Monopoly Profit Maximization and Sunk Costs

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1 Monopoly

We can maximize profit by choosing the quantity that maximizes profit:

$$\pi = \max_{Q} TR(Q) - TC(Q) = \max_{Q} P(Q) * Q - C(Q)$$

for which the optimality condition is:

$$MR(Q^*) = MC(Q^*))$$

1.1 Example

Suppose that Demand is defined by $Q = 5 - \frac{1}{2}P$, and supply is defined by $TC(Q) = 1 + Q^2$. Then on the cost side,

$$\begin{array}{l} \text{MC}(Q) = 2Q \\ \text{AC}(Q) = \frac{(1+Q^2)}{Q} \\ \text{AFC}(Q) = \frac{1}{Q} \\ \text{AVC}(Q) = Q \end{array}$$

To find the marginal revenue (MR), we need total revenue (TR). Since TR is a function of Q, we also need the inverse demand curve.

$$P = 10 - 2Q$$

$$TR(Q) = P \cdot Q = (10 - 2Q) \cdot Q$$

$$MR(Q) = \frac{d}{dQ}((10 - 2Q) \cdot Q) = 10 - 4Q$$

The Q that maximizes profits is defined by the FOC:

$$MR(Q^*) = MC(Q^*))$$

Implying for this special case:

$$10 - 4Q^* = 2Q^*$$

$$10 = 6Q^*$$

$$Q_M^* = \frac{5}{3}$$

The resultant price would be:

$$P_M^* = 10 - 2\left(\frac{5}{3}\right) = \frac{20}{3}$$

The competitive price would be determined by the intersection of demand and the supply curve (written as a function of Q):

$$P\left(Q^*\right) = MC\left(Q^*\right)$$

$$10 - 2Q^* = 2Q^*$$

$$Q_C^* = \frac{5}{2} > \frac{5}{3} = Q_M^*$$

$$MC\left(Q_M^*\right) = 2\frac{5}{3}$$

Now, lets find: Total Revenue (TR), Average Revenue (AR), Marginal Revenue (MR), Average Costs (AC), Average Variable Costs (AVC), Marginal Cost (MC), Elasticity (ε), the profit maximizing quantity, profit π, by filling in the following table.

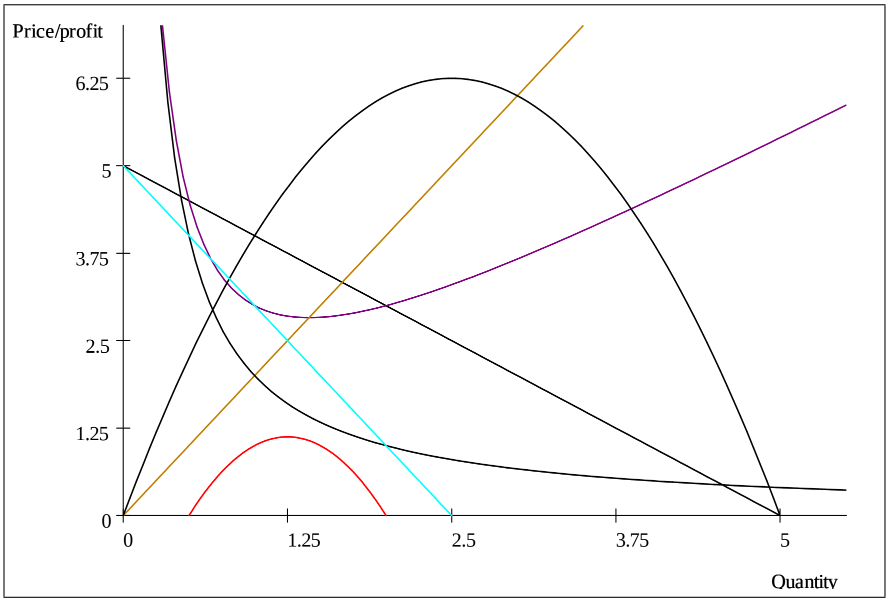
Q	**P**	**TR**	**AR**	**MR**	ε	**TC**	**AFC**	**AVC**	**AC**	**MC**	π
0	5	0	5	5	$-\infty$	1	∞	0	∞	0	-1
1	4	4	4	3	-4	2	1	1	2	2	2
2											
3											
4											
5											

Note where:

1. the elasticity is equal to 1,
2. the TR is maximized,
3. where profit is maximized,
4. where demand meets supply,

5. where AVC is minimized,

Please plot the table. The graph (Please label it.)



Red=
Green=
Blue=
Brown=
Purple=

1.2 Minimun Efficient Scale

Minimum efficient scale (MES) is the Q at which long run costs are minimized. Using the first order condition for a minimal;

$$MES(Q) = \frac{d}{dQ} AC(Q) = \frac{d}{dQ} \frac{\left(1+Q^2\right)}{Q}$$

$$= \frac{d}{dQ}\left(1+Q^2\right) \cdot \frac{1}{Q} + \frac{d}{dQ}\left(\frac{1}{Q}\right)\left(1+Q^2\right) = 0$$

$$= 2Q \cdot \frac{1}{Q} - \frac{1}{Q^2}\left(1+Q^2\right) = 2 - \frac{1}{Q^2}\left(Q^2+1\right) = 0$$

$$2 - \frac{1}{Q^2}\left(Q^2+1\right) = 0$$

The solutions are : $1, -1$. But Q must be positive so the MES is 1. (See graph above.)

1.3 General Form of Monopoly

1.3.1 Linear Costs

Suppose the inverse demand is $P = a - bQ$ and FC=fixed costs. (Note that "a" and "b" mean something completely different than in $Q = a - bP$.)

Then the profit maximization condition is:

$$\pi = \max_{Q} \{TR(Q) - TC(Q)\}$$

We can write it as:

$$\pi = \max_{Q} \{(P(Q) - c) \cdot Q - FC\}$$

$$\pi = \max_{Q} \{(a - bQ - c) \cdot Q - FC\}$$

FOC:

$$MR(Q) = a - 2bQ^* = c = MC(Q)$$

$$Q^* = \frac{a-c}{2b}$$

You can often check or completely avoid calculations by remembering this. To find the expression for profit, substitute the optimal quantity back in to the profit function.

$$\pi = \left(a - b\left(\frac{a-c}{2b}\right) - c\right) \cdot \left(\frac{a-c}{2b}\right) - FC$$

$$\pi = b\left(\frac{a-c}{2b}\right)^2 - FC = b \cdot Q^2 - FC$$

1.3.2 Quadratic Costs

Suppose, however, that TC has a quadratic term: $TC(Q) = cQ + dQ^2 + FC$. Then,

With linear demand:

$$\pi = \max_{Q} \left\{(a - bQ) \cdot Q - \left(cQ + dQ^2 + FC\right)\right\}$$

FOC:

$$a - 2bQ = c + 2dQ$$

$$Q^* = \frac{a-c}{2(b+d)}$$

Now the profit is:

$$\pi = \max_{Q} \left\{ (a - bQ) \cdot Q - \left(cQ + dQ^2 + FC \right) \right\}$$

$$\pi = (a - bQ^*) \cdot Q^* - \left(cQ^* + dQ^{*2} + FC \right)$$

$$\pi = ((a - c) - (b + d) Q^*) Q^* - FC$$

$$\pi = \left((a - c) - (b + d) \left(\frac{a - c}{2(b + d)} \right) \right) \cdot \left(\frac{a - c}{2(b + d)} \right) - FC$$

$$\pi = \left((a - c) - \left(\frac{a - c}{2} \right) \right) \cdot \left(\frac{a - c}{2(b + d)} \right) - FC$$

$$\pi = (b + d) \cdot \left(\frac{a - c}{2(b + d)} \right)^2 - FC$$

$$\pi = (b + d) \cdot Q^{*2} - FC$$

2 Sunk Costs

A fixed cost can be broken up in to a sunk (unavoidable) and unsunk (avoidable) component. AC=average total costs includes both. AAC=average avoidable costs includes variable and unsunk fixed costs. Lets talk through an example of sunk costs.

A cost is sunk if its unrecoverable,i.e., its there no matter what you do. The time cost in education is a sunk cost. So, suppose you graduated from UCD with a degree in Marxist economics and had trouble finding a job. No matter what you do after you graduate from UCD, you can't get your 4 years back. A cost is not sunk if you could do something to recover it,e.g., the cost of a textbook on Marxist economics is not sunk before the drop deadline. Part of the cost of a textbook is sunk after the drop deadline.

2.0.3 Sunk Cost and Bargaining

Suppose further that some English department (they are the only departments that regularly teach Marxist economics) approached you and offered you $25,000/year to teach Marxist economics. You balk at first and say, "That's ridiculous! I could have taken industrial organization courses and gotten a job for $50,000/year!" But, then they say, "That doesn't matter because that cost is sunk. You are lucky we didn't offer you $1 a year. Take it or leave it! Your outside option is the street, not $50,000/year."

If a cost is sunk, there is nothing you can do to recover it. Thus, it is irrelevant to whether you accept or reject the offer, since the cost will be there, regardless of whether or not you accept the offer. Sunk costs are irrelevant for calculating economic profits–once they are sunk.

In bargaining situations, the lowest offer your opponent will accept is her outside option from rejection. It's probably easiest to understand the working of sunk costs in bargaining situations by explicitly calculating a player's payoff from rejecting an offer.

3 Returns to Scale and Scale Economies (optional)

Returns to Scale measures the proportional increase in output from a given an increase in inputs.

Let $0 \leq \alpha$ then the production function $f(K, L)$ has:

Returns to Scale	General Expression	Cobb Douglas Example
Increasing Returns to Scale	$f(\alpha K, \alpha L) > \alpha f(K, L)$	$(\alpha K)^{\frac{3}{4}} \cdot (\alpha L)^{\frac{1}{2}} > \alpha \left(K^{\frac{3}{4}} \cdot L^{\frac{1}{2}}\right)$
Decreasing Returns to Scale	$f(\alpha K, \alpha L) < \alpha f(K, L)$	$(\alpha K)^{\frac{1}{4}} \cdot (\alpha L)^{\frac{1}{2}} < \alpha \left(K^{\frac{1}{4}} \cdot L^{\frac{1}{2}}\right)$
Constant Returns to Scale	$f(\alpha K, \alpha L) = \alpha f(K, L)$	$(\alpha K)^{\frac{1}{2}} \cdot (\alpha L)^{\frac{1}{2}} = \alpha \left(K^{\frac{1}{2}} \cdot L^{\frac{1}{2}}\right)$

On the other hand, single product **scale economies** $s = \frac{AC(Q)}{MC(Q)}$ tell us whether average costs are declining ($s > 1$), increasing ($s < 1$) or constant ($s = 1$) at a certain q. (Relate this to the graph of costs.)

For the example above:

$$s = \frac{AC(Q)}{MC(Q)} = \frac{\frac{1}{Q} + Q}{2Q} = \frac{1}{2Q^2} + \frac{1}{2}$$

Find the boundary Q :

$$s = \frac{1}{2Q^2} + \frac{1}{2} = 1$$

	General	Example
Economies of Scale	$s > 1$	if $Q < 1$
Diseconomies of Scale	$s < 1$	if $Q > 1$
Constant Economies of Scale	$s = 1$	if $Q = 1$

If a firm makes zero profits,i.e.,$MC = P$, then s measures the ratio of costs to revenues.

4 Marginal Costs and Average Total Cost (optional)

Here is a proof that when MC<ATC then, ATC is decreasing. If MC>ATC, then ATC is increasing. The slope of ATC is:

$$\frac{d}{dQ}ATC(Q) = \frac{d}{dQ}\left(\frac{1}{Q}(FC + VC(Q))\right)$$

$$= -\frac{1}{Q^2}(FC + VC(Q)) + \left(\frac{d}{dQ}(FC + VC(Q))\right)\frac{1}{Q}$$

$$= -\frac{1}{Q^2}(FC + VC(Q)) + (MC(Q))\frac{1}{Q}$$

The last expression is greater than zero iff:

$$MC(Q) > \frac{1}{Q}(FC + VC(Q))$$

$$MC(Q) > ATC(Q)$$

Hence, the slope of ATC is greater than zero iff MC>ATC. Similarly, the slope of ATC is less than zero if $MC < ATC$. Note that this also means that if

$$\frac{d}{dQ}ATC(Q) = 0$$

then

$$MC(Q) = ATC(Q)$$

This is the mathematical expression of the fact that MC=ATC at the minimum of ATC. You can use this instead of the product rule.