Residual Demand to Cournot

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January 24, 2008

1 Residual Demand to Duopoly

In the prior discussion, we had firm 2 maximizing profit with residual demand from "nonstrategic" competitive fridge with an upward sloping MC curve.

Now, we return to constant MC and build up to the case where firm 2 is maximizing profit with a best responding/strategic competitor, i.e., both firms 1 and 2 will be fully strategic in the end.

1.1 Notation:

Suppose we had the general inverse demand function:

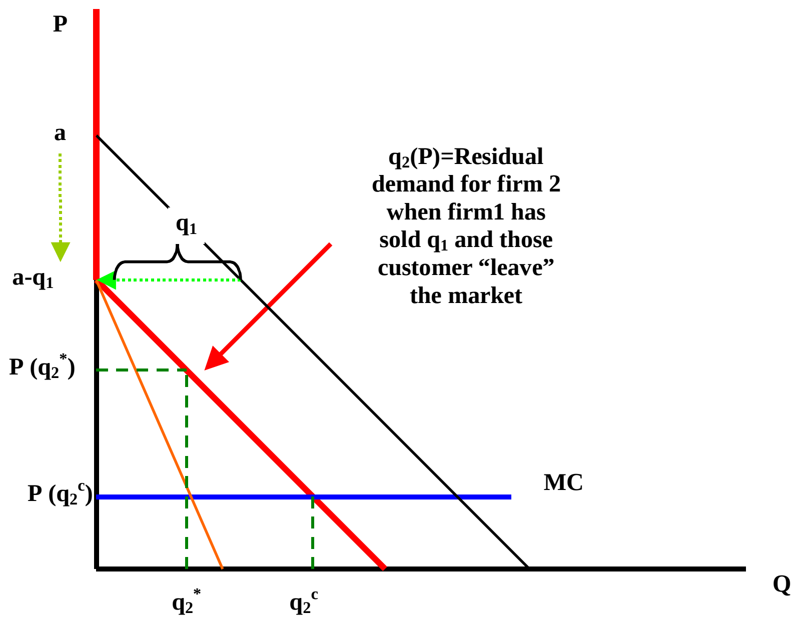
$$P = a - bQ = a - b(q_1 + q_2)$$

a =y intercept inverse demand
b =slope of inverse demand
MC for firm 1 $= c_1$
MC for firm 2 $= c_2$

and $c_1 = c_2 = c$ in the symmetric cost case.

1.2 The Monopolist within Cournot Competition

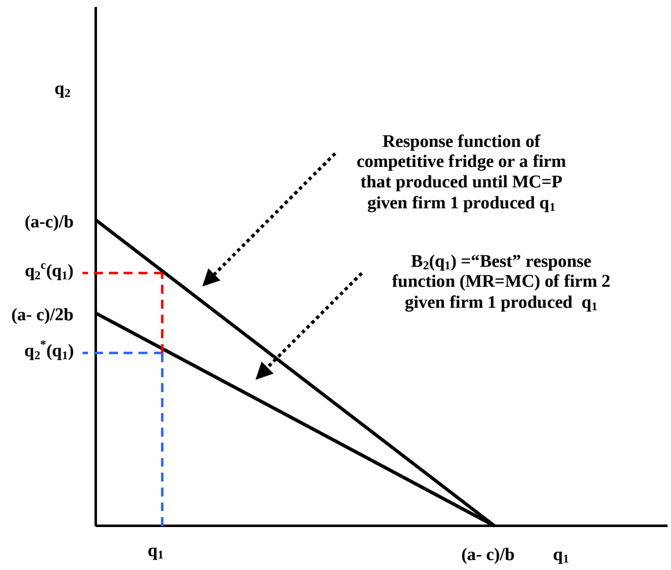
Lets start with the simple case where firm 1 has already sold q_1 and has left the market with its customers. That leaves residual demand which we discussed at the beginning of the residual demand notes. Note that with respect to its residual demand, firm 2 is a monopolist, producing up to q_2^* instead of the competitive quantity. In other words, firm 2 "under produces" given firm 1's production. Note also that this monopoly problem is not the equal shares monopoly problem (the solution of which is $q_1 = q_2 = \frac{a-c}{4b}$) even though we know that in equilibrium the firms will split the market, because in the duopoly problem, each firm reacts to the fact that its opponent under produces.



This diagram also shows that if we knew what q_1 was, we could solve the duopoly problem as a monopoly problem. But, unlike previous cases, where q_1 =supply curve of firm 1, from now on, firm 1 is going to also profit maximize with respect to q_2.To find either, we have to find both, the quantity where each firm is choosing their profit maximizing quantity given the quantity chosen by the other firm.

1.3 Best Response Function

The best response function is the solution of the monopoly problem above: $q_2^*(q_1)$ the quantity that maximizes firm 2's profit given a quantity chosen by firm 1. The best way to see a best response function is from a graph of each firms' quantity. Note that the axis are not P and Q, but q_1 and q_2.



Note that with the above demand $\frac{a-c}{2}$ would be the monopoly output for firm 2, if firm 1 produced 0 and that furthermore, if firm 1 produced up to $q_1 = a - c$, firm 2 would want to produce nothing, regardless of whether it was producing up to $MC = P$ nor $MC = MR$.

(I will show in section, how each firm underproduces, yielding its opponent a greater residual demand than it counted on, and that this process eventually converges.)

Now, let both firms compete on quantity.

1.4 Quantity Competition (Cournot Competition)

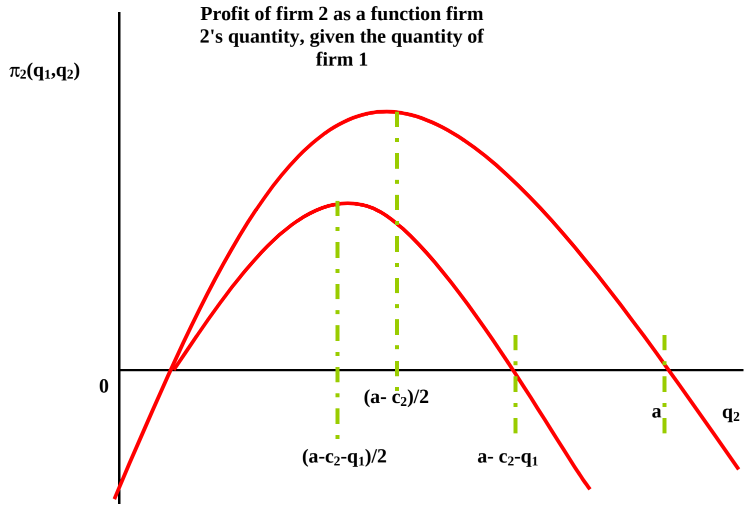
Now, we are going to make firm 1 is also profit maximizing/strategic. The game structure is as follows:

Players (firms)	$N = \{1, 2\}$
Strategies	$S_1 = S_2 = \{q : q \geq 0\}$
Payoffs for firm 1	$\pi_1(q_1, q_2) = \begin{cases} (P(Q) - c_1) \cdot q_1 & \text{if} \quad Q \leq a \\ -cq_1 & \text{if} \quad Q > a \end{cases}$
Payoffs for firm 2	$\pi_2(q_1, q_2) = \begin{cases} (P(Q) - c_2) \cdot q_2 & \text{if} \quad Q \leq a \\ -cq_2 & \text{if} \quad Q > a \end{cases}$

Assume as before that the inverse demand function is:

$$P(Q) = \left\{ \quad a - Q \quad \text{if} \quad Q \leq a \quad \right\}$$

where $Q = q_1 + q_2$. The residual demand is as above. The profit of firm 2 given some output choice of firm 1 is.



The higher curve in the above figure is the case where firm 2 is a monopoly.

The best response function is the quantity that 1 should set to maximize profit given a quantity of 2 :

$$\max_{q_2} \{(P(Q) - c) \cdot q_2\}$$

$$\max_{q_2} \{(a - b(q_1 + q_2) - c_2) \cdot q_2\}$$

FOC:

$$\frac{d}{dq_2}((a - b(q_1 + q_2) - c_2) \cdot q_2) = 0$$

$$a - 2bq_2 - bq_1 - c_2 = 0$$

$$q_2^* = \frac{a - c_2}{2b} - \frac{q_1}{2}$$

q_2^* is the best response function for firm 1 given any fixed quantity choice by firm 2. We can rewrite this:

$$q_2^* = B_2(q_1) = \left\{ \begin{array}{lll} \frac{a-c_2}{2b} - \frac{q_1}{2} & \text{if} & q_1 \leq a - c_2 \\ 0 & \text{if} & q_1 > a - c_2 \end{array} \right\}$$

Symmetrically for firm 1, when it is a strategically/profit maximizing firm.

$$q_1^* = B_1(q_2) = \left\{ \begin{array}{lll} \frac{a-c_1}{2b} - \frac{q_2}{2} & \text{if} & q_2 \leq a - c_1 \\ 0 & \text{if} & q_2 > a - c_1 \end{array} \right\}$$

In an equilibrium, $q_1^* = B_1(q_2^*)$ and $q_2^* = B_2(q_1^*)$: each firm is best responding to its opponent. In general, you have to solve both best response function simultaneously. Start with $q_2^* = B_1(q_1^*)$.

$$q_2^* = \frac{a-c_2}{2b} - \frac{1}{2}\left(\frac{a-c_1}{2b} - \frac{q_2^*}{2}\right)$$

$$q_2^* = \frac{1}{4}q_2^* - \frac{1}{4b}(a-c_1) + \frac{1}{2b}(a-c_2)$$

$$\frac{3}{4}q_2^* = \frac{a+c_1-2c_2}{4b}$$

$$q_2^* = \frac{a+c_1-2c_2}{3b}$$

With symmetric costs the equilibrium is: $(q_1^*, q_2^*) = \left(\frac{(a-c)}{3b}, \frac{(a-c)}{3b}\right)$.

However, with symmetric costs, there are several short cuts. Starting from the best response function of one firm,e.g., firm 2,

$$q_2^* = \frac{a-c}{2b} - \frac{q_1^*}{2}$$

you know that the equilibrium is symmetric, i.e., $q_1^* = q_2^*$:

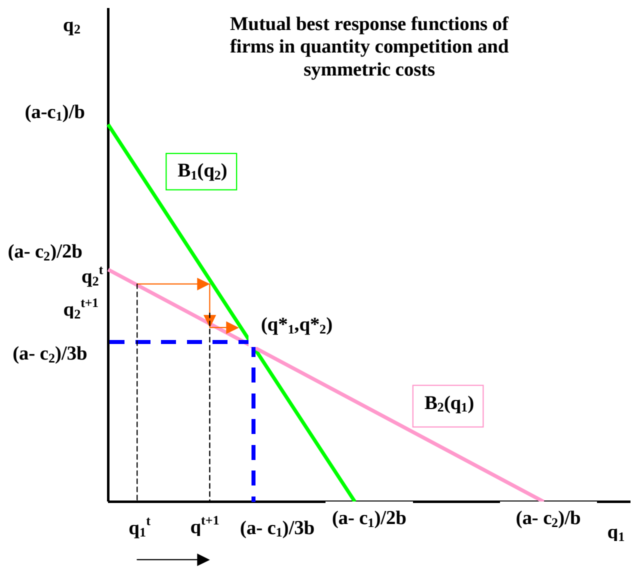
$$q_2^* = \frac{a-c}{2b} - \frac{q_1^*}{2}$$

Therefore again,

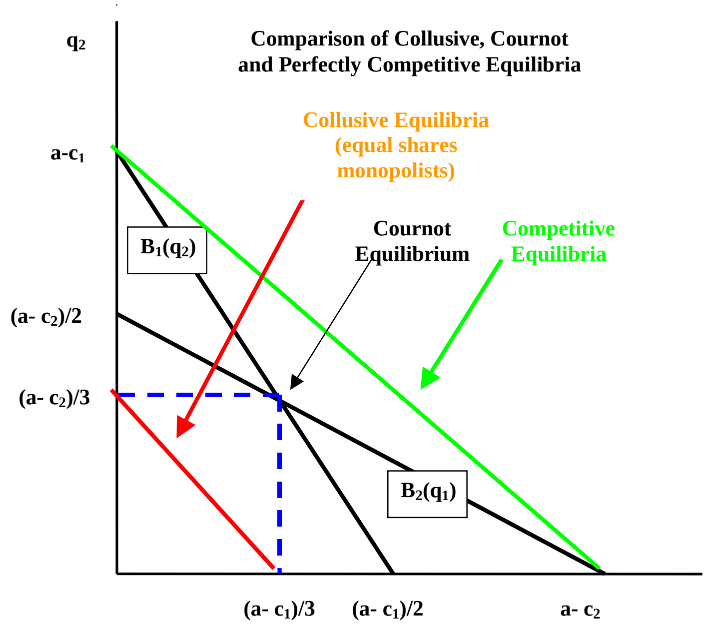
$$(q_1^*, q_2^*) = \left(\frac{(a-c)}{3b}, \frac{(a-c)}{3b}\right)$$

To understand what is going on, lets return our residual demand graph above. Recall that firm 2 was under producing given firm 1 produced q_1 when firm 2 responding by producing the monopoly quantity. Now, given firm 2 chose to produce q_2, firm 1 will have a residual demand, but larger than what it would otherwise, because of firm 2's "under" production. When firm 1 choose its profit maximizing quantity, it will be under producing as well, but facing a greater residual demand. Hence, by the same logic for ever turn t, $q_1^{t+1} > q_1^t$ and $q_2^{t+1} > q_2^t$. (Try to construct a numerical example to see how this works, e.g., $Q = 10 - P$, $MC_1 = MC_2 = 2, q_1 = 1$)

In this graph, you see that each firm is best responding to his opponent given the residual demand that his opponent is leaving it.



The equilibrium is where each player is playing a best response to his opponent. Therefore, we can find the equilibrium by plugging the best response of one player into that of the other.



1.5 Asymmetric Costs

When costs are not the same $c_1 \neq c_2$.

Quantity supplied by firm 1 given that of firm 2	$q_1 = B_1(q_2) = \frac{a-c_1}{2b} - \frac{1}{2}q_2$
Quantity supplied by firm 2 given that of firm 1	$q_2 = B_2(q_1) = \frac{a-c_2}{2b} - \frac{1}{2}q_1$
equilibrium quantities as function of demand and prices	$q_1^c = \frac{a-2c_1+c_2}{3b}, q_2^c = \frac{a-2c_2+c_1}{3b}$
Equilibrium Price and Quantity for the market	$P = a - bQ^c = \frac{a+c_1+c_2}{3}, Q^c = \frac{2a-c_1-c_2}{3b}$

$$\pi_i^c = (p^c - c_i)(q_i^c) - FC = \left(\frac{a-c_i-c_j}{3} - c_i\right)\left(\frac{a-2c_i+c_j}{3b}\right) - FC$$

$$\pi_i^c = \frac{(a-2c_i+c_j)^2}{9b} - FC = b(q_i^c)^2 - FC$$

Note the similarity with the expression for monopoly profits: $\pi_m = bQ^2 - FC$.

In equilibrium: More efficient firm gets larger share of market, but extra DWL from production by high cost firm.

1.6 N-firms with Symmetric Costs

We just substitute $q_1 + q_2$ in the preceding section with $\sum_{i=1}^{N} q_i$.

Then firm 1 maximizes profits:

$$\pi_1 = \max_{q_1}\{p(Q)q_1 - cq_1\} = \left[a - b\left(\sum_{i=1}^{N} q_i\right)\right]q_1 - cq_1$$

Substituting in $Q_{-1} = \sum_{i=2}^{N} q_i$ into the Cournot best Response function above:

$$B_1(Q_{-1}) = \frac{a-c}{2b} - \frac{1}{2}Q_{-1}$$

In the symmetric equilibrium, $q_1^* = q_2^* = q_3^* ..... = q^*$. Therefore,

$$Q_{-1}^* = \sum_{i=2}^{N} q_i^* = (N-1)\cdot q^*$$

Assuming symmetric costs:

$$q^* = \frac{a-c}{2b} - \frac{1}{2}(N-1)\cdot q^*$$

Solving, we get the following,

Each firm produces	$q^* = \frac{a-c}{(N+1)b}$
In aggregate they produce	$Q^* = Nq^* = \frac{N}{(N+1)} \frac{(a-c)}{b}$
The market price	$p^* = a - bQ^* = a - \frac{N}{(N+1)} \frac{(a-c)}{1}$,
Each firm's profit	$\pi_i^* = \frac{N}{(N+1)^2} \frac{(a-c)^2}{b} - FC = b\left(q^*\right)^2 - FC$

The following helps to understand how imperfect competition relates to CE from econ 100, but isn't likely to show up on an exam.

If there are fixed costs, then at some N, no more firms will want to enter, because they cannot recover their fixed costs. But, if FC=0, then N can go to infinity.

To get perfect competition, let $N \to \infty$.

Then each firm produces virtually nothing:

$$\lim_{N \to \infty} q^* = 0$$

But in aggregate they produce:

$$\lim_{N \to \infty} Q^* = \frac{a-c}{b}$$

and they charge cost:

$$p^e = \lim_{N \to \infty} \left\{ a - \frac{N}{(N+1)} \frac{(a-c)}{1} \right\} = \left\{ a - \frac{(a-c)}{1} \right\} = c$$

In the competitive equilibrium, as the number of firms go to infinity, price=MC=c.