Price Competition (Bertrand Competition)

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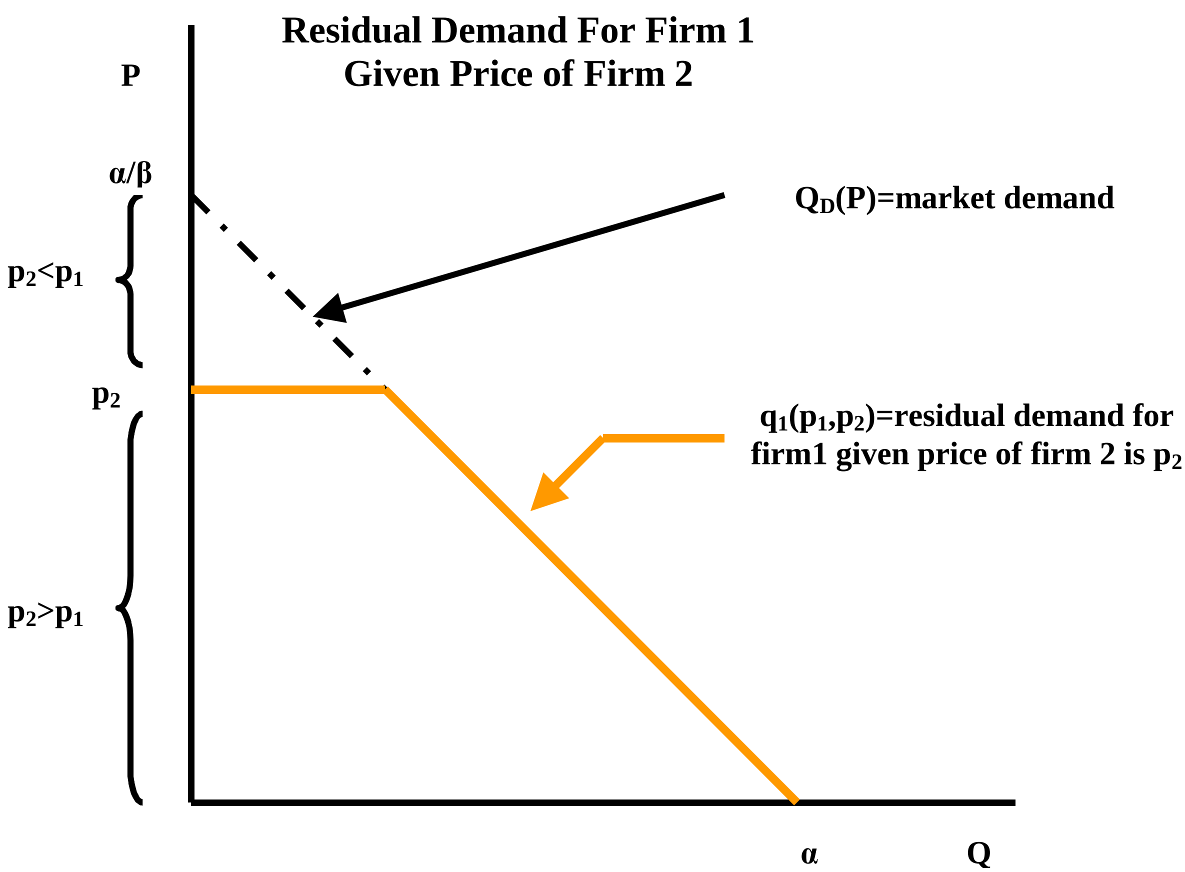
This model is peculiar because neither the profit nor the best response functions are continuous. Assume that the "market" demand function is:

$$Q(p) = \alpha - \beta p$$

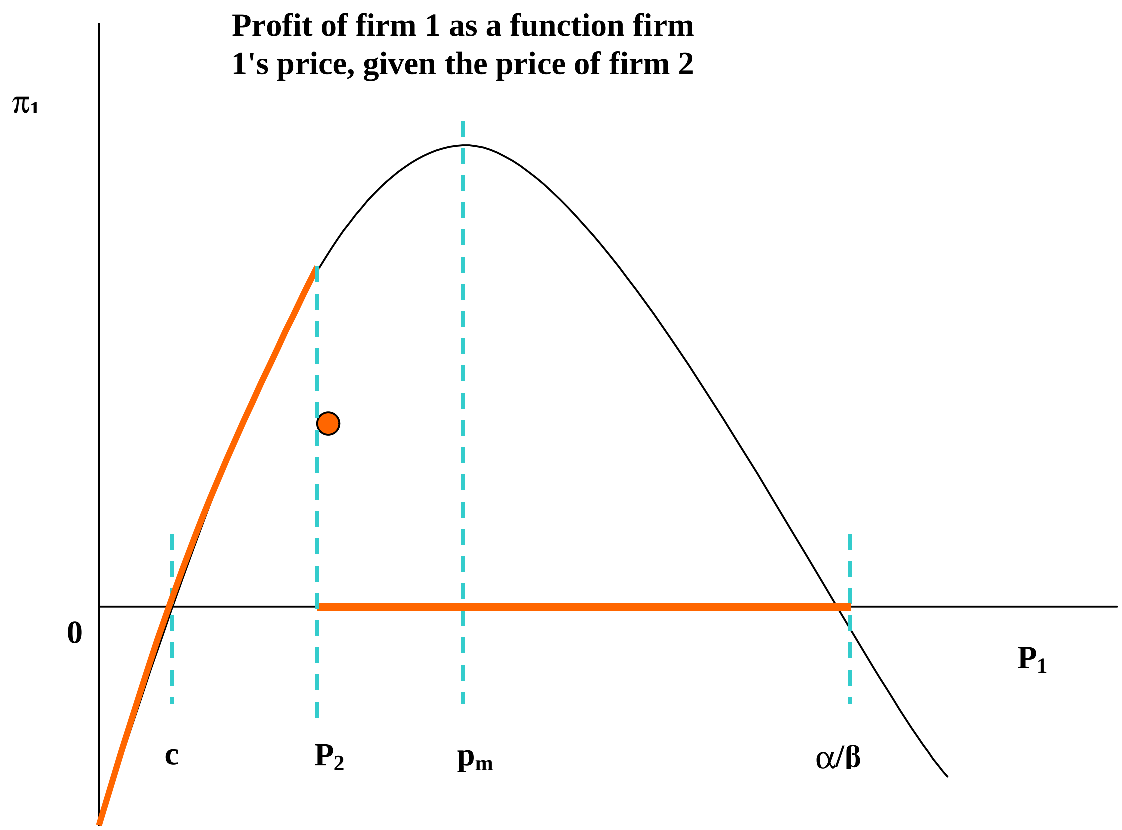
The demand function for firm 1 is:

$$q_1(p_1, p_2) = \left\{ \begin{array}{lll} Q(p_1) & \text{if} & p_1 < p_2 \\ \frac{1}{2}Q(p_1) & \text{if} & p_1 = p_2 \\ 0 & \text{if} & p_1 > p_2 \end{array} \right\}$$

Symmetrically for firm 2.



Players (firms)	$N = \{1, 2\}$
Strategies	$S_1 = S_2 = \{p : p \geq 0\}$
Payoffs	$\pi_1(p_1, p_2) = \left\{ \begin{array}{lll} (p_1 - c)\, Q(p_1) & \text{if} & p_1 < p_2 \\ \frac{1}{2}(p_1 - c)\, Q(p_1) & \text{if} & p_1 = p_2 \\ 0 & \text{if} & p_1 > p_2 \end{array} \right\}$



As Firm 1 raises prices, it goes from pricing below cost and losing money (furthest left) to just covering cost and making zero profits, to getting the whole market and making positive profits, until it matches p_2, at which point it splits the market. When its price is higher than p_2, it sells nothing and gets the whole market.

The best response function is the price that 1 should set to maximize profit given a price of 2 :

$$B_1(p_2) = \left\{ \begin{array}{lll} \{p_1 : p_1 > p_2\} & \text{if} & p_2 < c \\ \{p_1 : p_1 \geq p_2\} & \text{if} & p_2 = c \\ \varnothing & \text{if} & c < p_2 \leq p^m \\ \{p^m\} & \text{if} & p^m < p_2 \end{array} \right\}$$

Note that $\varnothing$ (the empty set) means that Firm 1 has no best response.

When firm 2 changes at or below cost, the best response of firm 1 is to charge something higher than firm 2. If firm 2 charges between cost and the monopoly price, firm 1 has no best response, because just up to the price that firm 2 changes, firm 1 gets the whole market, but just at the price that firm 2 change, firm 1 splits the market. This fact is illustrated by the increasing profit function which drops to an orange dot and then to a flat line at $p = 0$ zero in the above diagram.

0.1 Equilibrium

With symmetric costs (c, c) is equilibrium ,i.e.,

$$\pi_1 (p_1 = c, p_2 = c) \geq \pi_1 (p_1, p_2 = c), \text{for all } p_1$$

To see that this is an equilibrium, note that no firm has an incentive to deviate. Given $p_2 = c$, firm 1 will be losing money if $p_1 < p_2$, and would make no more profit if $p_1 \geq c$. Similarly for firm 2:

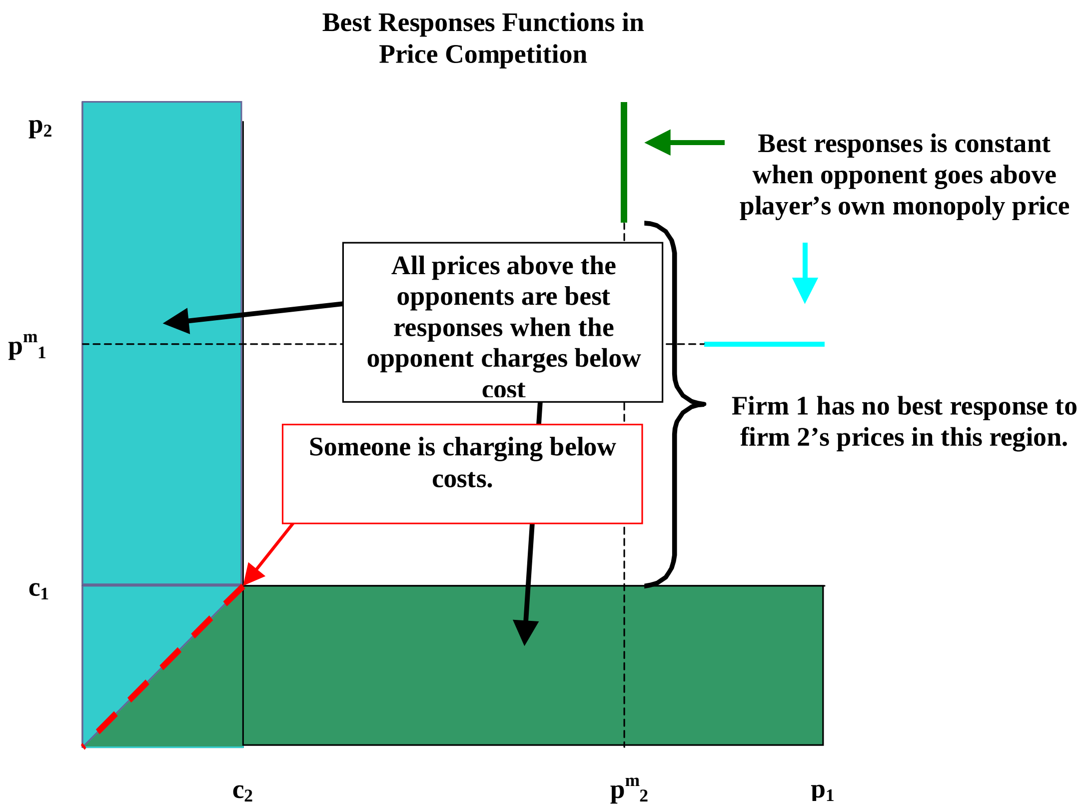
$$\pi_2 (p_1 = c, p_2 = c) \geq \pi_2 (p_1 = c, p_2), \text{for all } p_2$$

Quantity produced will be $Q = \alpha - c = q_1 + q_2 = \frac{a-c}{b}$, corresponding to the inverse demand $P = a - bQ$

0.2 Graphing The Best Response Functions–When Money Is Infinitely Divisible[1]

In the graph below, there is no best response if the price of the opponent is above cost but below its monopoly price: $c < p_2 \leq p^m$ =monopoly price, because for every $p_1 < p_2$ firm 1 could increase profits by increasing price. There is profitable deviation for firm 1 also when $p_2 \leq p_1$ because firm 1 could capture the whole market by going lower.

[1]For IO purposes, money is not often not infinitely divisible. Thererfore, you should read this graph only if you are interested in technical details.



Notice that firm 1 is reacting to when firm 2 is above firm 1's cost and monopoly price in the graph above.