Ph. D. Preliminary exam in Industrial Organization, September 2002

Answers to questions 1 and 2

1. Consider, for example, the following case. Hotelling's model where consumers have a quadratic transportation cost ($C = d^2$ where d is distance travelled), one firm. Each product involves a fixed cost of 0.16 and zero marginal cost. Consumers have a reservation price of 2.

Serving the whole market with one product costs 0.16, with two products costs 0.32 (the fixed cost twice).

Consider first the case where the monopolist locates only one product at $\frac{1}{2}$. The maximum price that allows the monopolist to serve the whole market is that price p which yields a delivered price of exactly 2 (the reservation price) at the extremes (at 0 and 1). Thus p solves $p + (\frac{1}{2})^2 = 2$, i.e. $p = \frac{7}{4} = 1.75$. The monopolist's profits would then be $1(1.75) - 0.16 = 1.59$.

Consider now the case where the monopolist locates two products at $\frac{1}{4}$ and $\frac{3}{4}$. The maximum prices p_1 and p_2 compatible with serving the whole market are such that the consumers at 0, $\frac{1}{2}$ and 1 pay exactly their reservation price. Thus $p_1 = p_2$ and this common price must satisfy $p + (\frac{1}{4})^2 = 2$, i.e. $p = \frac{31}{16} = 1.9375$. The monopolist's profits would be $1(1.9375) - 0.32 = 1.6175$.

Since 1.6175>1.59, the counter-argument of the four largest producers of RTE cereal is a valid argument, that is, it is not a contradictory argument. Whether it is true for the case of the RTE cereal industry remains an open question.

2. The Cournot equilibrium profits in region A where there are n firms are given by:

$$\pi_A^* = \frac{(a_A - c)^2}{(n+1)^2 b_A} - F$$

and in region B where there are m firms is:

$$\pi_B^* = \frac{(a_B - c)^2}{(m+1)^2 b_B} - F$$

Equilibrium requires

$$\pi_A^* \geq \frac{(a_B - c)^2}{(m+2)^2 b_B} - F \qquad (1)$$

$$and \quad \pi_B^* \geq \frac{(a_A - c)^2}{(n+2)^2 b_A} - F. \qquad (2)$$

When $F = 0$, $c = 1$, $b_A = b_B = 2$, $a_A = 100$, $a_B = 50$, $n = 20$ and $m = 10$ we have that $\pi_A^* =$ 11.11 and $\pi_B^* = 9.92$. The RHS of (1) is equal to 8.34 and the RHS of (2) is equal to 10.125. Thus inequality (2) is violated and hence we don't have an equilibrium.

When $n = 21$ and $m = 9$ we have that $\pi_A^* = 10.125$ and $\pi_B^* = 12$. The RHS of (1) is equal to 9.92 and the RHS of (2) is equal to 9.26. Hence inequalities (1) and (2) are satisfied and therefore $n = 21$ and $m = 9$ *is* an equilibrium.

2002 IO Prelim, September Round, Sketch of Answers

3. Conservation
   a. This is a standard problem. The solution is to spend zero on conservation of any kind, and set the monopoly price.
   b. Clearly the firm will spend on ineffective conservation rather than effective conservation, because it hurts profit less. The firm sets the monopoly price and spends $\frac{S_{\max}}{a}$ on ineffective conservation. This can be shown formally, which the student did just fine, except student did not show that the constraint must bind on the subsidy cap. To show this, examine Lagrangian:
   $$\mathrm{L} = PQ - C(Q) + (a-1)(R_e + R_i) + \lambda(S_{\max} - S)$$
   where $S = a(R_e + R_i)$. The FOC for R_i is
   $$(1-\lambda)a - 1 \leq 0$$
   with equality if $R_i > 0$ and inequality if $R_i = 0$. If the constraint on the subsidy is slack, then $\lambda = 0$ and from the FOC $a - 1 \leq 0$, which is false by assumption. So the constraint on the subsidy must bind.
   c. Here MC rises by b since there is an additional opportunity cost of producing. As long as $C'(Q) + b$ is greater than the monopoly price with those costs, the firm spends nothing on conservation at all. It faces a choice of how to reduce quantity to reap the subsidy: it can spend on R_e or raise P. Of course it will choose to raise price. This can be formalized, which the student did correctly.
   d. First, assume that the price cap does not bind. Then the answer is the same as for part c. If the price cap binds, then the only question is whether the firm wants to spend on R_e to further reduce output (clearly $R_i^* = 0$). The FOC for R_e is
   $$(\bar{p} - MC - b)\frac{\partial Q}{\partial R_e} - 1 < 0$$
   if $R_e^* = 0$. Thus the firm will spend on conservation only if $b > \bar{p} - MC - 1/\frac{\partial Q}{\partial R_e}$; in other words only if the subsidy is large enough to compensate for the lost profit.
4. Regulation
   a. The second best has the same definition as always: P=AC and efficient input mix.
   b. Standard A-J result. Note that you can prove the firm will not produce in the inelastic region of demand. There is no possibility of waste here (standard proof as in class).
   c. Indeterminate outcome, as usual.
   d. The firm cannot be induced to expand output into the inelastic region of demand, so the "nice" results for the ROC model do not apply here. The firm moves down the expansion path to the point where MR=0 and then wastes. Need not be close to 2nd best.
   e. Usual ROO result: no waste, efficient input mix, and arbitrarily close to 2nd best.

f. In equilibrium, the firm produces at the second best. Note with downward sloping AC curve, do not get to equilibrium in finite time.