1. Consider the following limited participation monetary model. Firms hire labor \((h_t)\) to produce output \((y_t)\) using the linear production function:

\[ y_t = \gamma h_t ; \quad \gamma > 0 \]

Output is sold at the nominal price of \(P_t\) dollars per unit of output. Firms pay labor in advance of production – they borrow the wage bill from financial intermediaries. Hence the cost of labor inputs is:

\[ \text{Cost} = R_t W_t h_t \]

where \(R_t\) denotes the (gross) nominal interest rate and \(W_t\) is the nominal wage. Financial intermediaries receive funds from two sources: Households make deposits \((I_t)\) at the beginning of the period before the current state of the world is known and the current monetary transfer (which determines the current state) is received from the government. The financial intermediary inelastically provides the funds in the form of loans to businesses to finance their wage bill. That is, financial intermediaries make no profits by assumption:

\[ I_t + g_t \bar{M}_{t-1} = W_t h_t \]

The income from the loans is distributed entirely to the households. Note this implies that the law of motion for the money supply is:

\[ \bar{M}_t = (1 + g_t)\bar{M}_{t-1} \]

Households make deposit, consumption, and labor decisions in order to maximize lifetime expected utility. Preferences are given by:

\[ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \ln c_t + A(1 - h_t) \right] \right\} \]

As mentioned above, households allocate part of their nominal wealth to the banking sector before knowing the current realization of the monetary growth rate. Then, after observing \(g_t\), they make consumption and labor decisions. It is assumed that the consumption is subject to a cash-in-advance constraint. Note that, as implied by the discussion above, current labor income can be used to finance current consumption.

The only source of uncertainty in the economy is due to the monetary growth rate; this random variable is assumed to be independently and identically distributed with \(E(g_t) > 0\). Given this environment, do the following:

a) Set up the firm’s and household’s maximization problem. Derive and interpret the associated necessary condition. In your answer, demonstrate how this model is distinguished from a typical cash-in-advance model.

b) Define a stationary monetary equilibrium in this economy.

c) Prove that the liquidity effect is present in this economy.

d) Prove that consumption and real wages are procyclical.

e) What is the correlation of savings (i.e. funds placed in the banking sector) and nominal interest rates?
2. In the analysis of monetary policy, it is currently common practice to use a monetary environment in which the behavior of the monetary aggregate is often ignored. (The next question presents an example of this approach.) In contrast, monetary theory typically imposes strong restrictions on the behavior of the money supply, nominal interest rates, and income, both nominal and real. Is this neglect of the monetary aggregate justified? In your answer, be specific in discussing the pros and cons of this approach.

3. Consider the following basic New Keynesian model

\[
\begin{align*}
  x_t &= E_t x_{t+1} - \left( \frac{1}{\sigma} \right)(i_t - E_t \pi_{t+1}) + u_t \quad (IS) \\
  \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t + e_t \quad (Inf) \\
  i_t &= \gamma \pi_t + \delta y_t \quad (PR)
\end{align*}
\]

where \( x_t \) is the output gap; \( \pi_t \) is the inflation rate; \( i_t \) is the nominal interest rate controlled by the central bank; and \( u_t \) and \( e_t \) are two uncorrelated and white noise disturbances. Answer the following questions concisely:

a) Comment on the micro-foundational origins of expression \((IS)\) and briefly discuss common alternative specifications. What consequences do these variations have on the optimal choices of \(\gamma\) and \(\delta\)?

b) Comment on the micro-foundational origins of expression \((Inf)\) and briefly discuss common alternative specifications. What consequences do these variations have on the optimal choices of \(\gamma\) and \(\delta\)?

c) Solve the model by expressing \((E_t \pi_{t+1}, E_t y_{t+1})\) as a function of \((\pi_t, y_t)\)' and the shocks only.

d) Discuss how it would affect the choices of \(\gamma\) and \(\delta\) if the monetary authority's only goal were output stability.

e) If \(e_t\) were a persistent (but not necessarily unit root) autoregressive process, discuss what effect this would have on the optimal choices of \(\gamma\) and \(\delta\).

f) Suppose the inflation equation becomes

\[ \pi_t = \beta \pi_{t-1} + e_t \]

instead. Does the new trade-off between inflation and output variability change your answer to (d)?

4. R. E. Lucas (1980) argues that economists,

... need to test them [models] as useful imitations of reality by subjecting them to shocks for which we are fairly certain how actual economies or parts of economies would react. The more dimensions on which the model mimics the answers actual economies give to simple questions, the more we trust its answers to harder questions.

Discuss the manner in which the monetary literature has made operational the "Lucas program." Include an evaluation of the success or failure of this program.