

Non-Price Discriminating Monopolist and Price Discriminating Monopolists with Two Demand Types

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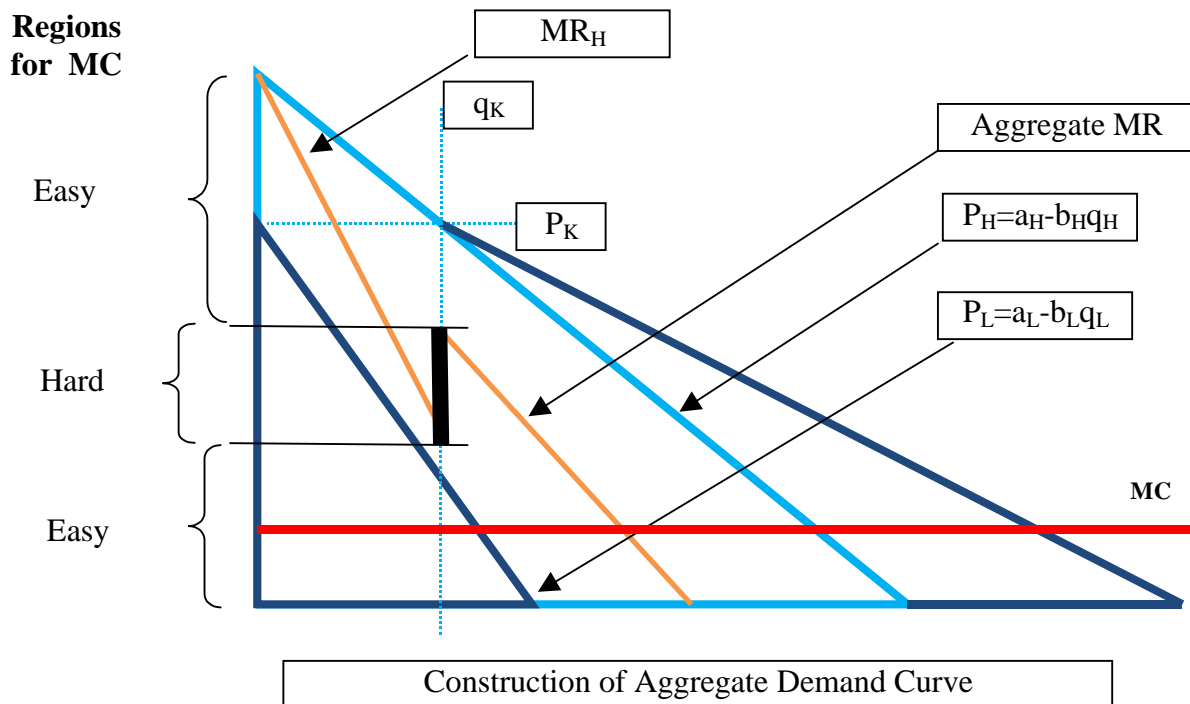
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1 Non-Price Discrimination

For a non-PD monopolist, there is a trade-off between increased revenue from increased quantities sold and decreased revenue from lower prices on all previous units. The net change in revenue depends upon elasticity and MC. This fact doesn't change when monopolist faces an aggregate demand. The only difference is that demand now has a kink, because there is more than one type of consumer. You can see how this works in the following graph given two types of inverse demands¹:

$$P_H = a_H - b_H q_H$$

$$P_L = a_L - b_L q_L$$



The profit maximization problem has an "easy case" and a "hard case". The above, where MC (red) goes below the black bar is the easy case.

¹See Aggregate Demand and Marginal Revenue notes for details.

1.1 Easy Case

In the easy case, MC only crosses aggregate marginal revenue, or MR_A at one point, i.e., not through the green spot in the graph above (Figure 1). On a test, you should check which case is allowed.

The problem with profit maximization with a kinked demand is that we need to find the relevant part of the (discontinuous) aggregate marginal revenue to equate to MC . With the easy cases, **we just have to test whether q^* in $MR(q^*) = MC(q^*)$ is below the kink: $q^* < q_K$, or beyond the kink: $q^* > q_K$.**

If you know its the easy case, start by first finding the quantity of the kink q_K . The price of the kink P_K will be at the intercept of the low demand type's demand curve= a_L . (see above graph). To find that kink quantity, solve for it using the demand curve of the high demand type and equate the price in it to the low demand intercept.

$$q_H = \frac{a_H - P_H}{b_H} = \frac{a_H - a_L}{b_H} = q_K.$$

Now, find the aggregate demand. Assume for the purpose of illustration that the inverse demands have the same slopes: $b_H = b_L = 1$. Below the kink price P_K , the aggregate demand is:

$$q = q_H + q_L = \frac{a_H - P_H}{b_H} + \frac{a_L - P_L}{b_L} = a_H + a_L - 2P$$

Note that both types of customers are buying at the same price because there is only one market, $P_H = P_L = P$. If we invert the inverse demand, we get the demand for $q \geq q_K$. The aggregate demand for $q < q_K$ is equal to the demand of the high type.

The whole inverse demand is then:

$$P = \left\{ \begin{array}{l} \frac{a_H + a_L - q}{2} \text{ for } q \geq q_K \\ a_H - q \text{ for } q < q_K \end{array} \right\}$$

Therefore, the whole MR

$$MR = \left\{ \begin{array}{l} \frac{a_H + a_L}{2} - q \text{ for } q \geq q_K \\ a_H - 2q \text{ for } q < q_K \end{array} \right\}$$

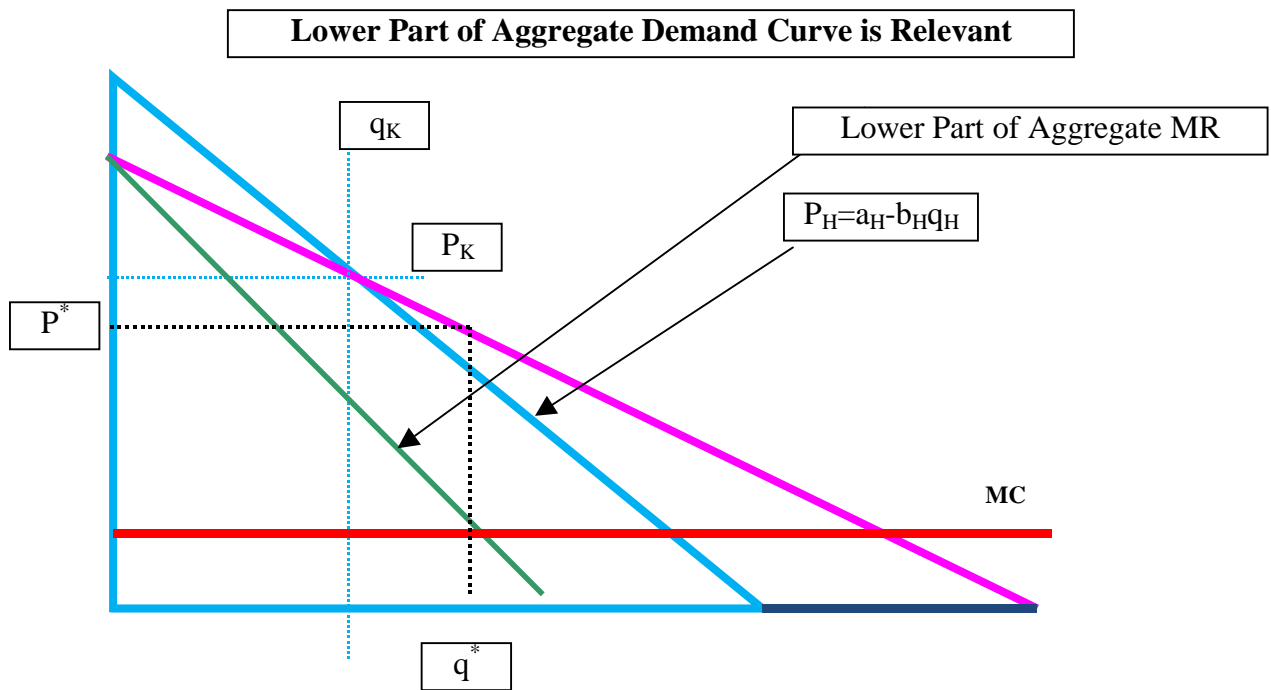
Now that we have found the kink, to find the optimal quantity, we have to equate the *relevant part of MR* to MC . $MR(q) = MC(q)$. Start with the top part of the MR curve (for $q \geq q_K$) and equate it to MC and solve for q^* .

$$MR(q^*) = \frac{a_H + a_L}{2} - q = MC(q^*)$$

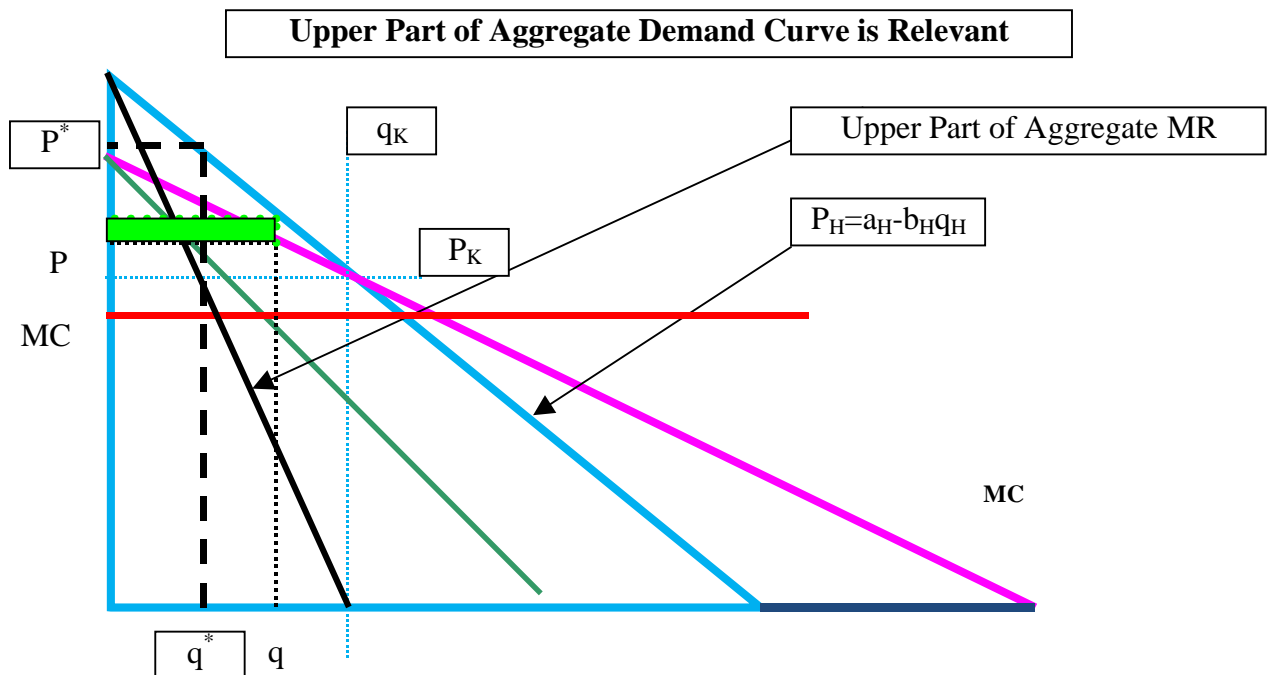
If $q^* \geq q_K$ then you have used the right part of the MR curve. If $q^* < q_K$ then you have the wrong q^* . Redo the calculation with $MR_H(q^*) = a_H - 2q = MC(q)$. Derive the rest, price, profit...as before.

1.1.1 Intuitions On What We Are Doing

The basic idea in pictures is to find out which demand curve is the *outer demand curve* for each case where $MR = MC$. The *outer demand curve* contains the inner demand curve and therefore, must be larger. That's the one the monopolist should use to maximize profit.



The green area below shows how the inner (purple) demand curve is not the right one. First, revenue profits will go up just by using the outer (blue) demand curve, while maintaining the same quantity. That's the green area. But, furthermore, we know that the profit maximizing price for that outer curve must use the $MC=MR_H$, i.e., its own demand curve. Therefore, the profit from $\pi(P^*) > \pi(P)$.



1.1.2 Dead Weight Loss When There Is A Kink

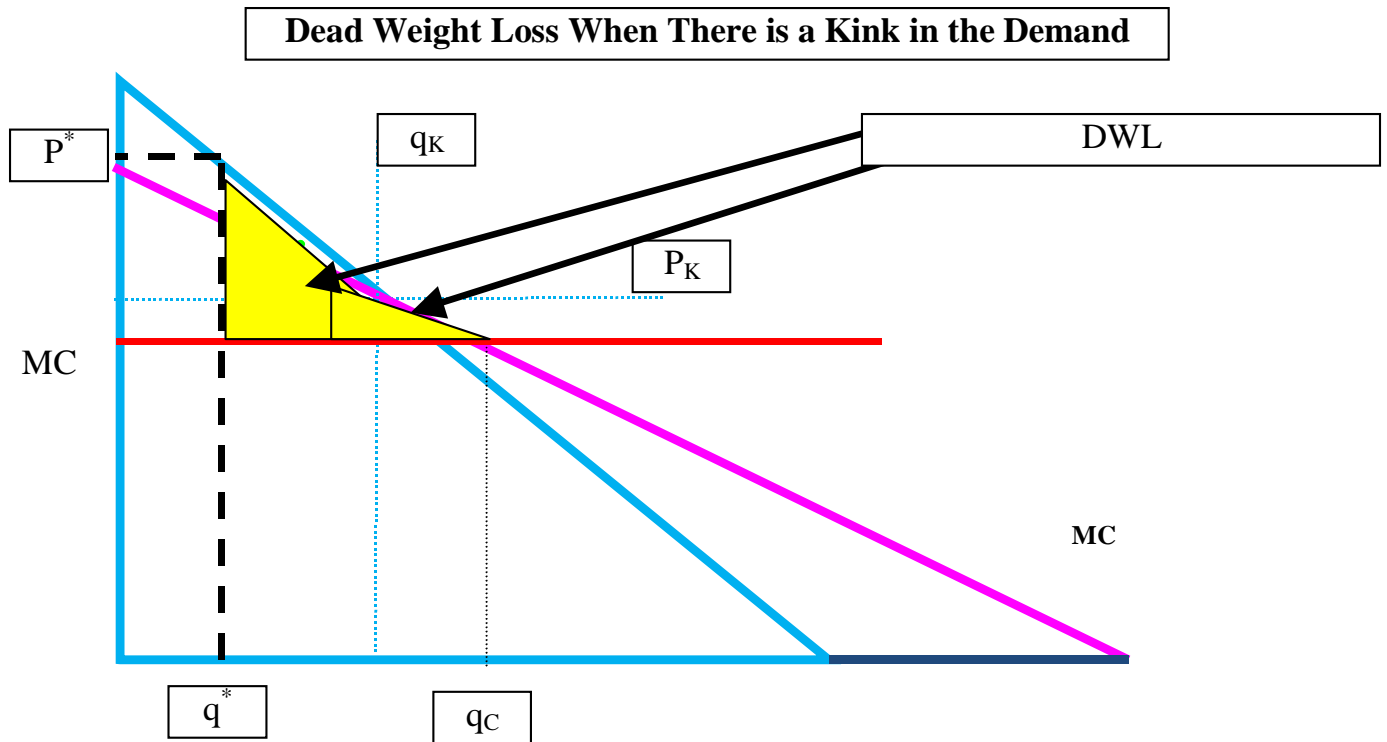


Figure 4

If q^* is below q_K then:

$$DWL = \frac{1}{2}(P^* - P_K) \cdot (q_K - q^*) + (P_K - MC) \cdot (q_K - q^*) + \frac{1}{2}(q_C - q_K) \cdot (P_K - MC)$$

1.2 Hard Cases

If you don't know it's the easy case, you have to test $MR=MC$ for both parts of the aggregate revenue curves.

What if the two parts of the MR curve in the kinked aggregate demand case were equal to MC at two qs , i.e., the MC curve goes through the gap between two parts of the aggregate demand curve (the black bar in figure 1) in the first graph above. (Its the graph between the two red lines below.) In this case, you have to find the profit for each q^* to see which maximizes profits—there is no absolute rule. To know which case you are in, you have to find the $MR = MC$ for both parts of the aggregate MR curve (see figure 1)

$$MR(q^*) = \left\{ \begin{array}{l} \frac{a_H + a_L}{2} - q \text{ for } q \geq q_K \\ a_H - 2q \text{ for } q < q_K \end{array} \right\} = MC(q^*)$$

1.3 Example

Lets see how this works through a numerical example. See figure 4 below.

The sleepy town of Couch Potato has expanded into the CP Metropolitan area. Now, Gresebomm's has 2 kinds of customers: those who have a high curiosity (H) for what a heart attack feels like and those who have a low curiosity (L). The price intercept for the high (H) demand consumer = $a_H = 2$ and price intercept for the low (L) demand consumer = $a_L = 1$. Let the slope of the inverse demand curves be equal to $b_H = b_L = 1$. Through a secret new invention, Gresebomm's now has zero MC .

Then the inverse demand of the H type is:

$$P_H = 2 - q_H$$

and that demand for the L type was

$$P_L = 1 - q_L$$

What would the be Gresebomms's profit maximizing price and quantity, if it could perfectly identify each type of consumer?

$$q_L = \frac{a - c}{2b} = \frac{1}{2}, p_L = \frac{1}{2}.$$

$$q_H = \frac{2}{2} = 1, p_H = 1.$$

Total quantity sold: $Q = q_H + q_L = \frac{3}{2}$
 The demand functions would be:

$$q_H = 2 - P_H$$

$$q_L = 1 - P_L$$

Aggregate demand (green, below) would be:

$$Q = q_H + q_L = \begin{cases} 2 - P & \text{if } P > 1 \\ 3 - 2P & \text{if } P \leq 1 \end{cases}$$

Inverting the lower part:

$$P = \frac{3}{2} - \frac{Q}{2}$$

The marginal revenue (red) would be:

$$MR(Q) = \begin{cases} 2 - 2Q & \text{if } Q < 1 \\ \frac{3}{2} - Q & \text{if } 1 \leq Q \end{cases}$$

Because marginal revenue is always positive for $Q \leq 2$, Total Revenue (dark blue) is always increasing and would be

$$P \cdot Q = \begin{cases} (2 - Q) \cdot Q & \text{if } Q < 1 \\ (2 - Q) \cdot Q + (1 - (Q - 1)) \cdot (Q - 1) & \text{if } 1 \leq Q \end{cases}$$

(The 2nd line of this revenue function comes from having to start at $Q \geq 1$.)

Suppose $MC = 0.4$ (horizontal purple line). Then the profit function will have two flat spots. To see which they are set both parts of $MR=MC$.

$$MR(Q) = \begin{cases} 2 - 2Q & \text{if } Q < 1 \\ \frac{3}{2} - Q & \text{if } 1 \leq Q \end{cases} = MC = 0.4$$

$$MR_H = 2 - 2Q = 0.4 \text{ at } Q_H = 0.8.$$

$$MR = \left(\frac{3}{2} - Q\right) = 0.4 \text{ at } Q = 1.1$$

Substituting each of these points into the profit function,

$$\pi(Q_H) = (2 - (.8) - .4)(0.8) = 0.64$$

$$\pi(Q) = \left(\frac{3}{2} - \frac{1.1}{2} - .4\right)(1.1) = 0.605$$

We see however, that the profit at $Q_A = 0.8$ is lower. Therefore, the monopolist will choose $Q_H = 0.8$. You can also see from the graph $\pi(Q) < \pi(Q_H)$ (purple curves).

Now, suppose $MC = 0.1$ (light blue)

$$MR_H = 2 - 2Q = 0.1, \text{ at } Q_H = 0.95$$

and

$$MR = \left(\frac{3}{2} - Q\right) = 0.1 \text{ at } Q = 1.4$$

The profits are then:

$$\pi(Q_H) = (2 - (.95) - .1)(0.95) = 0.90$$

with profit:

$$\pi(Q) = \left(\frac{3}{2} - \frac{1.4}{2} - .1\right)(1.4) - 0.98$$

Therefore, the monopolist will choose $Q = 1.4$. You can see from the graph that $\pi(Q) > \pi(Q_H)$ (light blue curves).

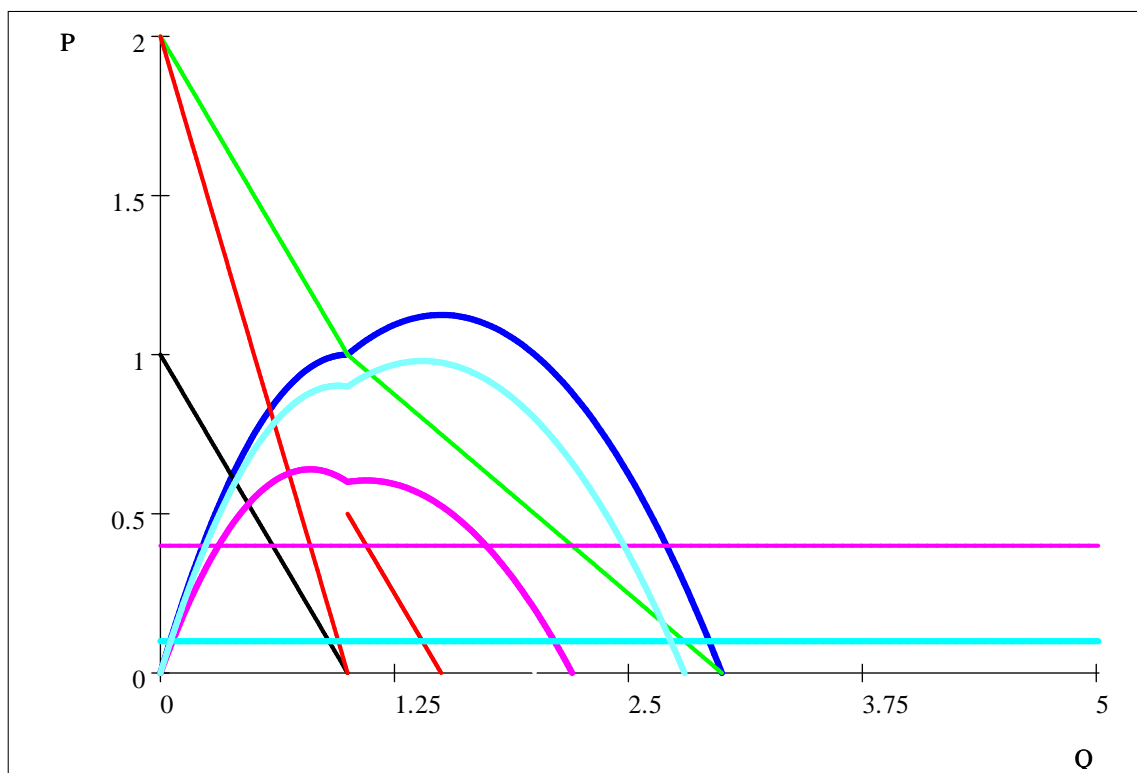


Figure 4: Revenue and Profit with $MC=0.1$ (Purple) and $MC=0.4$ (Blue) for Aggregate Demand (Green)

1.4 Price Discriminating Case

If the monopolist could distinguish between H and L , then, the monopolist can charge different prices for each group. (Then there won't be an aggregate demand for both markets, since that would require a common price.) If marginal costs were constant: $MC = c$, you could just set $MR_H(q_H) = c$ and $MR_L(q_L) = c$ to find the profit maximizing outputs q_H, q_L in each market. But, if marginal cost were not constant, e.g., $MC = cQ$ then, you would have to figure out the aggregate MR first. You would then set that $MR(Q^*) = MC(Q^*)$. This will give you the aggregate output Q^* . To find out what you would sell to H and L , you would set $MR_H(q_H) = MC(Q^*)$ and $MR_L(q_L) = MC(Q^*)$. See figure 5 below.

To find the Aggregate MR from:

$$MR_H = 2 - 2q_H$$

$$MR_L = 1 - 2q_L$$

we have to again make q a function of MR :

$$q_H = 1 - \frac{MR}{2}$$

$$q_L = \frac{1}{2} - \frac{MR}{2}$$

$$Q = q_H + q_L = 1 - \frac{MR}{2} + \frac{1}{2} - \frac{MR}{2} = \frac{3}{2} - MR$$

Since $MR=MC$, you could just substitute. So if $MC = \frac{1}{2}Q$ then,

$$Q = \frac{3}{2} - \frac{1}{2}Q$$

$$Q^* = 1$$

Now, equate MR_H to this MC .

$$MR_H = 2 - 2q_H = MC(1) = \frac{1}{2} \cdot 1 = \frac{1}{2}$$

$$MR_H = 2 - 2q_H = \frac{1}{2}$$

$$\frac{3}{2} = 2q_H$$

$$q_H = \frac{3}{4}$$

$$P_H = 2 - \frac{3}{4} = \frac{5}{4}$$

Now, equate MR_L to this MC .

$$MR_L = 1 - 2q_L = MC(1) = \frac{1}{2}$$

$$MR_L = 1 - 2q_L = \frac{1}{2}$$

$$q_L = \frac{1}{4}$$

$$P_L = 1 - \frac{1}{4} = \frac{3}{4}$$

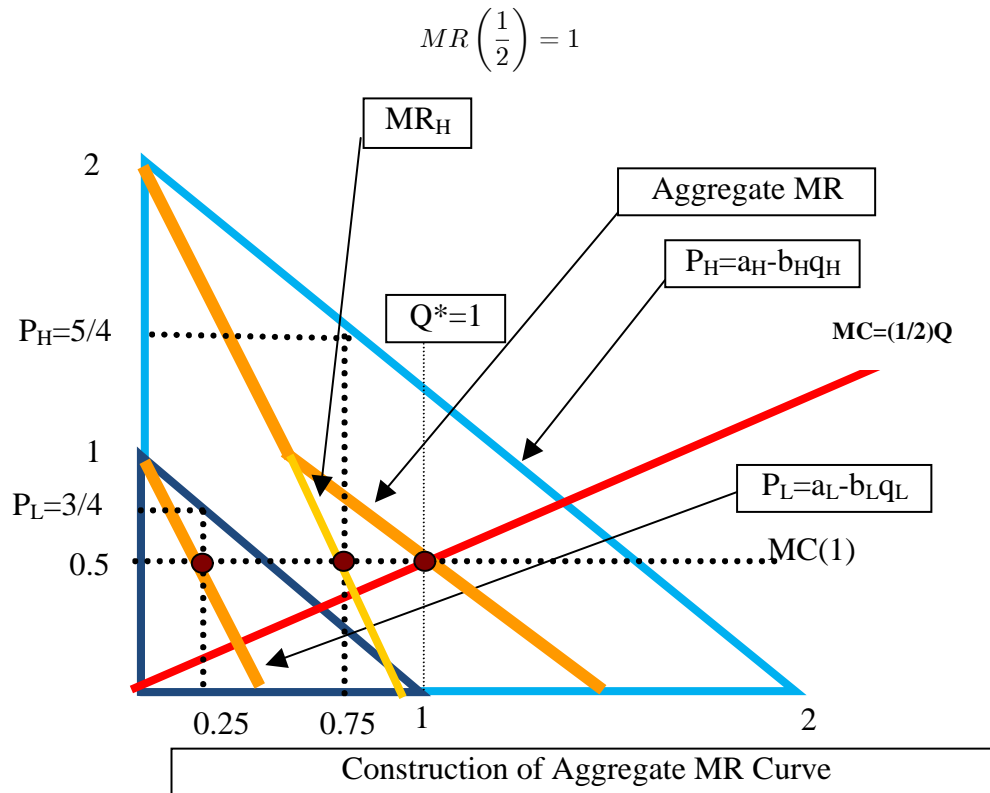
To graph this function, we need to find the kink in the aggregate MR, we need to find the q at which the aggregate $MR(q_K) = MR_H(q_K)$. To graph this function, we need to find the kink in the aggregate MR, we need to find the q at which the aggregate $MR(q_K) = MR_H(q_K)$

Thus,

$$MR = \frac{3}{2} - q = 2 - 2q$$

$$q_K = \frac{1}{2}$$

$$MR(Q) = \begin{cases} 2 - 2Q & \text{if } Q < \frac{1}{2} \\ \frac{3}{2} - Q & \text{if } \frac{1}{2} \leq Q \end{cases}$$



1.5 Summary and Comparison Between Price Discriminating and Non Price Discriminating Cases

In the non PD case, there is a jump at the kink quantity because the next sale will engage the 2nd market's demand curve, with no loss in revenue from prior sales to balance out the gain in quantity.² But in the PD case, prices can move independently. The monopolist will equalize MR across markets in the PD case. A unit will be sold in the lower demand market only when MR from the high demand market is just about to go below the low demand market's MR. Therefore, there shouldn't be a unit at which there is a jump in MR.

²See notes on Aggregate Demand and Marginal Revenue.