

Social Surplus

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1. Picallo is a monopolistic painter of "abstract art" who faces a demand of $Q = 10 - P$. He has two costs, a variable cost of $2Q$ for paint and canvas, and a fixed cost of 3 for a really nice studio. Bill and Ted, who paint houses for a living, after seeing Picallo's works—and what they sold—decided that it might be profitable for them to do some abstract art also. Like Picallo, they also need paint, canvas and a really nice studio in order to sell their works. (Let the general demand be $p = a - bQ$ and $MC = c$. **The general forms of profit and consumer surplus without numbers are optional. They allow for a simple expression of social surplus at the end. You can use these general forms to check your own calculations.**) How many entrants can this industry support?

- If Picallo were a monopolist, his profits would be:

$$\pi_P = b \left(\frac{a - c}{2b} \right)^2 - FC = \left(\frac{10 - 2}{2} \right)^2 - 3 = 13$$

- If Picallo and Bill were to compete Cournot-wise, each of their profits would be:

$$\pi_i = b \left(\frac{a - c}{3b} \right)^2 - FC = \left(\frac{8}{3} \right)^2 - 3 = \frac{37}{9}$$

- In general, the profits for each of n Cournot competitors when MC is constant must be greater than zero for firms to not want to exit:

$$\pi_i = b \left(\frac{a - c}{(n + 1)b} \right)^2 - FC \geq 0$$

$$\pi_i = b(q_i)^2 - FC^1 \geq 0$$

- Therefore, to find the maximal number of firms, solve for the zero profit condition of each firm, when n firms have entered: $\pi_i = \left(\frac{8}{(n+1)} \right)^2 - 3 = 0$ implies that $n = 3.6$. Thus, the abstract art industry can only support Picallo, Bill and Ted, who at that point approach starvation. (Hence, we

¹See my notes on Residual Demand to Cournot Competition

have a simple economic explanation of the market for starving artists!)

Note that total industry surplus = $\sum_{i=1}^n \pi_i = n \cdot \pi_i$ if all firms are identical.

2) Now, we want to consider social surplus = firm profits + CS.

- The consumer's surplus when Picallo is a monopolist is the area above the price below the demand curve:

$$CS = \frac{1}{2}b \left(\frac{a-c}{2b} \right) \left(\frac{a-c}{2b} \right) = \frac{1}{2} \left(\frac{8}{2} \right)^2 = 8.$$

- Social surplus then when Picallo is a monopolist is:

$$SS = \pi + CS = b \left(\frac{a-c}{2b} \right)^2 - FC + CS = 13 + 8$$

- The consumer surplus when it's Picallo and Bill competing is:

$$CS = \frac{1}{2}b \left(2 \left(\frac{a-c}{3b} \right) \right) \left(2 \left(\frac{a-c}{3b} \right) \right) = \frac{1}{2} \left(2 \left(\frac{8}{3} \right) \right)^2 = \frac{128}{9}$$

- In general consumer surplus is:

$$CS(n) = \frac{b}{2} \left(\frac{n(a-c)}{b(n+1)} \right)^2 = \frac{b}{2} n^2 (q_i)^2$$

- The general form of the social surplus where π_i is the profit from a single firm of n firms competing Cournot-wise is:

$$SS(n) = n\pi_i + CS(n)$$

$$SS(n) = nb(q_i)^2 - nFC + \frac{b}{2}n^2(q_i)^2 = \left(n + \frac{n^2}{2} \right) b(q_i)^2 - nFC$$

$$SS(n) = \frac{2n + n^2}{2(n+1)^2} \frac{(a-c)^2}{b} - nFC$$

- (This last equation needs to be checked.)

²See my notes on Residual Demand to Cournot Competition