University of California, Davis
Department of Economics
Econometrics

Date: July 8, 2003
Time: 3 hours
Reading Time: 20 minutes

PRELIMINARY EXAM FOR THE Ph.D. DEGREE

Answer 4 questions, at least one from each part. Closed Book exam.

PART I

Question 1. Point allocation: (a) 30%; (b) 20%; (c) 10%; (d) 20%; (d) 20%.

Consider the estimator $\widehat{\boldsymbol{\beta}}$ that minimizes

$$Q_N(\boldsymbol{\beta}) = \frac{1}{N}\sum_{i=1}^{N}(y_i - \exp(\mathbf{x}_i'\boldsymbol{\beta}))^2,$$

where $\boldsymbol{\beta}$ is a $k \times 1$ parameter vector and $\mathbf{x}_i$ is a $k \times 1$ stochastic regressor vector and it is assumed that $(y_i, \mathbf{x}_i)$ are iid over i.
In the true model

$$\begin{aligned} \mathrm{E}[y_i|\mathbf{x}_i] &= \exp(\mathbf{x}_i'\boldsymbol{\beta}_0) \\ \mathrm{V}[y_i|\mathbf{x}_i] &= \sigma_{i0}^2. \end{aligned}$$

State clearly any additional assumptions needed below.

(a) Obtain plim $Q_N(\boldsymbol{\beta})$ and hence prove the consistency of $\widehat{\boldsymbol{\beta}}$.
Hint: It is helpful to re-write $y_i - \exp(\mathbf{x}_i'\boldsymbol{\beta}) = [y_i - \exp(\mathbf{x}_i'\boldsymbol{\beta}_0)] + [\exp(\mathbf{x}_i'\boldsymbol{\beta}_0) - \exp(\mathbf{x}_i'\boldsymbol{\beta})]$
You need not formally verify any LLN and CLT used here.

(b) Obtain the limit distribution of $\sqrt{N}(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0)$.
You need not formally verify any LLN and CLT used here.

(c) Given your answer in part (b), provide complete details on a method to obtain consistent standard errors.

(d) Present in detail an iterative method that permits computation of $\widehat{\boldsymbol{\beta}}$.

(e) Suppose it is additionally assumed that $\sigma_{i0}^2 = \exp(\mathbf{x}_i'\boldsymbol{\beta}_0)$. Present a method to obtain an estimate of $\boldsymbol{\beta}$ that is more efficient than $\widehat{\boldsymbol{\beta}}$ given above.

Question 2. Point allocation: (a) 15%; (b) 15%; (c) 20%; (d) 15%; (e) 15%; (f) 20%.

Suppose that

$$\begin{aligned} y_i &= \exp(\mathbf{x}_i'\boldsymbol{\beta}) + u_i \\ u_i|\mathbf{x}_i &\sim \mathcal{N}[0, \exp(\mathbf{x}_i'\boldsymbol{\gamma})], \end{aligned}$$

where $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are $k \times 1$ parameter vectors, $\mathbf{x}_i$ is a $k \times 1$ stochastic regressor vector and it is assumed that $(y_i, \mathbf{x}_i)$ are iid over i..

(a) Give the objective function for a consistent joint estimator for $(\boldsymbol{\beta}, \boldsymbol{\gamma})$ based on $2k$ moment conditions.

(b) Suppose that the second assumption above does not hold. Instead $\mathrm{E}[u_i|\mathbf{x}_i] \neq 0$.
What condition must suitable instruments satisfy?
Give the formula for an objective function that yields a consistent estimate of $\boldsymbol{\beta}$ using these instruments.

(c) Give the asymptotic distribution for your estimator in (b).
Your derivation should be as brief as possible (Otherwise this could take a lot of time).

(d) Suppose we do not fully observe y_i above. Instead we only observe whether $y_i > 0$ or $y_i \leq 0$.
Give the formula for an objective function that yields consistent estimates of $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$.

(e) Suppose we do not fully observe y_i above. Instead we only observe positive values of y_i (and the associated $\mathbf{x}_i$).
Give the formula for an objective function that yields consistent estimates of $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$.

(f) Suppose we have panel data. There is no censoring or truncation. We specify that

$$\begin{aligned} y_{it} &= \alpha_i + \exp(\mathbf{x}_{it}'\boldsymbol{\beta}) + u_{it} \\ u_{it}|\mathbf{x}_{it} &\sim \mathcal{N}[0, \exp(\mathbf{x}_{it}'\gamma)], \end{aligned}$$

where independence over both $i = 1, ..., N$ and $t = 1, ..., T$ is assumed, N is small, $T \to \infty$, and $\alpha_1, ..., \alpha_N$ are unknown parameters along with $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$.
Give a method to obtain a consistent estimate of $\boldsymbol{\beta}$.

PART II FOLLOWS ON NEXT PAGE

PART II

Question 3. Point allocation: (a) 20%; (b) 10%; (c) 10%; (d) 20%; (e) 10%; (f) 10%; (g) 20%.

Consider the following DGP

$$\begin{aligned} x_t + \beta y_t &= u_{1t}; \qquad \text{where } u_{1t} = \theta u_{1t-1} + \varepsilon_{1t} \\ x_t + \alpha y_t &= u_{2t}; \qquad \text{where } u_{2t} = \rho u_{2t-1} + \varepsilon_{2t} \end{aligned}$$

with $|\rho| < 1$, and

$$\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \sim D\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \gamma \\ \gamma & \sigma_2^2 \end{pmatrix} \right]$$

where D denotes a generic distribution.

(a) Derive the degree of integratedness of the two series, x_t, and y_t, explicitly stating the parameter restrictions required in each case.

(b) Under what coefficient restrictions are x_t and y_t cointegrated?

(c) What are the cointegrating vectors in such cases?

(d) Choose a particular set of coefficients that ensures x_t and y_t are cointegrated and derive the error correction representation.

(e) Can all cointegrated systems be represented as an error-correction model? Explain.

(f) What are the problem/s of analyzing a VAR in the differences when the system is cointegrated?

(g) Suppose that you have the following DGP

$$\begin{aligned} x_{1t} &= \sum_{i=1}^{t} \varepsilon_{1i} + \varepsilon_{2t} \\ x_{2t} &= \frac{1}{2}\sum_{i=1}^{t} \varepsilon_{1i} + \varepsilon_{3t} \\ x_{3t} &= \varepsilon_{2t} \end{aligned}$$

(i) Is the process $X_t' = (x_{1t}, x_{2t}, x_{3t})$ I(1)? Justify

(ii) Calculate two orthogonal cointegrating vectors for the I(1) process X_t

Question 4. Point allocation: (a) 35%, (b) 10%, (c) 35%, (d) 25%.
Let

$$y_t = \beta t^{\alpha} + \varepsilon_t \qquad \varepsilon_t \tilde{} N(0, \sigma^2)$$

and α known. Given the normality of ε_t, the exact distribution of $\widehat{\beta}_{OLS}$ can be obtained. Answer the following questions:

(a) What is the distribution of $\widehat{\beta}_{OLS}$ for a generic α? Hint: $\lim_{T\to\infty} \sum_{t=1}^{T} t^v = T^{v+1}/(v+1)$

(b) What is the distribution of $\widehat{\beta}_{OLS}$ for $\alpha = 0$?

(c) What is the distribution of $\widehat{\beta}_{OLS}$ for $\alpha = 1/2$? How can you rescale the problem to obtain the distribution?

(d) What is the distribution of $\widehat{\beta}_{OLS}$ for $\alpha = -1$? Hint: $\lim_{T\to\infty} \sum_{t=1}^{T} t^{-2} = \pi^6/6$.

PART III FOLLOWS ON NEXT PAGE

PART III

Question 5. Point allocation: (a) 25%; (b) 25%; (c) 25%; (d) 25%.

Suppose that as a part of your dissertation you are estimating a linear regression model. You are concerned about parameter stability in your model and perform the F-tests suggested by Bai and Perron (Econometrica, 1998). Your results are printed below:

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*******************************************************
Output from the Bai and Perron testing procedures
*******************************************************
a) supF tests against a fixed number of breaks
----------------------------------------
The supF test for 0 versus 1 breaks (scaled by q) is: 8.02
The supF test for 0 versus 2 breaks (scaled by q) is: 17.72
The supF test for 0 versus 3 breaks (scaled by q) is: 15.13
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The critical values at the 5% level are (for k=1 to 3):
12.89 11.60  10.46
----------------------------------------
b) Dmax tests against an unknown number of breaks
----------------------------------------
The UDmax test is: 17.72
(the critical value at the 5% level is: 13.27 )
*******************************************************
The WDmax test at the 5% level is: 18.41
(The critical value is: 14.19 )
*******************************************************
c) supF(l+1|l) tests using global otimizers under the null
-------------------------------------------
The supF( 2 | 1 ) test is : 14.21
The supF( 3 | 2 ) test is : 7.81
**********************************************************
The critical values of supF(i+1|i) at the 5% level are (for i=1 to 3 ) are:
12.89 14.50  15.42
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(a) Draw conclusions about the number of breaks in your sample.

(b) Suppose you conclude from the Bai/Perron tests that there are some breaks in the parameters of your model and you want to incorporate these into your analysis. One of your options is to treat the breaks as deterministic as in the Bai/Perron approach. A second option is to model the breaks as random draws from some distribution using, for example, a Markov switching model. Outline the advantages and disadvantages of each of these approaches.

(c) This part is distinct from parts (a), (c), and (d). Consider a model with a log-likelihood function given by $L(X; \theta)$. Suppose you only observe a subset Y of the data, i.e. you observe $Y \subset X$. Outline how you could use the EM algorithm to estimate θ.

(d) This part is distinct from parts (a)-(c). Consider the problem of estimating $E(y|x)$ nonparametrically. Suppose that x is a $q \times 1$ vector of binary random variables. Explain what the curse of dimensionality is and explain why it is much less of a problem for binary x variables than continuous variables.

Question 6. Point allocation: (a) 25%; (b) 25%; (c) 25%; (d) 25%.

Suppose that the solution of a representative agents portfolio allocation problem is given by the following Euler equation

$$1 = E_t \left(\beta \frac{U'(C_{t+1})}{U'(C_t)} \frac{(P_{t+1} + D_{t+1})}{P_t} \right)$$

where C_t is consumption, P_t is the price of a financial asset, and D_t is the dividend paid out by the financial asset. Suppose that the agents utility function is given by

$$U_t = -C_t^{-1}$$

and you have quarterly time series data on P_t and D_t for one financial asset, and on C_t. In addition, you have data on the risk-free rate of return, R_t.

(a) Write down the sample moment conditions that you would use to obtain the GMM estimates of the unknown parameter β. Be careful to justify your choice of moments.

(b) Suppose that you estimate β using more than one moment condition. Given this set of moment conditions, explain how you would obtain the efficient GMM estimates.

(c) Suppose that you estimate β using more than one moment condition. Write down Hansen's J-statistic and explain how you would use it to make inference about the validity of the moment conditions and the model implied by the above Euler equation. If you were to reject the null hypothesis using Hansen's J-test, what would you do?

(d) Suppose that you estimate β using more than one moment condition. Explain how you would test for weak identification using the results of Stock and Wright (Econometrica, 2000).

END OF EXAM.