Bundling

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1 General Discussion of Bundling

There are three separate considerations for bundling:

1. Consumer Surplus Maximization (Incentive Compatibility)
2. Producer Profit Maximization
3. Social surplus Maximization (Minimization of dead weight loss)

1.1 Firms' profit:

Suppose the P's are prices for goods and c's are costs for 1 and 2. Then the firm's profit will be:

Sell	**Profit**
Good 1	$P_1 - c_1$
Good 2	$P_2 - c_2$
Bundle	$P_{12} - c_1 - c_2$

1.2 Consumer Surplus (CS)

CS must be greater than zero for the consumer to want to buy.

Sell	**Surplus**
Good 1	$u_1 - P_1 \geq 0$
Good 2	$u_2 - P_2 \geq 0$
Bundle	$u_1 + u_2 - P_{12} \geq 0$

To see why bundling works consider the following simple though abstract model.

Goods\Valuations	**Customer A**	**Customer B**
Good 1	b	a
Good 2	c	d

1.3 Pure Bundling Strategies

Here, since the consumer can only buy the bundle or not buy, the incentive compatibility (IC) condition is, the consumer prefers the bundle to nothing.

$$u_1 + u_2 - P_{12} \geq 0$$

Since the firm is not going to leave surplus it doesn't want to extract, it will set $P_{12} = u_1 + u_2$. Thus, the strategies and profits of the firm are:

	Pure Bundling Strategies	**Firm Profits**
1	**sell the bundle to both customers**	$2 \cdot \min\{b+c, a+d\} - 2(c_1 + c_2)$
2	**sell the bundle to one customer**	$\max\{b+c, a+d\} - (c_1 + c_2)$

1.4 Pure Component Sales Strategies

Here, the IC condition for the consumer when he can only buy the component is, the consumer prefers the component to nothing.

Good 1	$u_1 - P_1 \geq 0$
Good 2	$u_2 - P_2 \geq 0$

The firm will again set prices so to extract all surplus.

$P_2 = u_2$
$P_2 = u_1$

If there is more than one type of consumer, the firm must set the price to the valuation of lowest consumer that it wants to sell to.

	Pure Components Sales Strategies	**Firm Profits**
3	sell both to both customers	$2 \cdot \min\{a, b\} + 2 \cdot \min\{c, d\} - 2(c_1 + c_2)$
4	sell one to both and the other to one	$\max\{a, b\} + 2 \cdot \min\{c, d\} - (c_1 + 2c_2)$
5	sell one to both and the other to one	$2 \cdot \min\{a, b\} + \max\{c, d\} - (2c_1 + c_2)$
6	sell one to each	$\max\{a, b\} + \max\{c, d\} - (c_1 + c_2)$

You have to compare all of both forms to each other to know which is best. This makes sense because the most you can charge when you sell to both customers for the bundle is the sum of the valuations of the lowest demand customers. The most you can charge when you don't bundle, when you sell to both, is the minimum of what each is willing to pay for each good.

1.5 When is Pure Bundling (PB) More Profitable than Component Sales (CPS)?

If we ignore costs, then PB is always better than CPS when there is negative correlation of consumer valuations values–each consumer is willing to pay more for one good than the other consumer. Suppose $b > a$, $d > c$:

Goods\Valuations	**Customer 1**	**Customer 2**
Good 1	b	a
Good 2	c	d

Then the price of the bundle is greater than the sum of the prices:

$$2 \cdot \min\{b + c, a + d\} > 2 \cdot \min\{a, b\} + 2 \cdot \min\{c, d\}$$

if

$$2 \cdot \min\{b + c, a + d\} > 2a + 2c$$

if

$$2 \cdot (b + c) > 2a + 2c \text{ since } b > a$$
$$2 \cdot (a + d) > 2a + 2c \text{ since } d > c$$

The same inequality holds if the correlation went the other way $b < a$, $d < c$.

1.6 Mixed Bundling

With mixed bundling, the alternative is not nothing, but either the component or the bundle, since both are available.

	Goal of Firm	**Incentive Compatibility**
1	sell consumer the bundle	$u_1 + u_2 - P_{12} \geq \max\{u_1 - P_1, u_2 - P_2\}$
2	sell consumer good 1	$u_1 - P_1 \geq \max\{u_1 - P_1, u_1 + u_2 - P_{12}\}$
3	sell consumer good 2	$u_2 - P_2 \geq \max\{u_1 + u_2 - P_{12}, u_2 - P_2\}$

For consumer A this would mean:

	Goal of Firm	Incentive Compatibility
1	sell consumer A the bundle	$b + c - P_{12} \geq \max\{b - P_1, c - P_2\}$
2	sell consumer A good 1	$b - P_1 \geq \max\{b - P_1, b + c - P_{12}\}$
3	sell consumer A good 2	$c - P_2 \geq \max\{b + c - P_{12}, c - P_2\}$

Simplifying:

	Goal of Firm	Incentive Compatibility
1	sell consumer A the bundle	$b + c - P_{12} \geq \max\{b - P_1, c - P_2\}$
2	sell consumer A good 1	$b - P_1 \geq \max\{b - P_1, b + c - P_{12}\}$
3	sell consumer A good 2	$c - P_2 \geq \max\{b + c - P_{12}, c - P_2\}$

(Table A)

This problem may look complicated, but fortunately, there is a way to simplify the goal of the firm. Mixed bundling allows the firm to sell only component that raise more revenue than they cost the firm. Mixed bundling is therefore always as profitable as either pure bundling or component sales. So, when faced with a mixed bundling problem, look first for the consumer whose valuation of the good is less than the cost of producing that good. Sell everyone else the bundle. Set your prices so that no one wants to deviate from that arrangement. Often, mixed bundling is also better than pure bundling even if all consumers value the goods more than they cost to produce, because, often, the part of the consumer valuation beyond the bundle price is unextractible. See figure 1. Some times, if costs are high component sales can be even more profitable than pure bundling. See figure 2.

2 Worked out Example

1. Jack Jacques Rouseau (JJ), a famous french action adventure (AA) film maker is contemplating the release of 2 of his films on DVD: Cafe Mayhem and Subway Mayhem. The AA crowd has two types: those who get excited by cafe scenes (Talkers) and those who like subway scenes (Riders). His MC for making DVD's is 2. JJ has hired you as his economic consultant. He thinks that there are equal number of Talkers and Walkers, and that their valuations are in the following table. He wants to know if he should release Cafe Mayhem and Subway Mayhem in a pure bundle or separately, or in mixed bundle.

You can use your common sense to answer this question, but it may help to spell out all of the steps using what I have worked out in the General Discussion above.

	Talkers	Riders
Cafe Mayhem	b=3	a=1
Subway Mayhem	c=2	d=4

If costs are zero then the profits for the firm with each pricing strategy are:

	Pure Bundling Strategies	JJ's Profits
1	**sell the bundle to both customers**	$2 \cdot \min\{2+3, 1+4\} = 10$
2	**sell the bundle to one customer**	$\max\{2+3, 1+4\} = 5$

There are many forms of component sales. An exhaustive list of all the forms:

	Pure Components Sales Strategies	JJ's Profits
1	sell both DVDs to both customers	$2 \cdot \min\{1,2\} + 2 \cdot \min\{3,4\} = 2 \cdot 1 + 2 \cdot 3 = 8$
2	sell one to both and the other to one	$\max\{1,2\} + 2 \cdot \min\{3,4\} = 2 + 2 \cdot 3 = 8$
3	sell one to both and the other to one	$2 \cdot \min\{1,2\} + \max\{3,4\} = 2 \cdot 1 + 4 = 6$
4	sell one to each	$\max\{1,2\} + \max\{3,4\} = 2 + 4 = 6$

Suppose now that Cafe Mayhem costs 2. Bundling would be most profitable, if JJ didn't have to sell Cafe Mayhem to the Riders. JJ's profits would then be $P_{12} = 5$ from selling the bundle to Talkers and $P_2 = 4$ from selling Subway Mayhem to the Riders only. Now, you need to check the ICs and set P_1 so that neither the Talkers nor the Riders want to deviate. Substituting the values for the ICs in Table A.

	Goal of Firm	Incentive Compatibility
1	sell Talkers the bundle	$3+2-P_{12} \geq \max\{3-P_1, 2-P_2\}$
2	sell Riders Subway Mayhem	$4-P_2 \geq \max\{1-P_1, 1+4-P_{12}\}$

	Goal of Firm	Incentive Compatibility
1	sell Talkers the bundle	$3+2-5 \geq \max\{3-P_1, 2-4\}$
2	sell Riders Subway Mayhem	$4-4 \geq \max\{1-P_1, 1+4-5\}$

We see that $P_1 \geq 3$ for this scheme to work.

2.1 More General Examples

Once you understand what is going on, graphs can be very helpful.

In the following, I show the profits from component sales, pure bundling and mixed bundling. The valuations or the sum of the valuations, if it's a bundle price, are positive numbers. In each case, there are 4 consumers with valuations of $\{(3,9), (4,6), (6,4), (9,3)\}$ .The first number stands for the valuation of good 1 and the 2nd component stands for the valuation of good 2. The negative numbers stand for MC.

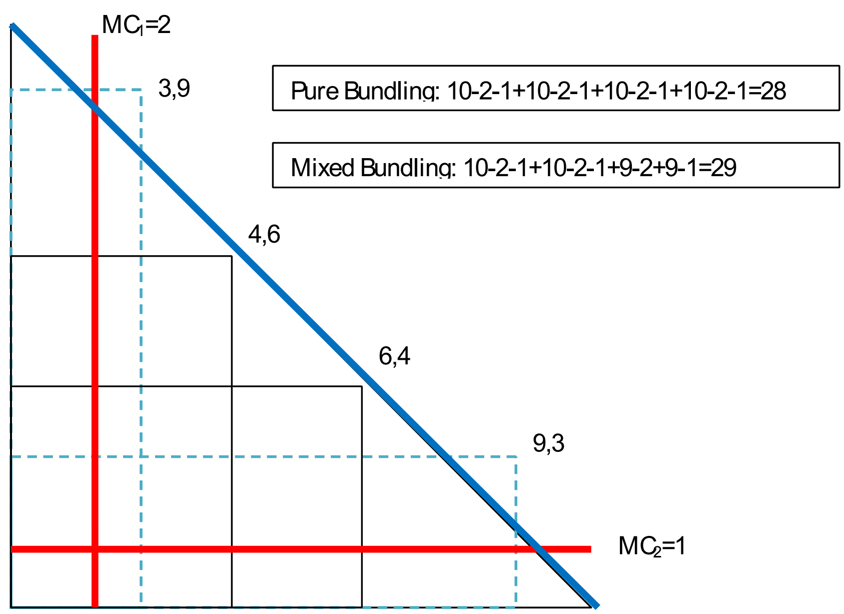

Fig. 2: Mixed Bundling is better than Pure Bundling (Bundle price in blue)

Figure 1

If marginal costs are high enough, then component sales is better than pure bundling and any nontrivial form of mixed bundling.

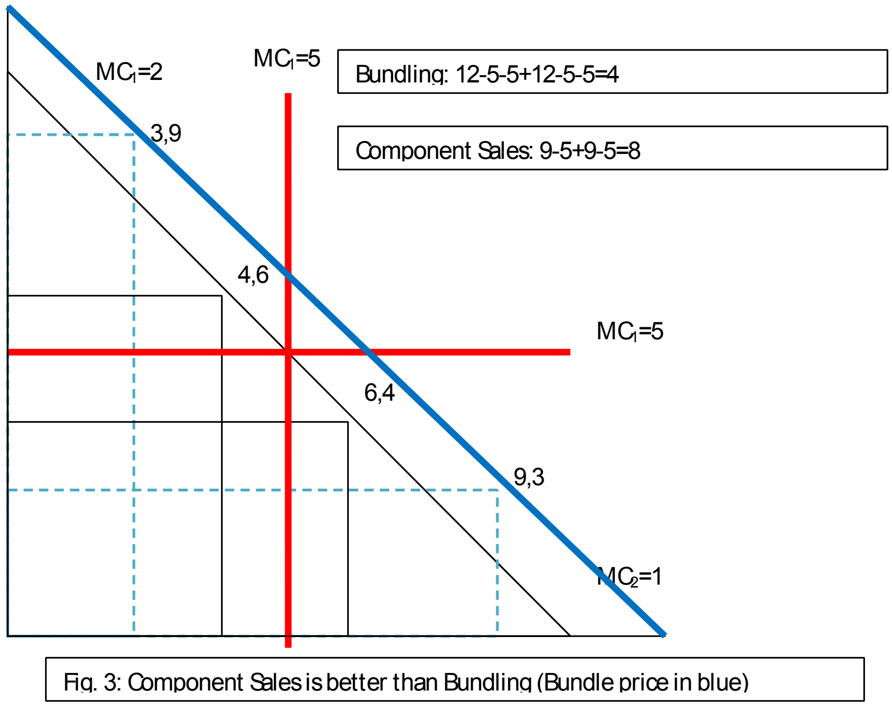

Figure 2

Pure bundling like other forms of monopoly pricing creates DWL. So does mixed bundling. You can see this by drawing in the CE, where $P_i = MC_i$ for goods $i = 1, 2$.Whatever price is not these creates DWL. However, unlike other forms of monopoly price, the DWL can result in too much production–when the bundle purchaser values only values one component of the bundle.