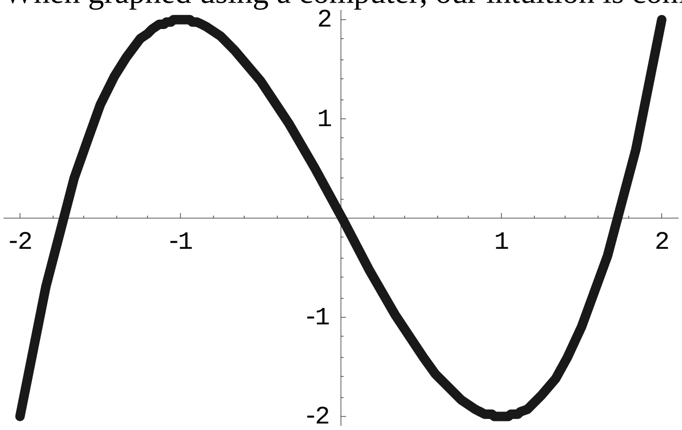
III.3 Graphing a Function

Step 1: Find all x where f'(x)=0. These are the critical points.
Step 2: Find all x where f''(x)=0.
Step 3: Form a Table that divides the domain into segments defined by the critical points and the points where the second derivative is zero.

Ex: $f(x)=x^3-3x$

	(-inf,-1)	-1	(-1,0)	0	(0,1)	1	(1,+inf)
f(x)							
f'(x)	+	0	-	-	-	0	+
f''(x)	-	-	-	0	+	+	+

When graphed using a computer, our intuition is confirmed:



III.4 Maximum and Minimum [Interior Solution]

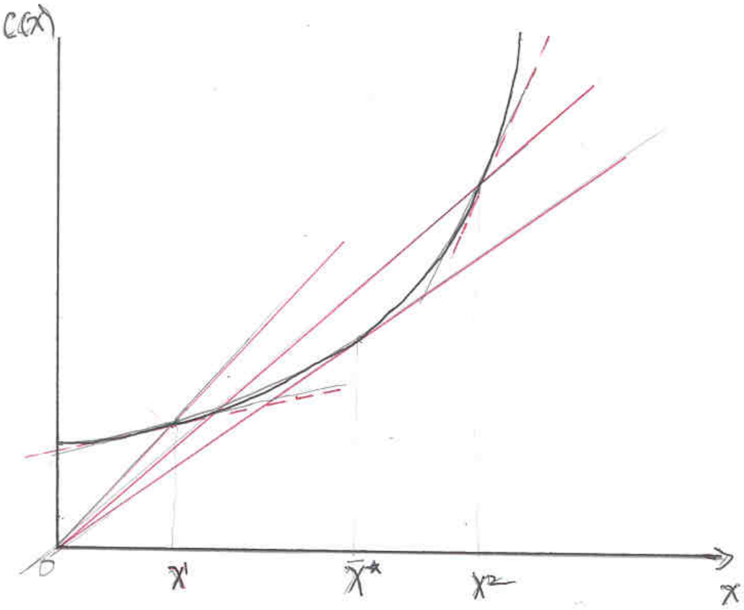
a. If $f'(x_0)=0$, $f''(x_0)<0$, then x_0 is a *relative maximum*
b. If $f'(x_0)=0$, $f''(x_0)>0$, then x_0 is a *relative minimum*
c. If $f'(x_0)=0$, $f''(x_0)=0$, then x_0 is neither

III.4.1 Global Maximum and Minimum

If $x^0 >0$ is the only critical point, the domain is the whole R^1, $f''(x_0)<0$ then it is a *global maximum*, $f''(x_0)>0$, then x_0 is a *global minimum*

III.5 Simple Economic Application on Derivatives

III.5.1 Cost Functions



III.5.2 Elasticity
What is the change in *y* when we change *x* by a small amount?

$$\varepsilon_{yx} = \frac{\%\Delta y}{\%\Delta x} = \frac{dy/y}{dx/x} = \frac{dy/dx}{y/x} = \frac{f'(x)}{y/x} = \frac{\text{marginal}}{\text{average}}$$

III.5.3 Profit maximization by a firm facing perfectly competitve output markets
$\Pi(Q) = PQ - C(Q)$

First Derivative: $\frac{d\Pi(Q^*)}{dQ} = P - C'(Q^*) = 0$

Second Derivative: $\frac{d^2\Pi(Q^*)}{dQ^2} = -C''(Q^*) < 0$, so critical point Q* is a global max.

[See excel example]

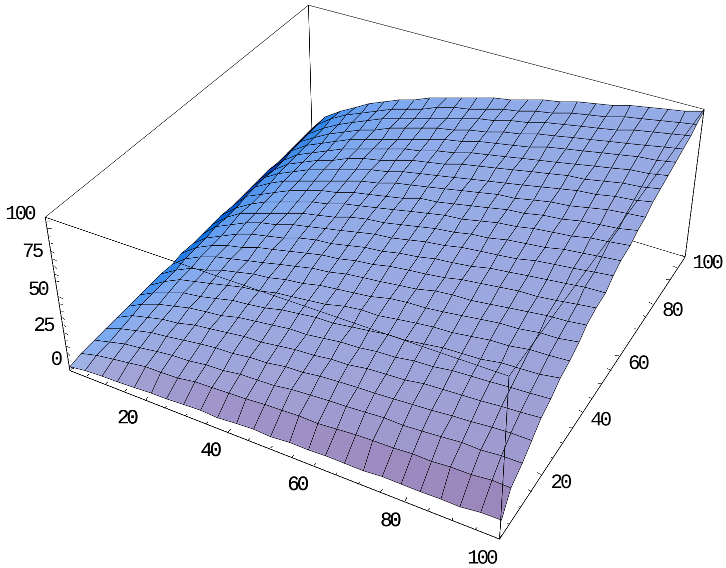
III.6 Exponents

a) $x^m x^n = x^{m+n}$
b) $(x^m)^n = x^{mn}$
c) $x^m / x^n = x^{m-n}$
d) $x^1 = x$
e) $x^0 = 1$

IV. 2-Variable Functions

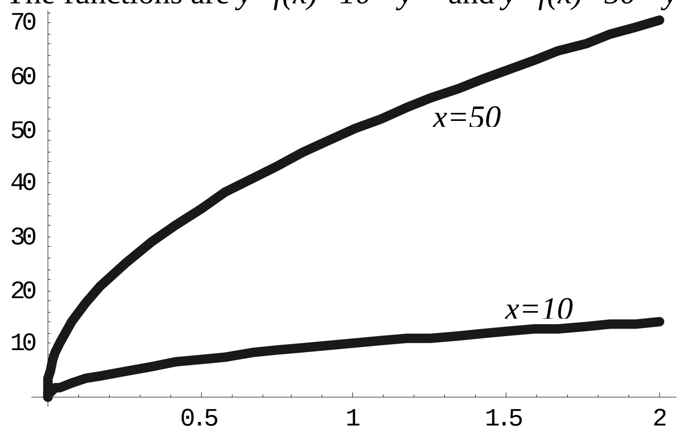
Example: *u=U(x,y)*
→ a (x,y) pair maps onto a number on R^1, here called ‘u’.

IV.1 An Example: $u=x^{1/2}y^{1/2}$



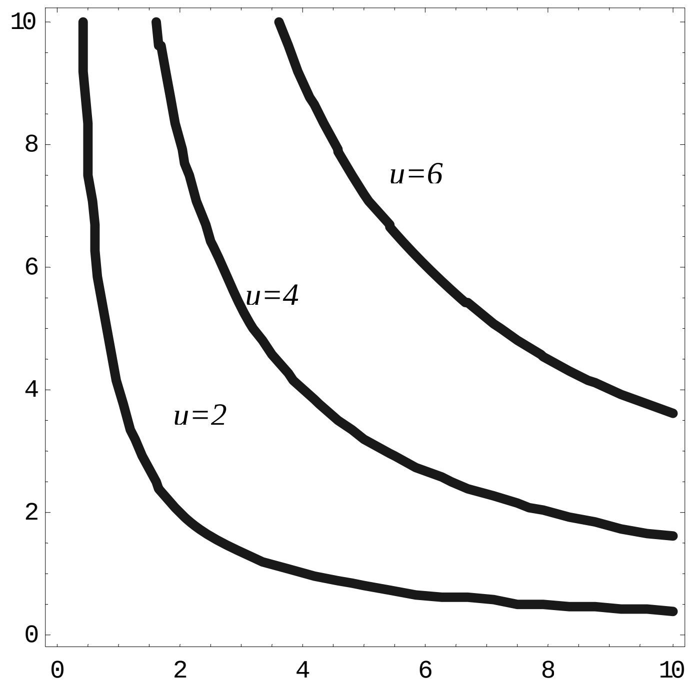
IV.2 Fix x=10 and then graph, fix x=50, and then graph:

The functions are $y=f(x)=10^{1/2}y^{1/2}$ and $y=f(x)=50^{1/2}y^{1/2}$



IV.3 Fix u=2, fix u=4, u=6:

What combinations of *x* and *y* will give us the same level of U?



IV.4 Partial Differntiation

For *u=U(x,y)*, we change x without changing the value of y

$$\frac{\partial U}{\partial x}(x,y) = \lim_{h \to 0} \frac{U(x_0 + h, y_0) - U(x_o, y_0)}{h}$$

Ex: u=$x^{1/2}y^{1/2}$. $\frac{\partial U}{\partial x}(x,y) = \frac{1}{2} x^{-1/2} y^{1/2}$

IV.5 Total Derivative

We are often interested in a small change around (x_0,y_0) when we change both x and y. How does this affect u?

Start from the partial derivative, and assume small changes, called *dx* and *dy:*

$$\frac{\partial U}{\partial x}(x\ \ , y)dx \approx U(x_0 + dx, y_0) - U(x_o, y_0)$$

It is natural to generalize:

$$U(x_0 + dx, y_0 + dy) - U(x_o, y_0) \approx \frac{\partial U}{\partial x}(x_0, y_0)dx + \frac{\partial U}{\partial y}(x_0, y_0)dy$$

Or, $\Delta U(x_o, y_0) \approx \frac{\partial U}{\partial x}(x_0, y_0)dx + \frac{\partial U}{\partial y}(x_0, y_0)dy$

This is the total Differential.

Application: Slope of Indifference Curves: