Ph. D. Preliminary exam in Industrial Organization, July 2003

Answers to questions 1 and 2

1. If a manager is offered a profit contract we call her a type P manager, while if she is offered a revenue contract we call her a type R manager. Solve the game backwards. There are four subgames: one where both managers are of type P, one where they are both of type R and two where one is of type P and the other of type R.

(1) Both managers are of type R. Then each manager will try to maximize the revenue of its firm. The game involves the following payoff functions:

$$R_1(q_1, q_2) \to q_1 \cdot (60 - q_1 - q_2) \qquad R_2(q_1, q_2) \to q_2 \cdot (60 - q_1 - q_2)$$

The Nash equilibrium of this game is given by $q_1 = q_2 = 20$ with a corresponding price of 20. The net income of the owner of the firm is $0.8\,[20\,(20) - 12\,(20) - 80] = 64$.

(2) Both managers are of type P. Then each manager will try to maximize the profit of its firm. The game involves the following payoff functions:

$$\Pi_1(q_1, q_2) \to q_1 \cdot (60 - q_1 - q_2) - 12 \cdot q_1 \qquad \Pi_2(q_1, q_2) \to q_2 \cdot (60 - q_1 - q_2) - 12 \cdot q_2$$

The Nash equilibrium of this game is given by $q_1 = q_2 = 16$ with a corresponding price of 28. The net income of the owner of the firm is $0.8\,[28\,(16) - 12\,(16) - 80] = 140.8$.

(3) One manager is of type P, the other is of type R. Then one manager will try to maximize the profit of its firm and the other will maximize revenue. The game involves the following payoff functions:

$$\Pi_1(q_1, q_2) \to q_1 \cdot (60 - q_1 - q_2) - 12 \cdot q_1 \quad \text{and} \quad R_2(q_1, q_2) \to q_2 \cdot (60 - q_1 - q_2)$$

The Nash equilibrium of this game is given by $q_1 = 12$, $q_2 = 24$. The net income of the owner of firm 1 is 51.2 and that of firm 2 is 166.4.

Thus the first-stage reduces to:

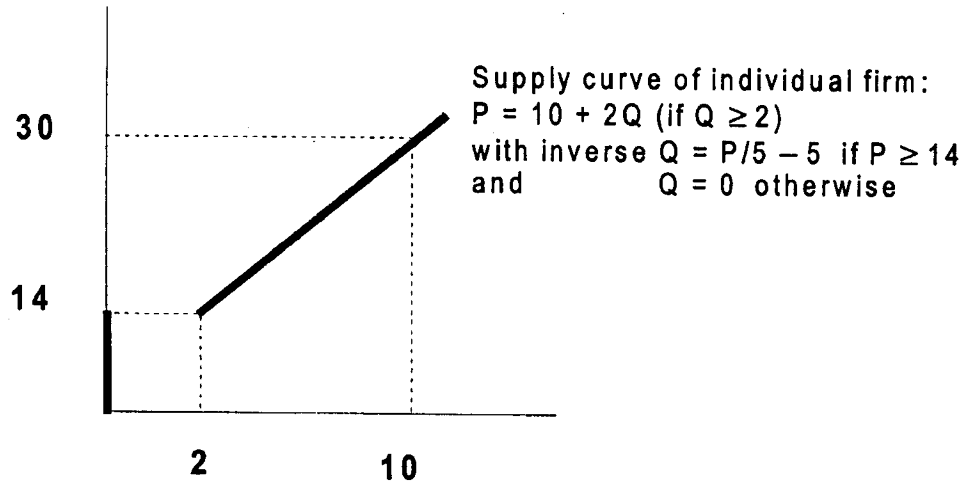
		Ms. Lazyhead P	Ms. Lazyhead R
Mr. Busybody	P	140.8, 140.8	51.2, 166.4
	R	166.4, 51.2	64, 64

Hence there is a unique subgame perfect equilibrium, where both choose a type R manager (this is a prisoner's dilemma situation: they would both be better off if they both chose a manager of type P).

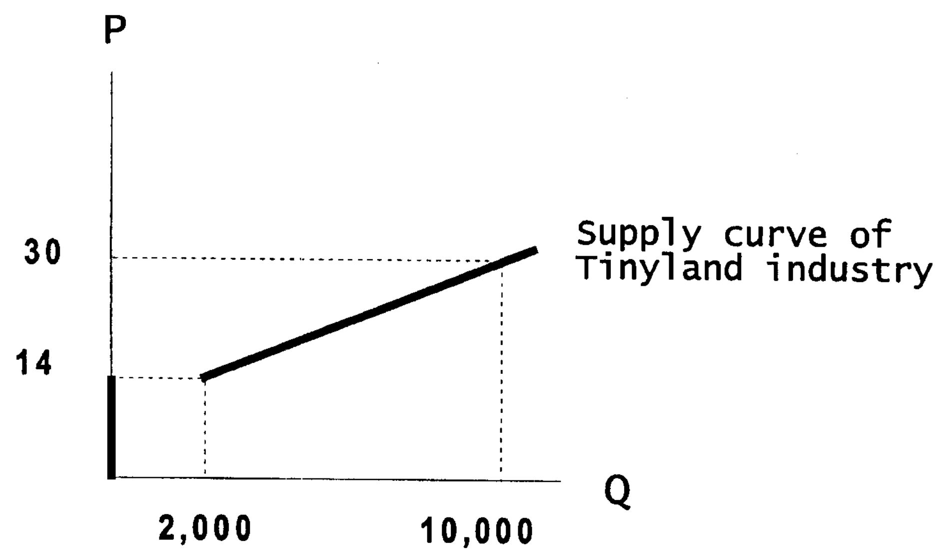
2. (a) $MC = 10 + 2Q$. Since there is a sunk cost equal to 6, the supply curve of a Tinyland firm coincides with that portion of its MC curve that starts at the quantity at which revenue minus variable cost is at least 4 (the recoverable portion of the fixed cost: 10 – 6). To calculate this quantity (taking into account that P = MC):

$$(10 + 2Q)Q - (10Q + Q^2) \geq 4$$

which gives $Q^2 \geq 4$, i.e. $Q \geq 2$ hence $P \geq 14$.



(b) The supply curve of the Tinyland industry is the horizontal sum of 1000 individual supply curves, thus



[to obtain note that it must be of the form a+bQ with a + 2000b = 14 and a + 10,000b = 30. Thus $P = 10 + Q/500$ with inverse $S(P) = 500P - 5{,}000$].

(c) If $P<14$, the dominant firm gets all the market and can sell $Q=30{,}000 - 200P$, while if $P \geq 14$, the dominant firm can sell

$$[30{,}000 - 200\,P] - [500\,P - 5{,}000] = 35{,}000 - 700\,P$$

(d) A monopolist sets $MR = 150 - \frac{Q}{100}$ equal to MC = 2. Solving $150 - \frac{Q}{100} = 2$ gives Qb=b14,800 and Pb=b76.

(e) If LCN sets a price greater than 14 its profit function is:

$$\pi(P) = (35000 - 700P)P - [250 + 2\,(35000 - 700P)].$$

Setting $d\pi/dP = 0$ we obtain

$$35{,}000 - 1{,}400\,P + 1{,}400 = 0$$

which gives P = 26 and Q = 16,800 (for LCN) and π = 403,950. Alternatively, LCN could set P slightly lower than 14 (with corresponding quantity slightly greater than 27,200) and get a profit of 326,150. Since this is less, the equilibrium will be P=26, with LCN producing 16,800 units and the Tinyland industry producing 8,000 units.

(f) The monopoly solution is P = 76, Q = 14,800 with profits of π = 1,094,950, consumer surplus of CS = 547,600 and totals surplus of TS = 1,642,550.
The price leadership solution is P = 26, total output of Q = 24,800 (of which 16,800 supplied by LCN), profits for LCN of π = 403,950 and consumer surplus of CS = 1,215,200 and totals surplus TS = 1,618,400. Thus the elimination of trade barriers has *reduced social welfare in Monoland*: the gain in consumer surplus is outweighed by the loss in profits to the home firm (LCN).

(g) Splitting LCN would give rise to a Cournot duopoly with profit functions

$$\Pi_1(q_1, q_2) \to q_1 \cdot \left(150 - \frac{1}{200} \cdot q_1 - \frac{1}{200} \cdot q_2\right) - 250 - 2 \cdot q_1 \quad \text{and}$$

$$\Pi_2(q_1, q_2) \to q_2 \cdot \left(150 - \frac{1}{200} \cdot q_1 - \frac{1}{200} \cdot q_2\right) - 250 - 2 \cdot q_2$$

The Cournot equilibrium is given by $q_1 = q_2 = \frac{29600}{3} = 9866.67$, $P = \frac{154}{3} = 51.33$, $\pi_1 = \pi_2 = \frac{4378550}{9} = 486505.56$, $CS = \frac{8761600}{9} = 973511.11$, $TS = \frac{17518700}{9} = 1946522.22$.

Section B

1 Question 3

This question deals with Ellison (Rand '94) and Borenstein and Shepard (Rand '96). Both of these papers empirically test the validity of certain theoretical models of collusion. Ellison tests two "classes" of models, the Rotemberg/Saloner/Haltiwanger/Harrington model and the Green and Porter model. Borenstein and Shepard test the Rotemberg/Saloner/Haltiwanger/Harrington model.

1.1 Set up

a. Briefly describe the Rotemberg-Saloner and Green and Porter models of tacit collusion. Pay particular attention to what is known by the firms and the behavior of demand. Also, characterize the movements of price in the market.

Sketch of Answer:

The Rotember Saloner model is a model of collusion under demand fluctuations and price competition. In each period firms observe the shock to demand and set prices such that no firm has an incentive to collude. The shocks to demand are iid; this implies that the expected future profits from colluding one period from now never changes. Therefore, the punishment from cheating is independent of the current demand shock. Because, at a given collusive price level, the profits from cheating are increasing in the shock (and the punishment is independent of a shock) higher shocks nessecitate reducing the collusive price. This leads to counter-cyclical pricing.

The Green and Porter model of collusion is a model of collusion under demand uncertainty and the shocks to demand not directly observed by the firms. Furthermore, the strategies of other firms are not observed. This creates an environment where prices may revert to non-cooperative levels. Consider the case where a firm observes a low market clearing price. The firm is unsure whether this is the result of a negative demand shock or cheating, since shocks and competitor strategies are not observed. To discourage cheating, it is necessary for the firm to revert to non-cooperative behavior if the market clearing price falls below some "trigger" level. In equilibrium, cheating never ocurrs, nevertheless firms must revert to non-cooperative behavior to discourage the possibility of cheating. Therefore, within a Green and Porter model periods of "price wars" will exist.

b. Discuss the main differences between the RS/HH and GP models. Is the nature of "price wars" the same in the two classes of models? If not, how do price wars differ and what within the theoretical models generates this difference?

Sketch of Answer:

The key difference is the nature of demand shocks. In the RS model the demand "shocks" are observed before deicsions are made and the decisions of your rivals are also observed. This allows the market clearing price to fluctuate so that no firm has an incentive to cheat. In the GP model the demand shocks *and your rivals decisions are not observed, so that there is uncertainty as to what caused a low market clearing price.*

c. Closely related to the Rotemberg and Saloner model is the Haltiwanger and Harrington model. Briefly discuss how the Haltiwanger and Harrington model differs from the Rotemberg and Saloner model.

Sketch of Answer:

The Haltiwanger and Harrington model assumes that demand is cyclical. This changes the equilibrium because, for example, during an upward movement in demand, the present discounted value of cheating is reduced since demand will be high in periods right after cheating; this increases the punishment from cheating. This effect is reversed during downward movements in demand. Therefore, for a given current *demand level firms can sustain higher prices when when demand is increasing than when demand is falling. This model also creates counter-cyclical prices and this effect is largest during "recessions".*

1.2 Ellison and Borenstein and Shepard

d. What empirical predictions of the RS/HH and the GP models does Ellison test?

Sketch of Answer:

To test the GP model, Ellison tests whether a deviations in the firms' market shares from their average market shares are associated with rightward shifts in supply. A market share that is larger than a firm's average market share may be due to the firm cheating (or, a positive demand shock for that firm). Again, because the firm's decision variable is not observed a large enough deviation may necessitate a price war.

To test the RS/HH model, Ellison constructs a "boom" variable that measures current demand relative to future demand. Ellison tests whether boom periods (periods of high demand) are less likely to lead to collusive state. This is somewhat related to the RS/HH model, but not completely. The RS/HH model predicts prices are a smooth function of demand shocks, the Ellison model assumes to regimes, so prices are move discretely with the "boom" variable.

e. Describe the context of the paper: What is the industry? What are the data? Characterize the time series properties of equilibrium prices in this industry? *A priori*, which theory seems to be most consistent with the industry and the data (and why)?

Sketch of Answer:

The paper uses data from the Joint Executive Board, a legal railway shipping cartel that operated in the lat 1800s. Ellison has data on weekly prices and quantities. The time series properties of prices suggest 2 or more different regimes. Prices were relatively stable in the \$7 to \$8 per ton range, but there were occasional price wars that pushed rates as low as \$2.50 per ton. This fact seems most consistent with the GP model since prices appear to revert to non-ccoperative levels.

f. How does the author propose to test the two theories? Is the model structural or reduced form? What is(are) the dependent variable(s). Are all of the dependent variables observed?

Sketch of Answer:

Ellison estimates a structural model with three dependent variables: supply (p), demand (q) and the decision to collude. Supply follows a Markov switching process in which the firms are allowed to switch from periods of collusion to periods of non-cooperative behavior. The decision to collude is treated as a latent variable; it is not observed.

g. What is the major econometric/conceptual problem that Ellison has to deal with (and spends a considerable amount of the paper dealing with it)? Can he completely rule this out?

Sketch of Answer:

The main problem Ellison has to worry about is whether there existed secret price cuts. He cannot completely rule these out, but spends a great deal of time searching for an omitted variable in his model.

h. How does Ellison's test of the RS/HH model differ from Borenstein and Shepard?

Sketch of Answer:

Ellison's test of the RS/HH model differs in a number of respects. Most notable, BS take a reduced for approach and looko at how equilibrium prices move with expected changes in demand and costs; Ellison's model looks at how expected changes in demand affect the supply equation of the industry. Ellison's model also assume that expected changes in demand cause discrete changes in supply, whereas in BS this effect is continuous.

i. What econometric difficulty (or difficulties) must Borensterin and Shepard confront? How do the authors deal with this problem (or problems)? Be as detailed as possible.

Sketch of Answer:

The biggest problem that BS have is that they are essentially regressing p on current demand and future demand, such as:

$$p_t = \alpha + \beta_1 Q_t + \beta_2 Q_{t+1} + \varepsilon_t \tag{1}$$

There test is to see if β_2 is significant. However, if ε_t is serially correlated then ε_t will be correlated with ε_{t+1} and Q_{t+1} will have explanatory power even if firms are not colluding. Therefore, they have control for this serial correlated so that they are sure that the effect of Q_{t+1} is from collusion and not spurious.

Of course, they must also worry about endogeneity of Q.

2 Question 4

The questions relates to Bresnahan and Reiss (JPE '91) and Berry (Econometrica '92). Both of these papers deal with the sources behind entry.

2.1 Bresnahan and Reiss and Berry

a. Describe the theoretical underpinning for Bresnahan and Reiss' concept of an entry threshold. What is the intuitive interpretation of their entry threshold ratio measure? What are its bounds, and what are the theoretical interpretations of these bounds?

Sketch of answer:

Consider the market for tire dealers. A small town with, say, one person cannot sustain a tire dealer. As the population of the town grows, so does demand (if the new residents have the same willingness to pay schedule, then demand will rotate out around the y-axis). At some point, demand is large enough such that 1 tire dealer could enter and earn zero profits. Suppose this occurs with 3000 people. As the town continues to grow, either the monopolist earns positive profits or another entrant will enter. The key to Bresnahan and Reiss' model is to see when this second entrant enter. Suppose the second firm enters when the town is 6000 in population. Assuming the two firms equally share the demand, this implies it took the same number of people to sustain the second firm as it did to sustain the monopolist. This implies the prices are the same when there are 1 and 2 firms; entry had no affect on prices. This would imply collusion.

Now suppose the second entrant enters when there are 8000 people. This implies that each firm is now servicing 4000 people (and earning zero profits). For this to be the case, prices must have fallen. Therefore, the ratio of the number of extra people needed for a duopolist and the number of people needed for a monopolist can tell us something about price without any price data! The larger the increase in the number of extra people needed for the dupolist, the greater the price decrease.

Now consider the extreme of perfect competition. As firms continue to enter and the market becomes more competitive, subsequent firms will need the same number of extra people as the last entrant. The ratio will then be 1. In a "typical" market, that is one where competition becomes more and more intense with more entry. The ratio will start above one and converge to one. In a market where collusion is sustainable for the first, say, 3 firms but then breaks down, the ratio will be 1 for the firms 2 and 3, and then increase above 1 and, again, converge to 1.

b. Describe the empirical model employed. What is their dependent variable? Do they have actual data on their dependent variable? If not, how do they confront this? What assumptions are made about the error term?

Sketch of answer:

Bresnahan and Reiss write down an equation for profits. They do not observe profits, and thus estimate a latent variable model. They observe that any potential entrant receives the same market-level idiosyncraticshock to profits (error term).

c. Berry also estimates a model of entry. How does his empirical model differ from Bresnahan and Reiss? It may be helpful to write out an equation. Is there any way to view it as a generalization of the BR model?

Sketch of answer:

Berry's model differs in two important respects. First, he introduces a second unobserved error term to profits – at the firm level. Second, he included firm level variables in the profit equation. In particular, profits are expressed as (with minor changes):

$$\pi_{ik}(N) = X_i\beta - \delta \ln(N) + Z_{ik}\alpha + u_{io} + u_{ik}$$

where Z_{ik} are firm level variables (firm k market i), u_{io} is a shock to profits for all firms entering market i and u_{ik} is a firm-level shock to profits of firm k in market i. If we were to omitt the Zs and u_{ik}, then this model can be viewed as the BR model.

d. Estimation of the full model is difficult and requires the use of simulation methods. Are there special cases for his model that makes estimation simpler? If so, what are they? Be as specific as you can.

Sketch of answer:

Yes. If $\delta = 0$ and $u_{io} = 0$, then we have a simple probit model. If $u_{ik} = 0$ and $\alpha = 0$, then this model yields an ordered probit model (e.g., BR). Should also discuss what they restrictions mean and whether they are realistic.

e. Describe the context of Berry's paper: What is the industry? What are the data?

Sketch of answer:

Berry analyzes the airline industry. In particular, he looks at entry decisions in the routes that connect the top 50 cities during the first and third quarters of 1980.

f. What feature of Berry's data allows him to estimate a more sophisticated model than Bresnahan and Reiss?

Sketch of answer:

Berry has the universe of potential entrants for each market. This allows him to include a firm specific error term in the profit equation.

g. One of the goals of Berry's paper is to estimate the effect of an airline's scale of operations at an airport on its profitability. What idea is this trying to capture? How does Berry incorporate this in his empirical model?

Sketch of answer:

Berry is trying to see whether a firms presence at one of the two airports and/or size of this presence affects profits, and thus entry. To incorporate this, he includes variables in the Zs that capture whether the firm has other flights at both of the two airports (an indicator variable), the share of flights out of both airports (continuous) and the number of destination served out of both airports (also continuous).

h. In general, what does Berry find?

Sketch of answer:

He finds that presence appears to more important than the size of that presence. Serving both airports with a near 0% market share increases profits more than serving one of the two cities with a 100% market share.

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Suggested Answers for Section C

5.

a. Last period: graph should show the AJ effect. The price cap constraint on the (L,K) graph is found as follows. Since p_t must be less than or equal to p_{t-1}, Q_t will be greater than or equal to Q_{t-1} (as long as p_{t-1} is below the monopoly price, which isn't a given with ROR regulation). So the constraint set, assuming there is no pure wasting of inputs, is all input bundles on and above (or to the right) the Q_{t-1} isoquant. The newly chosen input bundle is along the expansion path and on isoquant Q_{t-1} (again assuming p_{t-1} is below the monopoly price).

b. PC2 is equivalent to the FV mechanism, as is mentioned in Sibley's article on the ISS-R mechanism. The cap on p_{t-1} will not bind but the cap on A will. So the standard FV results apply. Converge toward the first best over time, with convergence quicker the higher is the discount rate.

c. PC2 is still equivalent to the FV mechanism, also as pointed out in Sibley's article. Customers always prefer the new tariff, which means the introduction of the optional tariff is immaterial and doesn't change the firm's actions.

d. The key to this part is to recognize that the option to choose tariff $([C_{t-1} - p_{t-1}Q_{t-1}]/N, p_{t-1})$ is the same option that consumers have in Sibley's ISS-R mechanism. So we can recast A_t as $A_t = \varphi_t - (p_{t-1}Q_{t-1} - C_{t-1})$, where φ_t is the firm's claim about the ΔCS_t resulting from p_t. Then Sibley shows that truthful revelation of φ_t is incentive compatible, consumers always choose the new tariff, and we move to first best immediately.

6. Spectrum auctions

a. Initial allocation matters when there is incomplete information about the private values that the firms place on the licenses. The usual bugaboo, that ex post bargaining may be costly, may also prevent efficient ex post allocation of licenses.

b. Even in the Coasian world with no bargaining costs, as Milgrom (JPE 2000) points out, when there incomplete information in a private-value environment, ex post bargaining cannot generally achieve efficient rearrangement of property rights.

c. See pp. 258ff of Milgrom (JPE 2000).

d. Based on pp. 261ff of Milgrom (JPE 2000). It is efficient (in the sense of maximizing the value of the license allocation) for 1 to win A and 2 to win B.

e. You'll get the efficient outcome, with 1 paying 1 or 2 for A and 2 paying 1 or 2 for B (how much is paid depends on the tie-breaking rule used after firms 1 and 3 bid 1 for license A, for example).

f. A high bid by bidder 1 on license A helps bidder 2 to acquire license B (because then firm 3 drops out and 2 doesn't have to raise its bid on B). Vice-versa for a high bid by bidder 2 on license B. Each bidder prefers that the other raise the total of the individual bids sufficiently to beat firm 3's bid.

g. The bi-matrix is:

		Firm 2	
		Raise bid on B	Don't raise bid on B
Firm 1	Raise bid on A	2,2	2,3
	Don't raise bid on A	3,2	0,0

Note the off diagonals are pure strategy equilibria, but you are to find the mixed strategy eq'm. Using the usual techniques, the symmetric strategy is for each firm to raise with probability 2/3. To find this, you can find the probability p that Firm 1 raises its bid on license A that makes Firm 2 indifferent between raising and not raising its own bid. You'll find that $p = 2/3$. Applying symmetry or solving the analogous problem for q, the probability that Firm 2 raises its bid on license B, shows that $q = 2/3$ also.

h. The probability of reaching an efficient solution in the mixed strategy eq'm is one minus the probability of reaching an inefficient solution. You would get an inefficient solution if neither 1 nor 2 raised on A or B, so that firm 3 wins both licenses. The probability of neither raising under the mixed strategy is 1/3 * 1/3 = 1/9. So the answer is 8/9. The interesting thing here is that the free riding problem can lead to an inefficient outcome when package bidding is allowed, whereas you'll have the efficient outcome for sure when package bidding is not allowed (as in part e).