Constant Shares Demand

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1 Constant Shares Demand

1.1 Numerical Example

To find the firm's constant or equal share of the demand curve, just take the total demand curve and divide it by the number of shares. For $Q = \alpha - \beta P$, the constant shares demand for each of n firms is:

$$q_i = \frac{Q}{n} = \frac{\alpha}{n} - \frac{\beta}{n} P$$

For example, suppose total market demand is: $Q = 10 - P$ and Firm 1 and 2's constant shares demands are $q_1 = q_2 = q = \frac{1}{2}(10 - P)$ and each of their marginal costs are $MC(q) = 2$, with $FC = 0$.

Then we can find the profit maximizing output for each of firm 1 and firm 2 by the $MC(q) = MR(q)$ for each of their shares of the demand.

Equal shares demand:

$$q = \frac{10 - P}{2} = 5 - \frac{P}{2}$$

The inverse demand is:

$$P = 10 - 2q$$

At profit max:

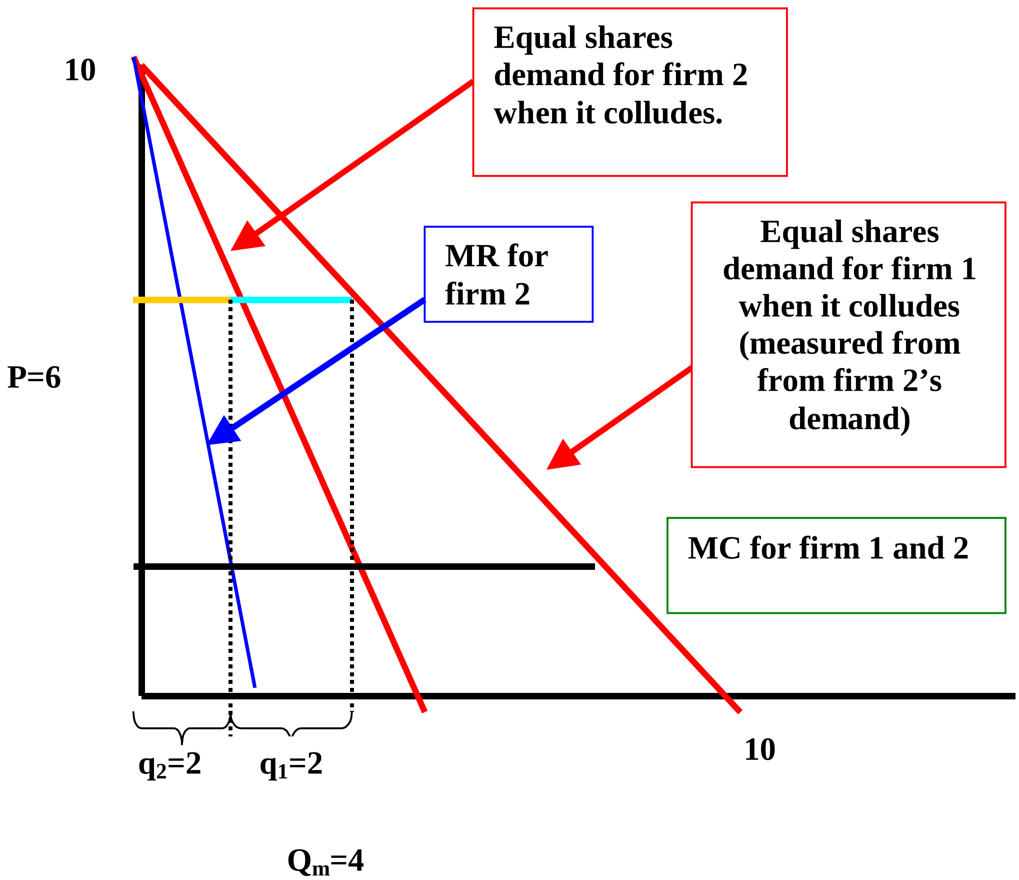
$$MR = 10 - 4q = 2$$

$$8 = 4q$$

$$q = 2$$

The industry output would be $Q = q_1 + q_2 = 4$.

$$P = 10 - 2 \cdot 2 = 6$$



2 Algebraic Derivation

Recall that the market demand was $Q = \alpha - \beta P$ and that this same market's inverse demand can be expressed as $P = a - bQ$ where $a = \frac{\alpha}{\beta}$ and $b = \frac{1}{\beta}$. Splitting the demand into equal shares, the demand facing each of firm 1 and 2 would be:

$q_1 = q_2 = \frac{1}{2}(\alpha - \beta P)$. Let $\alpha' = \frac{\alpha}{2}, \beta' = \frac{1}{2}\beta$. Then, demand for each firm would be $q_2 = q_1 = (\alpha' + \beta' P)$. The profit maximizing quantity would then be:

$$q^* = \frac{1}{2}(\alpha' - \beta' c) = \frac{1}{2}\left(\frac{\alpha}{2} - \frac{\beta c}{2}\right)$$

Recall that when the optimal quantity for a monopolist based on the parameters of the inverse demand is

$$Q^* = \frac{a - c}{2b}$$

If we split this optimal quantity into two parts, we would get

$$q_1 = q_2 = \frac{1}{2}\left(\frac{a-c}{2b}\right)$$

Substituting $a = \frac{\alpha}{\beta}$ and $b = \frac{1}{\beta}$:

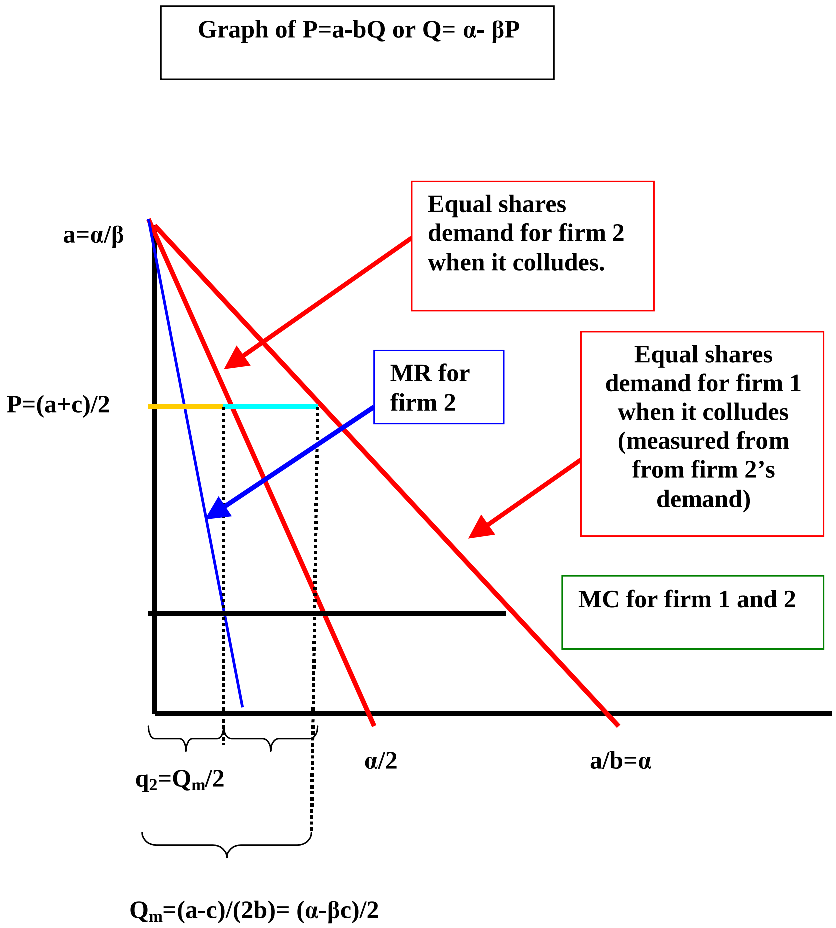
$$q_1 = q_2 = \frac{1}{2}\left(\frac{\frac{\alpha}{\beta}-c}{2\left(\frac{1}{\beta}\right)}\right) = \frac{1}{2}\left(\frac{\frac{\alpha}{\beta}}{2\left(\frac{1}{\beta}\right)} - \frac{c}{2\left(\frac{1}{\beta}\right)}\right)$$

$$q_1 = q_2 = \frac{1}{2}\left(\frac{\alpha}{2} - \frac{\beta c}{2}\right)$$

Recall from 2.1 of "NPV and Demand Growth" that the monopoly output for this demand is

$$Q^* = \frac{1}{2}(\alpha - \beta c)$$

Thus, we have shown that you can get the constant shares output either by dividing the demand in constant shares, $q_i = \frac{Q}{n} = \frac{\alpha}{n} - \frac{\beta}{n}P$ and finding the equilibrium output for each firm , as we did above, or by just dividing the monopoly output Q^*.The price in either case is: $P = a - bQ = \frac{a+c}{2}$.


Graph of P=a-bQ or Q= α- βP
Equal shares demand for firm 2 when it colludes.
a=α/β
MR for firm 2
Equal shares demand for firm 1 when it colludes (measured from from firm 2's demand)
P=(a+c)/2
MC for firm 1 and 2
α/2
a/b=α
q2=Qm/2
Qm=(a-c)/(2b)= (α-βc)/2