

ADVANCED ECONOMIC THEORY

Reading time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph. D. DEGREE

Answer four questions: at least one from each section

Section I

1. Three scientists, A, B and C agree that the space of “possible worlds” is $\Omega = \{1, 2, 3, 4, 5\}$. Initially they have a common prior that attaches equal probability to every state. Now they agree that A will perform the experiment represented by the following partition: $\{1,2,4\}, \{3,5\}$, B will perform the experiment represented by the following partition: $\{1,3,5\}, \{2,4\}$ and C will perform the experiment represented by the following partition: $\{2,3,4\}, \{1,5\}$. They are interested in establishing the probability of event $F = \{1,2,3\}$. After the experiment, each scientist will communicate her posterior probability of F to a common friend. The friend will compute the average of the posteriors and make a public announcement of this average (so that it becomes common knowledge). Each scientist will then use this information to update her posterior of F and communicate the new posterior to the common friend, who will then recompute the average and announce it publicly, and so on.

- (i) Replace the question marks in the following tables (e.g. in the second table, in the cell corresponding to state 2 and column A you have to write the posterior probability that scientist A attaches to event F given the information that she obtains if the true state is 2).
- (ii) If the true state is 1, which of the scientists (if any) will discover in the end that the true state is 1?
- (iii) If the true state is 2, which of the scientists (if any) will discover in the end that the true state is 2? [Note: there is one more question after all the tables.]

Time:	AFTER EXPERIMENT						
A's partition	<table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td>1</td><td>2</td><td>4</td></tr></table>	1	2	4	<table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td>3</td><td>5</td></tr></table>	3	5
1	2	4					
3	5						
B's partition	<table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td>1</td><td>3</td><td>5</td></tr></table>	1	3	5	<table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td>2</td><td>4</td></tr></table>	2	4
1	3	5					
2	4						
C's partition	<table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td>2</td><td>3</td><td>4</td></tr></table>	2	3	4	<table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td>1</td><td>5</td></tr></table>	1	5
2	3	4					
1	5						
Common knowledge partition	?						

Scientist →	A	B	C	Average
State Φ				
1	?	?	?	?
2	?	?	?	?
3	?	?	?	?
4	?	?	?	?
5	?	?	?	?

Time: AFTER FIRST ANNOUNCEMENT

A's partition	?
B's partition	?
C's partition	?
Common knowledge partition	?

Scientist →	A	B	C	Average
State Φ				
1	?	?	?	?
2	?	?	?	?
3	?	?	?	?
4	?	?	?	?
5	?	?	?	?

Time: **AFTER SECOND ANNOUNCEMENT**

A's partition	?
B's partition	?
C's partition	?
Common knowledge partition	?

Scientist →	A	B	C	Average
State Φ				
1	?	?	?	?
2	?	?	?	?
3	?	?	?	?
4	?	?	?	?
5	?	?	?	?

(iv) Give a precise statement of Aumann's Agreement Theorem and explain whether the hypotheses of the theorem are satisfied in this example at the end of the exchange (that is, using the last table above), concerning event F.

2. Two objects (e.g. a left shoe and a right shoe) are auctioned to 3 bidders: Ann, Bob and Carla. Each bidder assigns zero value to each *individual* object but values the pair at \$100. The bidding takes place as follows. There are two rounds. In each round each player can bid either \$0 or \$10 or \$20. At the end of the first round the following happens: (1) if one player bid **more** than the others, he/she gets the first object and *everybody* (including those who lost the first round) has to pay his/her bid; (2) if the highest bid was submitted by at least two players then Ann gets the object (*even if she was not* one of the highest bidders) and *everybody* has to pay his/her bid (even those who lost). Then we proceed to the next stage where the same rules apply. Thus, at the end of the second round the following happens: (1) if one player bid **more** than the others, he/she gets the second object and *everybody* (including those who lost the second round) has to pay his/her bid; (2) if the highest bid was submitted by at least two players then Ann gets the second object (*even if she was not* one of the highest bidders) and *everybody* has to pay his/her bid (even those who lost). [Note that in the second round the bids that are used to determine the outcome of the second round, i.e. who gets the second object, are **not** the cumulative bids (first-round bid + second-round bid) but just the second-round bids.]

- (a) Write the normal form of the second-round game in the case where in the first round Ann bid \$0, Bob bid \$10 and Carla bid \$10. Suppose that there is common belief in rationality: what strategy profile will be played in this second-round game?
- (b) Does any of the players have a dominant strategy *in the second round* ? If Yes, explain why, if No explain why not. [You have to cover all cases, not only the one given above under (a).]
- (c) What is the iterated dominant-strategy equilibrium in the second round?
- (d) Write the normal form for the first-round game assuming that everybody predicts the outcome of the second round to be the one given in part (c).
- (e) What are the pure-strategy Nash equilibria of the first round game under (d)?

Section 2

3. Consider an exchange economy with two periods ($t = 0, 1$) and uncertainty described by S states of nature at date 1. There are L goods at each date and agent i has the initial endowment $\omega^i \in \mathbf{R}_+^{L(S+1)}$. The preferences of each agent are characterized by a utility function $u^i : \mathbf{R}_+^{L(S+1)} \rightarrow \mathbf{R}$ which is smooth, strictly quasi-concave and has indifference surfaces which do not intersect the axes.

- (a) Define a spot-financial market equilibrium for the economy where $p = (p_0, \dots, p_S)$ is the vector of spot prices, $q = (q_1, \dots, q_J)$ the vector of security prices and V is the $S \times J$ matrix of payoffs of the securities (assume that the payoff of the securities is in units of good 1 whose price in each state is normalized to 1).
- (b) Suppose the financial markets are complete. Using the agents' first-order conditions at an equilibrium and the first-order conditions for Pareto optimality, show that the equilibrium is Pareto optimal.
- (c) Suppose as in (b) that the financial markets are complete, that there is only one good (income) at each date, that the first security is the riskless bond ($V^1 = (1, \dots, 1)$) and that agents have VNM utility functions

$$u^i(x^i) = v_0^i(x_0^i) + \sum_{s=1}^S \rho_s v^i(x_s^i)$$

where $\rho_s > 0$ is the probability of state s ($s = 1, \dots, S$) and $v^i > 0$, $v^{i''} < 0$. Show that the equilibrium price of security j can be written as

$$\bar{q}_j = \frac{E(V^j)}{1 + \bar{r}} + \text{cov}(V^j, \frac{\bar{\pi}}{\rho})$$

where \bar{r} is the equilibrium interest rate and $\bar{\pi}$ the vector of state prices implicit in the prices of the securities. Explain the economics of the risk premium.

- (d) Let $w_1 = (w_1, \dots, w_S)$ where $w_s = \sum_{i=1}^I \omega_s^i$ denotes aggregate output in state s . Show that there exists a strictly decreasing function g such that the equilibrium risk premium can be written as

$$\bar{q}_j = \frac{E(V^j)}{1 + \bar{r}} + \text{cov}(V^j, g(w_1))$$

Interpret this formula.

4. Consider an exchange economy with two periods ($t = 0, 1$) and uncertainty described by S states of nature at date 1. There is one good at each date and agent i has the initial endowment $\omega^i \in \mathbf{R}_+^{(S+1)}$. The preferences of each agent are characterized by a utility function $u^i : \mathbf{R}_+^{L(S+1)} \rightarrow \mathbf{R}$ which is smooth, differentiably strictly quasi-concave and has indifference surfaces which do not intersect the axes. There are J securities with a payoff matrix V at date 1. Unlike the previous question, the financial markets are incomplete ($J < S$).

- (a) Write out the definition of a financial market equilibrium and the first-order conditions which are satisfied at an equilibrium.
- (b) Explain from the first-order conditions why an equilibrium is not likely to be Pareto optimal.
- (c) Suppose now that you want to prove formally that an equilibrium with incomplete markets is typically suboptimal. Explain what you are able to prove and give a (precise) outline of the proof.
- (d) What are the non-generic cases that you know where a financial market equilibrium is Pareto optimal despite the fact that the financial markets are incomplete?

SECTION 3

5. i) State Savage's representation theorem, restricting your attention to acts with a finite number of consequences. State each axiom in a formally precise manner, and interpret its content intuitively.

ii) Savage's theorem contains a representation of "qualitative probability" relations with particular structure; state this representation, and point out what additional structure is added.

iii) Let \mathbf{N} denote the set of natural numbers and Σ denote the family of all sets $A \subseteq \mathbf{N}$ such that A or its complement A^c is finite. Define a relation \trianglerighteq on the family of *finite* subsets of Σ that orders sets lexicographically according to their greatest elements: $A \equiv B$ iff $A = B$, and $A \triangleright B$ iff, for some $k \geq 0$, the k largest elements of A and B agree, and the $k+1$ st element of A is larger than the $k+1$ st element of B , or if B has only k elements. For example, $\{5\} \triangleright \{4, 3, 2\}$, $\{5, 4, 3\} \triangleright \{5, 4, 2\}$, and $\{5, 4, 3\} \triangleright \{5, 4\}$.

Show that this ordering can be extended to a qualitative probability on Σ .

Is this extension unique?

Can this qualitative probability be represented by a finitely additive probability measure?

6. i) State a version of Rabin's "Calibration Theorem"
- ii) Which descriptive challenge to EU theory does Rabin's Theorem formalize?
- iii) Briefly describe the basic ingredients of Prospect Theory.
- iv) How does Prospect Theory predict the choice patterns referred to in part ii)?
- v) Briefly state the Equity Premium Puzzle, using the Hansen-Jagganathan bounds.
- vi) Describe the potential relevance of Rabin's Theorem and Prospect Theory to the Equity Premium puzzle.