Random Entry

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Suppose that firm B was considering entering into an industry now monopolized by firm A, in any one of n periods, with probability p_t every period, $t \leq n$. What profit should A expect to make?

In every period, A will either be a monopolist and make monopolist profits π_M, or a duopolist and make duopolist π_D–supposing that A and B never cooperate. Lets look at the simple case when the probability of entry is fixed $p_t = p$. (The same principle applies in the general case, when p_t is not fixed.)

We can calculate the expected value by starting in the first case: B enters in the first period, A makes a duopoly profit from the first period onwards. (See figure 1)This can happen with probability p and A will make duopoly profits for n periods.

$$\text{Entry in Period 1 Profits=} p \cdot n \cdot \pi_D$$

If B enters in the 2nd period, A will make monopoly profits in the first period and duopoly profits from the 2nd period onwards for $n-1$ periods. This can happen with probability $(1-p) \cdot p$, the odds of there being no entry in the first period and entry in the 2nd.

$$\text{Entry in Period 2 Profits=} (1-p) \cdot \pi_M + (1-p) \cdot p \cdot (n-1) \cdot \pi_D$$

B enters in the $k+1$'th period with probability $(1-p)^k \cdot p$. A will make monopoly profits for k periods and duopoly profits for the rest of the $n-k$ periods.

$$\text{Period } k \text{ Profits=} (1-p)^k \cdot k \cdot \pi_M + (1-p)^k \cdot p \cdot (n-k) \cdot \pi_D$$

Expected profit for firm A when there are n possible periods of entry and entry occurs in the k'th period :

$$\begin{aligned} E(\pi) &= p \cdot n \cdot \pi_D + (1-p) \cdot \pi_M + (1-p) \cdot p \cdot (n-1) \cdot \pi_D + (1-p)^2 \cdot 2 \cdot \pi_M \\ &\quad + (1-p)^2 \cdot p \cdot (n-2) \cdot \pi_D + ... + (1-p)^k \cdot k \cdot \pi_M + (1-p)^k \cdot p \cdot (n-k) \cdot \pi_D \end{aligned}$$

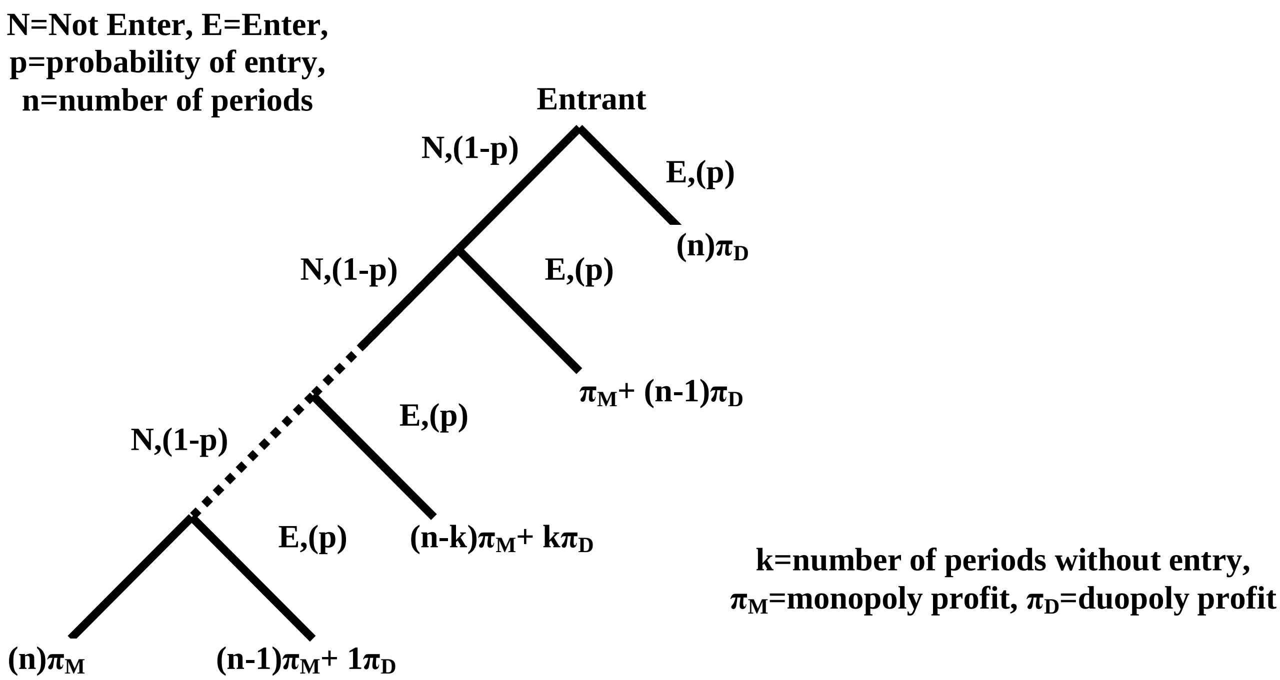


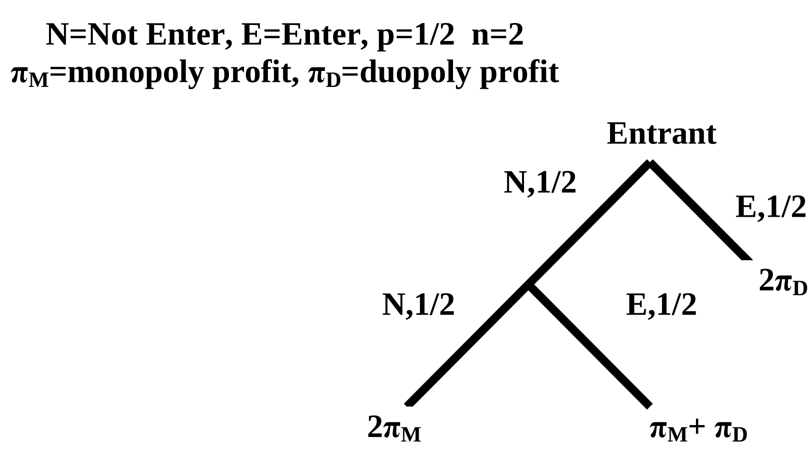
Figure 1

Probability\Period of Entry	1	2	3	...n
p	E	E	E	E
$(1-p)\cdot p$	N	E	E	E
$(1-p)^2\cdot p$	N	N	E	E
.....				
$(1-p)^n\ p^{n-k}$	N	N	N	...

0.1 Numerical Example:

Suppose that the number of possible periods of entry is $n = 2$ the probability of entry is $p = \frac{1}{2}$, when no entry has already occured. The expected profit of firm A is then:

$$E(\pi) = \frac{1}{2}\cdot 2\cdot \pi_D + \left(\frac{1}{2}\right)^2 \cdot (\pi_M + \pi_D) + \frac{1}{2}\cdot 2(\pi_M)$$



Probability\Period of Entry	1	2
$p = \frac{1}{2}$	E	E
$(1-p) \cdot p = \frac{1}{4}$	N	E
$(1-p)^2 = \frac{1}{4}$	N	N
.....		

0.2 Uniform Distributions

The probability that you will draw numbers X below x that are uniformly distributed between a and b:

$$P(X \leq x) = \int_a^x \frac{1}{b-a} dt = \left[\frac{1}{b-a}\right]_a^x = \frac{x-a}{b-a}$$

Suppose $a = 0$, $b = 1$, then $P(X \leq x) = x$.