University of California, Davis
Department of Economics
**Econometrics**

Date: September 10, 2002
Time: 3 hours
Reading Time: 20 minutes

PRELIMINARY EXAM FOR THE Ph.D. DEGREE

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Answer 4 questions, at least one from each part. Closed Book exam.

## PART I

**Question 1.** Point allocation: (a) 20%; (b) 20%; (c) 20%; (d) 20%; (d) 20%.

The density for the logistic distribution is

$$f(y) = \frac{\exp(-(y-\gamma))}{[1+\exp(-(y-\gamma))]^2},$$

where $-\infty < y < \infty$, $-\infty < \gamma < \infty$ and it can be shown that $\mathrm{E}[y] = \gamma$ and $\mathrm{Var}[y] = \pi^2/3$.

Suppose we have a random sample $(y_i, \mathbf{x}_i)$, $i = 1, ...n$, where $\mathbf{x}_i$ is a $k \times 1$ nonstochastic regressor vector and $y_i$ has the above density with

$$\gamma_i = \mathbf{x}_i'\boldsymbol{\beta},$$

where $\boldsymbol{\beta} = \boldsymbol{\beta}_0$ in the data generating process.

**(a)** Give the log-likelihood function.

**(b)** Give the first-order conditions for the MLE $\widehat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}_0$.

**(c)** Obtain the limit distribution of $\sqrt{n}(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0)$.
Obtain this as easily as you can. You need not formally verify any LLN and CLT used here, but state any assumptions made.

**(d)** Does consistency of $\widehat{\boldsymbol{\beta}}$ require correct specification of the density of $y$, or can consistency be obtained under weaker assumptions on $y$? Explain your answer. A brief explanation will do and there is no need to apply LLN or CLT.

**(e)** Suppose you only had access to a nonlinear least squares program, rather than maximum likelihood. Explain how you would obtain consistent estimates of $\boldsymbol{\beta}$. Would this estimator be fully efficient? Explain.

**Question 2.** Point allocation: (a) 20%; (b) 20%; (c) 20%; (d) 20%; (e) 20%.

For the random variable $y^*$ with exponential distribution the density function, distribution function, mean, variance and truncated mean are given by

$$\begin{aligned} f(y^*) &= \gamma \exp(-\gamma y^*), \qquad y^* > 0, \qquad \gamma > 0 \\ F(y^*) &= 1 - \exp(-\gamma y^*), \\ \mathrm{E}[y^*] &= 1/\gamma; \mathrm{Var}[y^*] = 1/\gamma^2; \mathrm{E}[y^* | y^* > c] = c + 1/\gamma \end{aligned}$$

We incorporate regressors by specifying

$$\gamma_i = \exp(\mathbf{x}_i'\boldsymbol{\beta}),$$

where $\mathbf{x}_i$ and $\boldsymbol{\beta}$ are $k \times 1$ vectors. Throughout we assume a sample of size $n$ with independence over $i$.

**(a)** Suppose $\mathbf{x}'\boldsymbol{\beta} = \alpha + \delta z$ and we obtain estimates $\widehat{\alpha} = 0.3$ and $\widehat{\delta} = 0.4$. Give a meaningful interpretation of how the conditional mean of $y^*$ changes as $z$ changes.

**(b)** Suppose we fully observe $\mathbf{x}_i$ but do not fully observe $y_i^*$. Instead we observe

$$\begin{aligned} d_i &= 1 \qquad \text{if } y_i^* \geq 2 \\ d_i &= 0 \qquad \text{if } y_i^* < 2. \end{aligned}$$

Give the objective function for the MLE of $\boldsymbol{\beta}$ given data $(d_i, \mathbf{x}_i)$, $i = 1, ..., n$.

**(c)** Suppose we fully observe $\mathbf{x}_i$ but do not fully observe $y_i^*$. Instead we observe

$$\begin{aligned} y_i &= y_i^* \qquad \text{if } y_i^* \geq 2 \\ y_i &= 0 \qquad \text{if } y_i^* < 2. \end{aligned}$$

Give the objective function for the MLE of $\boldsymbol{\beta}$ given data $(y_i, \mathbf{x}_i)$, $i = 1, ..., n$..

**(d)** Suppose we observe only

$$y_i = y_i^* \qquad \text{if } y_i > 2,$$

and the associated $\mathbf{x}_i$ when $y_i > 2$. i.e. there is no record of either $\mathbf{x}_i$ or $y_i$ when $y_i^* < 2$. Show how to consistently estimate $\boldsymbol{\beta}$ using a least squares package.

**(e)** Suppose we have panel data when $y_{it}$ is exponentially distributed with parameter $\gamma_{it}$. There is no censoring or truncation. We specify that

$$\gamma_{it} = \exp(\alpha_i + \mathbf{x}_{it}'\boldsymbol{\beta}), \quad i = 1, ..., n, \quad t = 1, ..., T,$$

where $\alpha_1, ..., \alpha_n$ and $\boldsymbol{\beta}$ are parameters to estimate. State whether or not it is possible to consistently estimate $\boldsymbol{\beta}$, and if it is possible say how you would do so, in the following situations:
**(i)** $n$ is small and $T \to \infty$.
**(ii)** $T$ is small and $n \to \infty$.

PART II FOLLOWS ON NEXT PAGE

## PART II

**Question 3.** Point allocation: 50% for the asymptotic limit of the numerator; 40% for the asymptotic limit of the denominator; 10% for appropriately combining these limits and matching the corect answer.

For a sample $\{y_0, y_1, ..., y_T\}$, the Sargan-Barghawa (1983) statistic is defined as

$$SB = \frac{\frac{1}{T^2}\sum_{t=0}^{T} y_t^2}{\frac{1}{T}\sum_{t=1}^{T} \Delta y_t^2}$$

which can be used to test the unit root null hypothesis and can be shown to be the reciprocal of the Durbin-Watson statistic. Show that if $\{y_t\}$ is given by

$$\begin{aligned} y_t &= y_{t-1} + \varepsilon_t \\ \varepsilon_t &= \rho\varepsilon_{t-1} + u_t; u_t \sim i.i.d. \\ E(u_t) &= 0; E(u_t^2) = \sigma^2; |\rho| < 1 \end{aligned}$$

then

$$SB \xrightarrow{d} \frac{1-\rho^2}{(1-\rho)^2} \int_0^1 W(r)^2 dr$$

*Hint:*

$$\begin{aligned} \sum_{t=0}^{T} y_t^2 &= \sum_{t=1}^{T} y_{t-1}^2 + y_{T^2}, \\ & \frac{y_T^2}{T^2} \xrightarrow{p} 0 \end{aligned}$$

**Question 4.** Point allocation: (a) 20 points, (b) 15 points, (c) 25 points, (d) 15 points, (e) 25 points.

Consider the bivariate cointegrated VECM

$$\Delta \mathbf{y}_t = \mathbf{c} + \boldsymbol{\alpha}\boldsymbol{\beta}' \mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t; \qquad \varepsilon_{it} \overset{i.i.d.}{\sim} (0, \sigma^2)$$

where $\boldsymbol{\alpha} = (\alpha_1, 0)'$ and $\boldsymbol{\beta} = (1, -\beta_2)'$. Equation by equation, the system is given by

$$\begin{aligned} \Delta y_{1t} &= c_1 + \alpha_1(y_{1t-1} - \beta_2 y_{2t-1}) + \varepsilon_{1t} \\ \Delta y_{2t} &= c_2 + \varepsilon_{2t} \end{aligned}$$

Answer the following questions:

(a) From the cointegrated VECM representation above, derive the VECM representation

$$\Delta \mathbf{y}_t = \mathbf{c} + \Pi \mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t$$

and the VAR(1) representation

$$\mathbf{y}_t = \mathbf{c} + \mathbf{A}\mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t$$

i.e. determine the elements of the matrices $\Pi$ and $\mathbf{A}$.

(b) Based on the given values of the elements in $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, determine $\boldsymbol{\alpha}_\perp, \boldsymbol{\beta}_\perp$ such that $\boldsymbol{\alpha}'\boldsymbol{\alpha}_\perp = 0$ and $\boldsymbol{\beta}'\boldsymbol{\beta}_\perp = 0$.

(c) Using the Granger Representation Theorem determine that $\psi(1) = \boldsymbol{\beta}_\perp (\boldsymbol{\alpha}_\perp' \mathbf{I}_2 \boldsymbol{\beta}_\perp')^{-1} \boldsymbol{\alpha}_\perp$, where $\psi(L)$ is the moving average polynomial corresponding to the VECM system above and $\mathbf{I}_2$ is the identity matrix of order 2. *Hint:* you may show this result simply by showing that $\psi(1)$ is orthogonal to the cointegrating space.

(d) Using the Beveridge-Nelson decomposition and the result in (c), determine the common trend in the VECM system.

(e) Show that $\boldsymbol{\beta}'\mathbf{y}_t$ follows an AR(1) process and show that this AR(1) process is stable provided that $-2 < \alpha_1 < 0$. What can you say about the system when $\alpha_1 = 0$?

PART III FOLLOWS ON NEXT PAGE

## PART III

**Question 5.** Point allocation: (a) 25%; (b) 25%; (c) 25%; (d) 25%.

Suppose that as a part of your dissertation you are estimating a linear regression model by ordinary least squares. You are concerned about parameter stability in your model and perform the F-tests suggested by Bai and Perron (Econometrica, 1998). Your results are printed below:

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Output from the Bai and Perron testing procedures

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a) supF tests against a fixed number of breaks

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The supF test for 0 versus 1 breaks (scaled by q) is: 6.32
The supF test for 0 versus 2 breaks (scaled by q) is: 18.64
The supF test for 0 versus 3 breaks (scaled by q) is: 17.55

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The critical values at the 5% level are (for k=1 to 3):
12.89 11.60 10.46

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b) Dmax tests against an unknown number of breaks

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The UDmax test is: 18.64
(the critical value at the 5% level is: 13.27 )

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The WDmax test at the 5% level is: 20.06
(The critical value is: 14.19 )

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c) supF(l+1|l) tests using global optimizers under the null

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The supF( 2 | 1 ) test is : 28.43
The supF( 3 | 2 ) test is : 7.17

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The critical values of supF(i+1|i) at the 5% level are (for i=1 to 3 ) are:
12.89 14.50 15.42

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(a) Draw conclusions about the number of breaks in your sample.

(b) Assume that the errors in your model follow a GARCH(1,1) process. How does this affect the properties of the above tests? Be specific.

(c) Suppose you conclude from the Bai/Perron tests that there are some breaks in the parameters of your model and you want to incorporate these into your analysis. One of your options is to treat the breaks as deterministic as in the Bai/Perron approach. A second option is to model the breaks as random draws from some distribution using, for example, a Markov switching model. Outline the advantages and disadvantages of each of these approaches.

(d) Suppose you estimate the markov switching and deterministic breaks models discussed in part (c), but you suspect that neither model adequately fits the data. Explain in detail how you could use semiparametric (or nonparametric) analysis to gain insight into the stability of your model.

**Question 6.** Point allocation: (a) 25%; (b) 25%; (c) 25%; (d) 25%.

Suppose that the solution of a representative agents portfolio allocation problem is given by the following Euler equation

$$1 = E_t \left( \beta \frac{U'(C_{t+1})}{U'(C_t)} \frac{(P_{t+1} + D_{t+1})}{P_t} \right)$$

where $C_t$ is consumption, $P_t$ is the price of a financial asset, and $D_t$ is the dividend paid out by the financial asset. Suppose that the agents utility function is given by

$$U_t = \ln(C_t)$$

and you have quarterly time series data on $P_t$ and $D_t$ for one financial asset, and on $C_t$. In addition, you have data on the risk-free rate of return, $R_t$.

(a) Write down the sample moment conditions that you would use to obtain the GMM estimates of the unknown parameter $\beta$. Be careful to justify your choice of moments.

(b) Suppose that you estimate $\beta$ using more than one moment condition. Given this set of moment conditions, explain how you would obtain the efficient GMM estimates.

(c) Suppose that you estimate $\beta$ using more than one moment condition. Write down Hansen's J-statistic and explain how you would use it to make inference about the validity of the moment conditions and the model implied by the above Euler equation. If you were to reject the null hypothesis using Hansen's J-test, what would you do?

(d) Suppose that you estimated $\beta$ using a least squares regression of 1 on $X_{t+1}$, where $X_{t+1} = (C_t/C_{t+1})(P_{t+1} + D_{t+1})/P_t$. Show that this estimator is inconsistent and explain intuitively why it is inconsistent.

END OF EXAM.