Sequential Quantity Competition (Stackelberg Duopoly)

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February 8, 2008

In Stackelberg, the Leader moves first and chooses its profit maximizing quantity, given the anticipated reaction of the Follower. The Follower then chooses its quantity.

The ability to commit to a quantity makes a firm a "leader", and gives it an advantage over the follower. Another way to think of this is: if a firm can observe the strategy of its opponent, before it makes its own choice, then it is a follower. (A nuance: choosing "capacity" as opposed to quantity, in the first period may be observed by the opposing firm, but that doesn't make the chooser a leader, because they could still play Cournot (simultaneously) in the 2nd period, because the "leader" may not use all of its capacity. The Leader has to choose its actual quantity and the Follower has to observe that quantity in order for it to be a Stackelberg situation.)

The Stackelberg equilibrium is intermediate between monopoly and Cournot. We start solving it the same way as Cournot, but beginning with the Follower, say firm 2.

$$\max_{q_2} \{(P(Q) - c) \cdot q_2\} - FC$$

Firm 2 maximizes profit given the already chosen quantity of firm 1.

$$\pi_2 = \max_{q_2} \{(a - b(\bar{q}_1 + q_2) - c_2) \cdot q_2\} - FC$$

FOC:

$$\frac{d}{dq_2}((a - b(\bar{q}_1 + q_2) - c_2) \cdot q_2) = 0$$

$$a - 2bq_2 - b\bar{q}_1 - c_2 = 0$$

$$q_2^* = \frac{a - c_2}{2b} - \frac{\bar{q}_1}{2}$$

q_2^* is the best response function for firm 1 given any fixed quantity choice by firm 2. We can rewrite this:

$$q_2^* = B_2(q_1) = \left\{ \begin{array}{lll} \frac{a-c_2}{2b} - \frac{\bar{q}_1}{2} & \text{if} & q_1 \leq a - c_2 \\ 0 & \text{if} & q_1 > a - c_2 \end{array} \right\}$$

Now, when firm 1 chooses, it uses firm 2's best response function to anticipate the quantity that firm 2 will choose for any quantity that it firm 1 will choose.

$$\max_{q_1} \{(a - b(q_1 + q_2^*(q_1)) - c_1) \cdot q_1\} - FC$$

Assuming symmetric costs,

$$\max_{q_1} \pi_1 = \max_{q_1} \{(p(q_1 + B_2(q_1)) - c) q_1\} = \max_{q_1} \left\{ \left[a - b\left(q_1 + \frac{a-c}{2b} - \frac{1}{2} q_1 \right) \right] q_1 - c q_1 - FC \right\}$$

FOC:

$$a - b2q_1^* - \frac{a-c}{2} + bq_1^* - c = 0$$

$$q_1^* = \frac{1}{2b}(a - c)$$

Then

$$q_2^* = \frac{a-c}{2b} - \frac{\frac{1}{2b}(a-c)}{2}$$

$$q_2^* = \frac{1}{4b}(a - c)$$

Plugging these back into the profits of their respective firms with symmetric costs $c_1 = c_2 = c$:

$$\pi_1 = \left[a - b\left(\frac{1}{2b}(a-c) + \frac{a-c}{2b} - \frac{1}{2}\left(\frac{1}{2b}(a-c) \right) \right) - c \right] \frac{1}{2b}(a-c) - FC$$

$$\pi_1 = \left[(a-c) - \frac{3}{4}(a-c) \right] \frac{1}{2b}(a-c) - FC$$

$$\pi_1 = \left[\frac{1}{4}(a-c) \right] q_1 - FC$$

$$\pi_1 = \frac{b}{2}\left[\frac{1}{2b}(a-c) \right] q_1 - FC$$

$$\pi_1 = \frac{b}{2} q_1^2 - FC$$

$$\pi_2 = (a - b(q_1 + q_2) - c) \cdot q_2 - FC$$

$$\pi_2 = \left(a - b\left(\frac{1}{2b}(a-c) + \left[\frac{1}{4}(a-c)\right]\right) - c\right) \cdot \left[\frac{1}{4b}(a-c)\right] - FC$$

$$\pi_2 = \left(\frac{1}{4}(a-c)\right) \cdot \frac{1}{4b}(a-c) - Fc$$

$$\pi_2 = b\left(\frac{1}{4b}(a-c)\right) \cdot q_2 - FC$$

$$\pi_2 = bq_2^2 - FC$$

0.1 Comparison Between Stackelberg (sequential moves) and Cournot Competition (simultaneous moves)

Let q^c =Cournot quantity q^S =Stackelberg quantity in the following and $\pi^c = b\left(q^c\right)^2$ the profit from symmetric cost Cournot competition.

Leader produces more than symmetric costs Cournot:	$q_1^s = \frac{a-c}{2b} = \frac{3}{2}q^c > q^c = \frac{a-c}{3b}$
Leader profit is higher and follower profit lower than Cournot:	$\pi_1^s = \frac{(a-c)^2}{8b} > \pi^c, \pi_2^s = \frac{(a-c)^2}{16b} < \pi^c$
Follower produces less than symmetric costs Cournot:	$q_2^s = \frac{a-c}{4b} = \frac{3}{4}q_1^c < q^c = \frac{a-c}{3b}$
Price is lower than Cournot:	$p^s = \frac{a+3c}{4} < \frac{a+2c}{3} = p^c$
Stackelberg market quantity is higher than Cournot:	$Q^s = \frac{3(a-c)}{4b} > \frac{2(a-c)}{3b} = Q^c$

Notice that Stackelberg Leader's and Follower's profits are functions of their respective quantities.

$$\pi_1^s = \frac{(a-c)^2}{8b} = \frac{b}{2}\left(\frac{a-c}{2b}\right)^2 = \frac{b}{2}\left(q_1^S\right)^2$$

$$\pi_2^s = \frac{(a-c)^2}{16b} = b\left(\frac{a-c}{4b}\right)^2 = b\left(q_2^S\right)^2$$

We can't draw the best response graphs for Stackelberg like Cournot, because only the 2nd mover is best responding to the first.