

# RATIONALIZABLE SCREENING AND DISCLOSURE UNDER UNAWARENESS\*

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## Abstract

This paper analyzes a principal-agent procurement problem in which the principal is unaware some of the agent’s marginal costs. Since she does not conceive of all relevant types, her planned menu of contracts may be suboptimal to the agent. Communication arises naturally as some types of the agent may have an incentive to raise the principal’s awareness of marginal cost types before a contract menu is offered. The resulting menu must not only reflect the principal’s change in awareness, however: She also learns about the agent’s types, as not all of them need have incentives to raise her awareness. We capture this reasoning through an extensive-form version of cautious rationalizability with beliefs on marginal cost types restricted to logconcavity and “reverse” Bayesianism (Karni and Vierø, 2013). We show that if initially the principal is only unaware of some low marginal cost types, she is not made aware of all types and there is *bunching at the top*. If the principal is only unaware of some high marginal cost types, however, all types will fully raise her awareness. Thus, the principal is happily made aware of inefficiencies but kept tacitly in the dark about some efficiencies. The possibility of disclosure improves ex post social welfare but does not always eradicate welfare frictions due to asymmetric awareness.

**Keywords:** Hidden information; contract design; principal-agent model; procurement; games with unawareness; extensive-form rationalizability;  $\Delta$ -rationalizability; iterated admissibility; common belief in rationality.

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# 1 Introduction

In many real-life contracting situations, the agent hired by a principal is much more experienced in the task than the principal; this is often the reason why an agent is hired in the first place. Examples are an economics professor that hires a contractor to remodel a house, or a firm hiring an investment bank to prepare for stock offering. In other situations, contracting is complex and involves trade secrets or research & development. For instance, when procuring novel, highly specialized assets such as security or weapon systems, government agencies may not be aware of all the technological details, let alone the contractors' cost structure.

These are scenarios in which the agent is aware of some relevant events that the principal has not thought about at all at the time of writing the contract. However, the standard approach to contract design under hidden information (Mirrlees, 1971; Mussa and Rosen, 1978; Baron and Myerson, 1982; Maskin and Riley, 1984) poses that there is common awareness between the principal and the agent of all possible types and that their distribution is commonly known—the principal simply does not observe the agent's realized type. Typically, the principal offers a menu of contracts such that all types of the agent accept the contract that is meant for them. However, the agent may find that the menu offered by a principal with limited awareness about types is unappealing, or perhaps overly generous. For instance, contractors may find themselves expected to deliver output quantities at prices that fail to account for their actual cost structure; so, they may choose to misrepresent themselves or downright reject the principal's offer. Similarly, a monopoly regulator may not be privy to certain details of the monopolist's technology and thus set inadequate regulation standards.

This paper analyzes a procurement problem where the principal is unaware of some of the agent's marginal cost types. Before the actual screening problem, the agent may choose to change the principal's awareness. Indeed, communication arises naturally in the problems we study, as some agent types may have incentives to raise the principal's awareness of marginal cost types. Such communication does not only change the principal's awareness, however: Having been made aware of “new” types, she also reasons about which types of the agent (of which she is now aware) would have incentives to raise her awareness in the first place.

We model the interaction between agent and principal as an extensive-form games with unawareness (Heifetz, Meier, and Schipper, 2013; Halpern and Rêgo, 2014; Feinberg, 2021), and we capture the principal's reasoning about agents' types through a version of extensive-form cautious rationalizability. In order to link the principal's beliefs over marginal cost types across different awareness levels, we impose “reverse” Bayesian updating (Karni and Vierø, 2013; Dominiak and Tserenjigmid, 2018). In our context, reverse Bayesianism means that the relative likelihood of any marginal cost types  $\theta$  and  $\theta'$  that the principal conceived of initially remains the same upon becoming aware of an additional type  $\theta''$ , as long as  $\theta$  and  $\theta'$  are not ruled out by the agent's incentives. We further restrict beliefs over agent types to be logconcave, which implies that hazard rates are nondecreasing—a standard assumption in the screening literature. In our context, logconcavity represents the common belief that the principal's marginal beliefs over marginal cost types are unimodal over their support.

We show that, if the principal is unaware only of higher marginal cost types, then all types of the agent have an incentive to fully raise her awareness. However, if the principal is unaware only of events that decrease marginal costs, then she is not made fully aware: None of the types of the agent of which she is initially aware has incentives to raise her awareness about

other types, and if a type of the agent of which she is unaware prefers to raise her awareness, he will not do so to a level that includes his own type. Consequently, there is *bunching at the top for types of which the principal remains unaware*. In any case, the principal offers an optimal menu of contracts to all types of which she is or has become aware. The possibility of disclosure improves ex post social welfare but does not always eradicate welfare frictions due to asymmetric awareness.

Altogether, our solution concept has features of extensive-form rationalizability (Pearce, 1984; Battigalli, 1997; Heifetz, Meier, and Schipper, 2013); it captures a notion of caution (Heifetz, Meier, and Schipper, 2021) that is similar to iterated admissibility (Brandenburger, Friedenberg, and Keisler, 2008); and it incorporates further restrictions that are akin to  $\Delta$ -rationalizability, extensive-form best response sets, and self-admissible sets (see Battigalli and Siniscalchi, 2003; Battigalli and Friedenberg, 2012; Battigalli and Prestipino, 2013; Brandenburger, Friedenberg, and Keisler, 2008; Brandenburger and Friedenberg, 2010). We are not aware that the discrete screening problem has been solved previously with a rationalizability notion that incorporates log-concavity on types as a belief restriction.

Although games with unawareness are relatively new, this is not the first paper to study contracting under unawareness. von Thadden and Zhao (2012) study a principal-agent moral hazard problem in which the principal is aware of actions of which the agent is unaware. When contemplating whether or not to make the agent aware, the principal faces a trade-off between getting a better action and saving on information rents due to additional incentive compatibility constraints. Auster (2013) studies a principal-agent moral hazard problem in which the principal is aware of contingencies of which the agent is unaware but whose realization is informative about the agent's actions. In the optimal contract, the principal faces a trade-off between exploiting the agent's unawareness and using said contingencies in order to provide incentives. Filiz-Ozbay (2012) studies a risk neutral insurer who is aware of some contingencies that the insuree is unaware. The insurer has an incentive to mention only some contingencies in a contract while remaining silent on others.

In all of the papers above, in contrast to our paper, the principal has a higher awareness level than the agent. Auster and Pavoni (2021) and Lei and Zhao (2021) feature an agent with higher awareness level than the principal but in the context of optimal delegation. In Auster and Pavoni (2021), the agent is aware of both the set of his actions and their performance, and only reveals extreme actions. In Lei and Zhao (2021), the agent only partially reveals contingencies of which the principal is unaware. Principals who are unaware of more contingencies delegate a large set of projects. Ma and Schipper (2012) show that the arguments for welfare irrelevance of indescribable contingencies by Maskin and Tirole (1999) extend to persistent asymmetric information settings but not to asymmetric unawareness in a buyer-seller model. The closest paper to ours within the Maskin and Tirole (1999) paradigm is Herweg and Schmidt (2020), who study a procurement problem in which the seller may be aware of some design flaws. They propose an efficient two-stage mechanism with a neutral arbitrator that does not require an ex ante description of design flaws. For their argument to work it is crucial that any design flaws are verifiable ex post. However, we know from empirical studies (e.g., Bajari, Houghton, and Tadelis, 2014) that there is often disagreement ex post. Grant, Kline, and Quiggin (2012) discuss disagreements arising from asymmetric awareness in contracting. Finally, in the field of value-based business strategy, Bryan, Ryall, and Schipper (2021) devise cooperative games with incomplete information and unawareness for studying surplus creation

and surplus appropriation within contracting relationships.

Since communication arises naturally in our principal-agent problem as some types of the agent may have an incentive to raise the principal’s awareness, our paper is also related to disclosure in games with unawareness. Heifetz, Meier, and Schipper (2021) show that the unraveling argument breaks down in disclosure games à la Milgrom and Roberts (1986) when receivers may be unaware of some signalling dimension. In such a case, the receiver is unable to infer anything about the sender’s type from the absence of a signal. This has been experimentally tested in Li and Schipper (2018). Schipper and Woo (2019) apply this insight to electoral campaigning to discuss microtargeting of voters and negative campaigning.

The closest work to ours on adverse selection without unawareness are Pram (2021) and Ali, Lewis, and Vasserman (2022). Both papers consider a screening problem in which the agent can disclose verifiable evidence about his type before the principal commits to a mechanism. Pram (2021) characterizes environments in which verifiable evidence is welfare improving, namely whenever in the mechanism without disclosure some types would be excluded. Ali, Lewis, and Vasserman (2022) also find that the agent can benefit from prior disclosure of evidence, but it depends on that partial disclosure being feasible. Somewhat related is Sher and Vohra (2015), who consider disclosure in a monopolists’ price discrimination problem. The seller commits to a mechanism with evidence-contingent prices; i.e., disclosure occurs *after* commitment to the mechanism. Such timing would be much less compelling in the face of unawareness, however, as the principal is unaware of potential evidence that could be disclosed. Hidir and Vellodi (2021) study buyer’s cheap-talk prior trade and buyer optimal market segmentation consistent with their information revelation.

Our setting can be viewed as one with disclosure by the agent *prior* to the principal committing to the mechanism. However, instead of disclosure of information, we consider disclosure of awareness. In some sense, disclosure of awareness is the opposite of disclosure of information: Awareness of more types is akin to increasing uncertainty about the possible types that the principal faces.

Since we use a rationalizability notion with belief restrictions as solution concept, our work is also related to the mechanism design literature on implementation in rationalizable strategies. Perhaps the closest work to ours here is Ollár and Penta (2017), who study full implementation in rationalizable strategies with belief restrictions. A direct comparison is not straightforward: Ollár and Penta (2017) work with general belief restrictions and smoothness assumptions while we focus on log-concave marginal beliefs but in a discrete setting. On top of it, we allow for unawareness.

The paper is organized as follows: The next section outlines the model and our solution concept. In Section 3, we study the case in which the principal is initially unaware of lower marginal cost types only, both in a setting with three types and in a setting with an arbitrary number of finite types. Section 4 follows up with the analysis for the case in which the principal is initially unaware of higher marginal cost types only. We conclude with a discussion in Section 5. Proofs are relegated to appendices.

## 2 Model

### 2.1 Screening Game with Unawareness

A principal ( $P$ , “she”) wants to procure  $q \geq 0$  units of output from an agent ( $A$ , “he”). The principal’s utility of output is given by a twice continuously differentiable function  $v$  with  $v'(q) > 0$ ,  $v''(q) < 0$  and  $v(0) = 0$ ; we also impose the Inada condition  $\lim_{q \rightarrow 0} v'(q) = +\infty$ . The net utility or payoff for the principal from procuring  $q$  units from the agent in exchange for payment  $t \geq 0$  is  $u_P(q, t) = v(q) - t$ . We assume that contracts are bounded:  $q, t \leq b$  for some bound  $b > 0$ .

The agent’s marginal cost of production depends on the initial move of nature. Let  $\bar{\Theta}$  be the nonempty finite set of all marginal cost types; for simplicity, we set  $\bar{\Theta} = \{1, \dots, n\}$  for some natural number  $n > 1$ . The profit of an agent of type  $\theta$  from selling  $q$  units to the principal and collecting payment  $t$  is then  $u_A(q, t, \theta) = t - \theta q$ .

To represent asymmetric awareness of marginal costs, we assume that the agent is aware of all marginal cost types in  $\Theta_A = \bar{\Theta}$ —no matter what his type is—while the principal is initially unaware of some types:  $\Theta_P \subsetneq \bar{\Theta}$ .<sup>1</sup> Although our representation of the principal’s awareness level is unidimensional, we think of the agent’s type more as a score aggregating the impact of various events affecting marginal costs reminiscent of “pseudotypes” in scoring auctions; see, for instance, Asker and Cantillon (2006) or Bajari, Houghton, and Tadelis (2014). For instance, the agent’s marginal cost may be result of two components and the principal is only aware of one of them.

The game starts with Nature drawing the marginal cost type  $\theta$  of the agent from  $\bar{\Theta}$ . The agent is privately informed about his type and then decides whether to disclose the existence of some subset of cost types to the principal by sending her a message  $\Theta$  such that  $\Theta_P \subseteq \Theta \subseteq \bar{\Theta}$ . He can only communicate types of which he himself is aware. Moreover, the agent cannot reduce the principal’s awareness.

The principal’s initial awareness level,  $\Theta_P$ , and the message by the agent,  $\Theta$ , determines her interim awareness and subsequent choice of contract menus. She cannot reason about types she does not conceive of interim. We model this by considering poorer descriptions of the game trees in which nature draws initially only among types of which the principal is aware. There is a forest of game trees, one tree for each possible space of draws of nature she could become aware after disclosure by the agent. These trees are ordered by the richness of their set of moves of nature. See Figure 1 for an example with  $\bar{\Theta} = \{1, 2, 3, 4\}$ ,  $\Theta_P = \{1, 2\}$ , and three trees:  $T_{\bar{\Theta}}$  is the tree with full awareness,  $T_{\Theta_P}$  is the tree with the principal’s initial awareness level, and the intermediate tree is the tree where the agent made the principal aware of cost type 3. Depending on the disclosure, the principal lives in one of those trees as indicated by the information sets; if the agent remains silent, then her awareness is given by the lowest tree. In Figure 1, we can also appreciate another non-standard feature of extensive-form games with unawareness: A player’s information set at a history may consist of histories in a lower tree, signifying the fact that players may be unaware of some moves. See Heifetz, Meier, and

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<sup>1</sup>Letting the agent’s awareness depend on his type would make it easier for the principal to infer the agent’s type upon becoming aware. Our goal is to focus on the informativeness of disclosing awareness by itself, so we do not allow this. See Pram (2021) and Ali, Lewis, and Vasserman (2022) for disclosure of information only.

Schipper (2013) for details on extensive-form games with unawareness.

After disclosure, the principal offers a menu of contracts to the agent. He picks one of the contracts or the outside option, whose payoff is normalized to zero, and the game ends.<sup>2</sup>

## 2.2 Solution Concept

In order to define our solution, we first specify strategies and belief systems. Formally, let  $H_i$  be player  $i$ 's set of information sets, for  $i = A, P$ , across all trees. A *strategy* of player  $i$  assigns to each of  $i$ 's information sets an action available at that information set. For the principal, these actions are simply the offer of a menu of contracts designed for the types of which she is aware at the given information set; for the agent, the actions consist of a set of types of which to make the principal aware, and a choice of contract in the menu offered by the principal. Strategies are denoted by  $(s_i(h_i))_{h_i \in H_i}$ . Let  $S_i$  be the set of strategies of player  $i$  and  $S_{-i}$  be the set of strategies of player  $i$ 's opponent. Note that the agent has perfect information everywhere; his information sets are singleton.

A strategy  $s_i$  reaches information set  $h_i$  if there exists an opponent's strategy  $s_{-i}$  and move of nature such that the path induced by  $(s_i, s_{-i})$  and the move of nature leads to the information set  $h_i$ . Denote by  $T_{h_i}$  the tree in which information set  $h_i$  is located. Similarly, we denote by  $\Theta_{h_i}$  the set of marginal cost types of which the principal is aware in the tree in which information set  $h_i$  is located. Finally, let  $s^T$  be the  $T$ -partial strategy restricted to information sets in tree  $T$  and any tree poorer than  $T$ , with  $S^T$  denoting the set of  $T$ -partial strategies.

As the agent has perfect information about the move of nature, he forms beliefs only about the principal's strategy. A *belief system* of the agent is given by:<sup>3</sup>

$$\beta_A = (\beta_A(h_A))_{h_A \in H_A} \in \prod_{h_A \in H_A} \Delta(S_P^{T_{h_A}}),$$

which is a profile of beliefs—a belief  $\beta_A(h_A) \in \Delta(S_P^{T_{h_A}})$  for each information set  $h_A \in H_A$  about the principal's strategies in the  $T_{h_A}$ -partial game—with the following properties:

- (i) *Non-delusion*:  $\beta_A(h_A)$  reaches  $h_A$ ; i.e.,  $\beta_A(h_A)$  assigns probability 1 to the set of strategy profiles of the principal that reach  $h_A$ .
- (ii) *Bayesianism*: If  $h_A$  precedes  $h'_A$ , then  $\beta_A(h'_A)$  is the conditional belief derived from  $\beta_A(h_A)$  whenever possible.

The principal forms beliefs both over strategies of the agent and moves of nature. Her *belief system* is given by a profile of beliefs:

$$\beta_P = (\beta_P(h_P))_{h_P \in H_P} \in \prod_{h_P \in H_P} \Delta(S_A^{T_{h_P}} \times \Theta_{h_P})$$

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<sup>2</sup>We could allow for the principal to offer first an initial menu of contracts, after which she might be made aware of additional marginal cost types of the agent and offers a possibly revised menu of contracts. This would not change our results in any essential way, so we opt for the simpler description of the game.

<sup>3</sup>For any compact set  $K$ , let  $\Delta(K)$  denote the set of Borel probability measures on  $K$ . Note that the principal's set of strategies is infinite but compact.

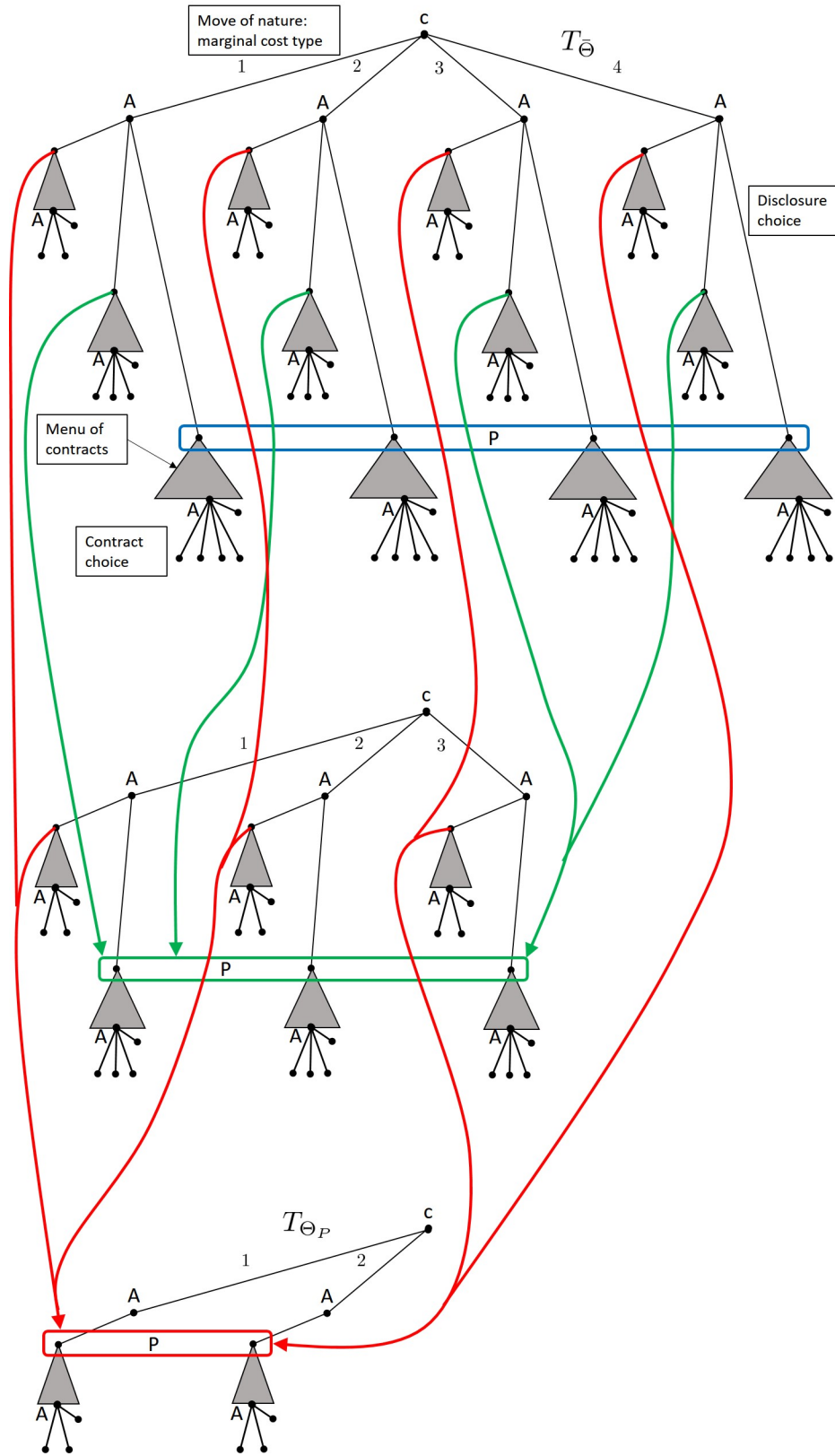


Figure 1: Example of the Game Form

which has the following properties:

- (i) *Non-delusion*:  $\beta_P(h_P)$  reaches  $h_P$ ; i.e.,  $\beta_P(h_P)$  assigns probability 1 to the set of strategy profiles of the agent and moves of nature that reach  $h_P$ .
- (ii) *Logconcavity*: Denote by  $p_{\Theta_{h_P}} := \text{marg}_{\Theta_{h_P}} \beta_P(h_P)$  the marginal probability on marginal cost types in  $\Theta_{h_P}$  and by  $\text{supp } p_{\Theta_{h_P}} = \{\theta : p_{\Theta_{h_P}}(\theta) > 0\}$ , its support. Let  $\theta^{(i)}$  denote the order statistics of the  $\text{supp } p_{\Theta_{h_P}}$ , and let  $p_{\Theta_{h_P}}^i = p_{\Theta_{h_P}}(\theta^{(i)})$ . We require that  $p_{\Theta_{h_P}}$  is logconcave; i.e., if  $|\text{supp } p_{\Theta_{h_P}}| > 2$ , for all  $i = 2, \dots, |\text{supp } p_{\Theta_{h_P}}| - 1$ ,

$$p_{\Theta_{h_P}}^i p_{\Theta_{h_P}}^{i+1} \geq p_{\Theta_{h_P}}^{i+1} p_{\Theta_{h_P}}^{i-1}.$$

- (iii) *“Reverse” Bayesianism*: Let  $\Theta_{h_P} \subseteq \Theta_{h'_P}$ . If  $\beta_P$  is such that the principal at  $h'_P$  cannot rule out any type in  $\Theta_{h_P}$  from the agent disclosing  $\Theta_{h'_P}$ , then the marginals of the principal’s belief systems  $p_{\Theta_{h_P}}$  and  $p_{\Theta_{h'_P}}$  satisfy “reverse” Bayesianism: For all  $\theta, \theta' \in \Theta_{h_P}$  in the support of both  $p_{\Theta_{h_P}}$  and  $p_{\Theta_{h'_P}}$ ,

$$\frac{p_{\Theta_{h_P}}(\theta')}{p_{\Theta_{h_P}}(\theta)} = \frac{p_{\Theta_{h'_P}}(\theta')}{p_{\Theta_{h'_P}}(\theta)}.$$

The second condition implies non-decreasing hazard rates (see Lemma 8 in the appendix), a standard condition typically assumed in screening problems and other problems of information economics (Mussa and Rosen, 1978; Spence, 1980; Baron and Myerson, 1982; Maskin and Riley, 1984; Matthews and Moore, 1986; Bagnoli and Bergstrom, 2005). Monotone hazard rates facilitate solving for the optimal menu of contracts subject to the agent’s incentive constraints. Note that we weaken log-concavity to apply only to non-zero probabilities, which means that the marginal beliefs of the principal on marginal cost types are unimodal with respect to all marginal cost types that get assigned strictly positive probability.

The third condition says that, after becoming aware of additional types in  $\Theta_{h'_P}$ , the relative likelihood of types that she has been aware of at  $\Theta_{h_P}$  should remain the same, provided that they are not ruled out. Such a condition has been suggested and axiomatized by Karni and Vierø (2013) for updating beliefs of a single decision maker upon becoming aware (see also Dominiak and Tserenjigmid, 2018).<sup>4</sup> This condition is less compelling in a game theoretic setting in which, conditional on an information set, a player may not only increase her own awareness but at the same time infer from the opponent’s actions some information about the latter’s types as well. However, note that the assumption is mute for types that are assigned zero probability upon becoming aware (because, for instance, they can be ruled out from raising the principal’s awareness). This condition allows us to link first-level beliefs and relate menus of contracts across trees, and also facilitates the analysis of rational disclosure decisions by the agent. In addition to the restrictions discussed above, we sometimes also impose a tie-breaking condition

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<sup>4</sup>Their axioms feature invariant risk preferences; i.e., risk preferences that do not change with changes in awareness. This is also an implicit assumption made in games with unawareness. The assignment of payoffs to terminal histories does not depend on the game tree in the forest but just on the terminal history as long as the terminal history exists in the tree.



and common belief of this tie-breaking condition. We discuss the nature of this condition as well as its importance for some of our results later on in the text.

For any player  $i = A, P$ , a strategy  $s_i$  is *sequentially rational* at information set  $h_i$  with belief  $\beta_i(h_i)$  if either  $s_i$  does not reach  $h_i$  or there does not exist a strategy  $s'_i$  that coincides with  $s_i$  for all information sets of player  $i$  preceding  $h_i$  and yields a higher expected payoff given  $\beta_i(h_i)$ .

Let  $B_i$  denote player  $i$ 's set of belief systems. We apply a cautious version of extensive-form rationalizability with first-order marginal beliefs on types restricted by logconcavity and reverse Bayesianism.

**Definition 1 ( $\Delta$ -Prudent Rationalizable Strategies)** For each player  $i \in \{A, P\}$ , define inductively<sup>5</sup> the following sequences of belief systems and strategies:

$$\begin{aligned}
 R_i^0 &= S_i \text{ and, for } k \geq 1, \\
 B_A^k &= \left\{ \beta_A \in B_A : \begin{array}{l} \text{For every } h_A, \beta_A(h_A) \text{ has full support on the set of all} \\ s_P \in R_P^{k-1, T_{h_A}} \text{ that reach } h_A, \text{ provided this set is non-empty.} \end{array} \right\} \\
 B_P^k &= \left\{ \beta_P \in B_P : \begin{array}{l} \text{For every } h_P, \text{ the support of } \beta_P(h_P) \text{ is given by the set of all} \\ (s_A, \theta) \in R_A^{k-1, T_{h_P}} \times \Theta_{h_P} \text{ that reach } h_P \text{ if this set is non-empty.} \end{array} \right\} \\
 R_i^k &= \left\{ s_i \in R_i^{k-1} : \begin{array}{l} \text{There exists a belief system } \beta_i \in B_i^k \text{ with which, for every} \\ \text{information set } h_i \in H_i, s_i \text{ is sequentially rational at } h_i. \end{array} \right\}
 \end{aligned}$$

The set of player  $i$ 's  $\Delta$ -prudent rationalizable strategies is:

$$R_i^\infty = \bigcap_{k=1}^{\infty} R_i^k.$$

We call our solution concept  $\Delta$ -prudent rationalizability because it features restrictions on first-order beliefs. Rationalizability with such restrictions has been called  $\Delta$ -rationalizability by Battigalli and Siniscalchi (2003) and Battigalli and Prestipino (2013), and extensive-form best response sets and directed rationalizability by Battigalli and Friedenberg (2012). Heifetz, Meier, and Schipper (2021) use the name prudent rationalizability for a cautious version of extensive-form rationalizability in order to distinguish it from cautious rationalizability in Pearce (1984), who presents another but related solution concept. Our solution concept also includes features of iterated admissibility and self-admissible sets (Brandenburger, Friedenberg, and Keisler, 2008; Brandenburger and Friedenberg, 2010). Meier and Schipper (2012) show the equivalence of prudent rationalizability to iterated elimination of conditional weakly dominated strategies and to a version of iterated admissibility conditional on partial-games of the extensive-form game with unawareness.

This solution concept is easier to interpret than standard equilibrium solution concepts like perfect Bayesian equilibrium or sequential equilibrium. It captures common cautious (strong) belief in rationality (see Battigalli and Siniscalchi, 2002; Guarino, 2020) and common belief in

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<sup>5</sup> Although our game is infinite due to the principal's infinite strategy set, there is no need to consider transfinite induction because strategy sets are compact and utility functions are continuous (Bernheim, 1984, and Battigalli, 2003). In fact, we show in the proofs that the maximal reduction obtains after a few finite levels.

reverse Bayesianism and in logconcavity of marginals on types. Important for our purpose, it does not assume that players are automatically certain of a ready-made convention of play upon becoming aware, like any equilibrium concept assumes, as it is not clear how this could emerge under unawareness.<sup>6</sup> Note that since it is an iterated elimination procedure on strategies, it yields predictions for every finite level  $k$  of mutual (strong) belief. This is akin to level- $k$  reasoning in experimental game theory and should turn out useful for future experimental tests of our theory.<sup>7</sup>

### 3 Unawareness of Low Marginal Costs Only

#### 3.1 Three Marginal Cost Types

Before we discuss a more general model, consider the case with three types,  $\bar{\Theta} = \{1, 2, 3\}$ , in which the principal is unaware of the lowest-possible marginal cost:  $\Theta_P = \{2, 3\}$ . Thus, there are two trees:  $T_{\bar{\Theta}}$ , representing full awareness, and  $T_{\Theta_P}$ , representing the principal's initial awareness level.

We characterize  $\Delta$ -prudent rationalizable strategies level-by-level. At level 1, since the principal has full support beliefs, she offers a menu of exactly two contracts in the lower tree and a menu of exactly three contracts in the upper tree. Each menu has to maximize her expected profit given some full support belief over marginal cost types and strategies of the agent. The difference is that at the lower tree she is aware of two types only while at her information set in the upper tree she can take into account all three marginal cost types. The fact that in the upper tree she has been made aware of  $\theta = 1$  by the agent does not allow her to exclude any agent type because at level 1 no restrictions are implied yet on the agent's strategies. In the lower tree, she can offer any optimal menu of two contracts in  $([0, b]^2)^2$  with the belief that puts sufficiently large probability to one type of the agent accepting only the one contract and rejecting all others, and the other type accepting only the other contract and rejecting all others. Analogously, in the upper tree, she can offer any optimal menu of three contracts.

The agent at level 1, when faced by a menu of contracts, selects a contract that maximizes his payoff unless none satisfies his participation constraint, in which case he selects the outside option. As shown in the appendix, the agent's payoff function satisfies decreasing differences in quantities and marginal cost types (Lemma 2), hence his optimal contract quantity is decreasing in his type (Lemma 3). Note that each of the last information sets of the agent in both trees are singleton. Thus, he knows the menu of contracts offered.

Regarding raising the principal's awareness: At his first information set in the upmost tree, any action by the agent can be optimal with an appropriate belief over the principal's contracts. He may make the principal aware of  $\theta = 1$  if he believes with sufficiently high probability that he gets a better deal; he may also keep the principal in the dark about  $\theta = 1$  if he believes with sufficiently high probability that it would result in a worse deal. This is because no restrictions

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<sup>6</sup>See Schipper (2021) for a discussion of equilibrium notions under unawareness.

<sup>7</sup>See Li and Schipper (2020, 2018) for experimental tests of prudent rationalizability without restrictions in disclosure games.

on the principal's strategies can be assumed at level 1.

At level 2, the principal is certain of first-level  $\Delta$ -prudent rationalizable strategies of the agent. Thus, she is certain that the agent observes participation constraints, self-selects into a contract in the menu according to his incentives, and that chosen quantities are monotone in the agent's type. Since level 1 imposes no restrictions on the agent's decision w.r.t. disclosure, both disclosing and not are rationalizable for every type of the agent at level 1. Any belief system  $\beta_P \in B_P^2$  for the principal must put strictly positive probability on any type making her aware when reaching her information set in the upper tree  $T_{\bar{\Theta}}$ . Our solution concept requires in this case that her marginal beliefs satisfy reverse Bayesianism.

With such beliefs, any second-level  $\Delta$ -prudent menu of contracts must satisfy the following. The menu of contracts offered in tree  $T_{\bar{\Theta}}$  solves:

$$\max_{(q_{\bar{\Theta}}^{\theta}, t_{\bar{\Theta}}^{\theta})_{\theta=1,2,3}} \sum_{\theta=1}^3 p_{\bar{\Theta}}(\theta) \left( v \left( q_{\bar{\Theta}}^{\theta} \right) - t_{\bar{\Theta}}^{\theta} \right)$$

subject to incentive compatibility constraints  $IC_{\bar{\Theta}}$ : For  $\theta = 1, 2$ ,

$$t_{\bar{\Theta}}^{\theta} - \theta q_{\bar{\Theta}}^{\theta} \geq t_{\bar{\Theta}}^{\theta+1} - \theta q_{\bar{\Theta}}^{\theta+1}$$

and the participation constraint  $PC_{\bar{\Theta}}$ :

$$t_{\bar{\Theta}}^3 - 3q_{\bar{\Theta}}^3 \geq 0,$$

while the principal's menu of contracts in the lower tree  $T_{\Theta_P}$  solves:<sup>8</sup>

$$\max_{(q_{\Theta_P}^{\theta}, t_{\Theta_P}^{\theta})_{\theta=2,3}} p_{\Theta_P}(2) \left( v \left( q_{\Theta_P}^2 \right) - t_{\Theta_P}^2 \right) + p_{\Theta_P}(3) \left( v \left( q_{\Theta_P}^3 \right) - t_{\Theta_P}^3 \right)$$

subject to  $IC_{\Theta_P}$ :

$$t_{\Theta_P}^2 - 2q_{\Theta_P}^2 \geq t_{\Theta_P}^3 - 2q_{\Theta_P}^3$$

and  $PC_{\Theta_P}$ :

$$t_{\Theta_P}^3 - 3q_{\Theta_P}^3 \geq 0.$$

In stating these problems, we make use of the fact that the principal is certain that the agent is rational, hence the incentive compatibility and participation constraints. We also make use of the fact that, as the principal's marginal on cost types is logconcave, solutions will satisfy a monotonicity constraint: Higher marginal cost types select lower quantities (see Appendix A). With such a monotonicity constraint, only local upward (in terms of marginal costs) incentive compatibility constraints need to be considered, instead of all incentive compatibility constraints (see Lemmas 4 and 5 in Appendix A).

The solutions to the principal's programs are characterized as follows (see Appendix A). For the principal's information set in  $T_{\bar{\Theta}}$ , optimal outputs  $\hat{q}_{\bar{\Theta}}^{\theta}$  for  $\theta = 1, 2, 3$  must satisfy:

$$v' \left( \hat{q}_{\bar{\Theta}}^1 \right) = 1,$$

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<sup>8</sup>Recall that  $p_{\Theta}(\theta)$  denotes the principal's marginal belief on type  $\theta \in \Theta$  at her information set where she is aware of the types in  $\Theta$ .

$$v'(\hat{q}_{\bar{\Theta}}^\theta) = \theta + \frac{\sum_{i < \theta} p_{\bar{\Theta}}(i)}{p_{\bar{\Theta}}(\theta)};$$

transfers are given by:

$$\begin{aligned} \hat{t}_{\bar{\Theta}}^3 &= 3\hat{q}_{\bar{\Theta}}^3, \\ \hat{t}_{\bar{\Theta}}^\theta &= \theta\hat{q}_{\bar{\Theta}}^\theta + \sum_{i > \theta} \hat{q}_{\bar{\Theta}}^i. \end{aligned}$$

For her information set in  $T_{\Theta_P}$ , the optimal outputs satisfy:

$$\begin{aligned} v'(\hat{q}_{\Theta_P}^2) &= 2, \\ v'(\hat{q}_{\Theta_P}^3) &= 3 + \frac{p_{\Theta_P}(2)}{p_{\Theta_P}(3)}; \end{aligned}$$

transfers are:

$$\begin{aligned} \hat{t}_{\Theta_P}^3 &= 3\hat{q}_{\Theta_P}^3, \\ \hat{t}_{\Theta_P}^2 &= 2\hat{q}_{\Theta_P}^2 + \hat{q}_{\Theta_P}^3. \end{aligned}$$

Moreover, as the principal's beliefs are required to satisfy reverse Bayesianism, we can conclude that for  $\theta = 2, 3$ ,

$$\hat{q}_{\bar{\Theta}}^\theta < \hat{q}_{\Theta_P}^\theta.$$

This inequality follows from Lemma 12 (i) in the appendix, but it can also be seen directly in this example. Take, for instance  $\theta = 3$ ; since  $v''(q) < 0$ , it suffices to show that  $v'(\hat{q}_{\bar{\Theta}}^3) > v'(\hat{q}_{\Theta_P}^3)$ . Under prudent beliefs,  $p_{\bar{\Theta}}(\theta) > 0$  for all  $\theta \in \bar{\Theta}$ , and so:

$$\frac{p_{\bar{\Theta}}(1)}{p_{\bar{\Theta}}(3)} + \frac{p_{\Theta_P}(2)}{p_{\Theta_P}(3)} > \frac{p_{\Theta_P}(2)}{p_{\Theta_P}(3)}.$$

By reverse Bayesianism,

$$\frac{p_{\Theta_P}(2)}{p_{\Theta_P}(3)} = \frac{p_{\bar{\Theta}}(2)}{p_{\bar{\Theta}}(3)}$$

and so:

$$\begin{aligned} \frac{p_{\bar{\Theta}}(1)}{p_{\bar{\Theta}}(3)} + \frac{p_{\bar{\Theta}}(2)}{p_{\bar{\Theta}}(3)} &> \frac{p_{\Theta_P}(2)}{p_{\Theta_P}(3)} \\ 3 + \frac{p_{\bar{\Theta}}(1) + p_{\bar{\Theta}}(2)}{p_{\bar{\Theta}}(3)} &> 3 + \frac{p_{\Theta_P}(2)}{p_{\Theta_P}(3)} \\ v'(\hat{q}_{\bar{\Theta}}^3) &> v'(\hat{q}_{\Theta_P}^3), \end{aligned}$$

as desired.

For the agent, any level-1 rationalizable strategy is also level-2 rationalizable. At level 3, the agent is certain of level-2 rationalizable strategies of the principal. The highest marginal cost type,  $\theta = 3$ , is indifferent between disclosing his existence or not: He rejects the optimal contract in  $T_{\Theta_P}$  in favor of his outside option, and he is held to his outside option under the optimal contract in  $T_{\bar{\Theta}}$ . In order to facilitate the analysis, we will use as a tie-breaking assumption that type  $\theta = 3$  does *not* disclose. While we do not model this disclosure cost explicitly, type  $\theta = 3$  would never disclose if disclosure bears a cost, even an infinitesimal one. We further discuss the formal role of this assumption in our results in the next subsection.

For  $\theta = 2$ , the intermediate marginal cost type, the disclosure decision is less trivial. He is certain of level-2  $\Delta$ -prudent rationalizable strategies of the principal. Thus, with any level-3 belief system, he is certain that the principal's menu of contracts offered upon becoming aware satisfies the monotonicity property of quantities for each type across trees implied by reverse Bayesianism.

We claim that, with any such a belief system, type  $\theta = 2$  prefers not to raise the principal's awareness. Recall that  $\hat{q}_\Theta^3 < \hat{q}_{\Theta_P}^3$ , an implication of reverse Bayesianism (Lemma 12 (i)). So,

$$\begin{aligned} 3\hat{q}_\Theta^3 - 2\hat{q}_\Theta^3 &< 3\hat{q}_{\Theta_P}^3 - 2\hat{q}_{\Theta_P}^3 \\ \hat{t}_\Theta^3 - 2\hat{q}_\Theta^3 &< \hat{t}_{\Theta_P}^3 - 2\hat{q}_{\Theta_P}^3 \end{aligned}$$

By (binding) incentive compatibility, the payoffs on each side of the inequality equal the payoffs for  $\theta = 2$  under the contracts meant for him in each menu. It follows that:

$$u_A(\hat{q}_\Theta^2, \hat{t}_\Theta^2, 2) < u_A(\hat{q}_{\Theta_P}^2, \hat{t}_{\Theta_P}^2, 2).$$

The low cost type,  $\theta = 1$ , of which the principal is initially unaware, does not want to alert the principal of his existence. To see this, note that at level 3, he is certain of level-2  $\Delta$ -prudent rationalizable strategies of the principal. Thus, he is certain that the principal's menu of contracts offered upon becoming aware satisfies the monotonicity property of quantities for each type across trees implied by reverse Bayesianism of the principal. With

$$u_A(\hat{q}_\Theta^2, \hat{t}_\Theta^2, 2) < u_A(\hat{q}_{\Theta_P}^2, \hat{t}_{\Theta_P}^2, 2)$$

and  $\hat{q}_\Theta^2 < \hat{q}_{\Theta_P}^2$  by reverse Bayesianism,

$$\begin{aligned} u_A(\hat{q}_\Theta^2, \hat{t}_\Theta^2, 2) + \hat{q}_\Theta^2 &< u_A(\hat{q}_{\Theta_P}^2, \hat{t}_{\Theta_P}^2, 2) + \hat{q}_{\Theta_P}^2 \\ \hat{t}_\Theta^2 - 2\hat{q}_\Theta^2 + \hat{q}_\Theta^2 &< \hat{t}_{\Theta_P}^2 - 2\hat{q}_{\Theta_P}^2 + \hat{q}_{\Theta_P}^2 \\ \hat{t}_\Theta^2 - \hat{q}_\Theta^2 &< \hat{t}_{\Theta_P}^2 - \hat{q}_{\Theta_P}^2 \end{aligned}$$

By (binding) incentive compatibility, the left-hand side of the inequality becomes:

$$\begin{aligned} \hat{t}_\Theta^1 - \hat{q}_\Theta^1 &< \hat{t}_{\Theta_P}^2 - \hat{q}_{\Theta_P}^2 \\ u_A(\hat{q}_\Theta^1, \hat{t}_\Theta^1, 1) &< u_A(\hat{q}_{\Theta_P}^2, \hat{t}_{\Theta_P}^2, 1). \end{aligned}$$

The left-hand side of the last inequality is the payoff for type  $\theta = 1$  under the optimal contract in  $T_\Theta$ , while the right-hand side is his payoff in  $T_{\Theta_P}$ , where he chooses the contract meant for type  $\theta = 2$ . Thus, the lowest-cost type prefers to remain under the principal's radar.

For the principal, all level-2 rationalizable strategies are also level-3 rationalizable since there is no change of strategies of the agent at level 2. Consequently, all level-3 rationalizable strategies of the agent are also level-4 rationalizable. Finally, since none of the types raises the principal's awareness at level 3, she must fall back on level-2 prudent rationalizable strategies of the agent upon becoming aware, and no further reduction of her strategy set occurs at level 4. This concludes the analysis.

To summarize: If the principal is unaware of low marginal cost events only, then in any  $\Delta$ -prudent rationalizable outcome she offers a menu of contracts for all types of the agent of which she is initially aware, since *none of the agent's types raises her awareness*. Thus, the principal remains unaware of the low marginal cost events. There is *bunching at the top* as the low marginal cost type selects the same contract as the intermediate marginal cost type.

### 3.2 Any Finite Number of Marginal Cost Types

We now consider more generally any finite number of marginal cost types, which allows us to address new questions: Is it still the case that there is no disclosure whatsoever when more types are allowed? If the agent raises the principal's awareness, the existence of how many types does he disclose?

In this section, we assume that  $\Theta_P = \{j, \dots, n\}$  for some  $1 < j < n$ . Thus, the principal is initially unaware of the marginal cost types  $\{1, \dots, j - 1\}$ . We call this model the model with unawareness of low marginal costs only. In this model, we invoke the tie-breaking assumption from the previous subsection: When a type of the agent is indifferent between raising the principal's awareness or not, we assume that he does not. This pertains especially to the type with the highest marginal cost type, who is indifferent between raising the principal's awareness or not because he is always held to his outside option.<sup>9</sup>

With a slight abuse of notation, let  $\Theta(\theta)$  denote the set of types that type  $\theta \in \bar{\Theta}$  discloses to the principal at said type's information set in the solution. Recall that  $\Theta_P \subseteq \Theta(\theta) \subseteq \bar{\Theta}$  for all  $\theta \in \bar{\Theta}$ .

**Proposition 1** *Consider the model in which the principal is unaware of low marginal cost events only:  $\Theta_P = \{j, \dots, n\}$  for some  $1 < j < n$ . Assume that it is commonly believed upon becoming aware that the highest marginal cost type does not raise the principal's awareness. In any  $\Delta$ -prudent rationalizable outcome, the following holds:*

- (i) *The principal does not become aware of all of the agent's types:  $\Theta(\theta) \subset \bar{\Theta}$  for all  $\theta \in \bar{\Theta}$ .*
- (ii) *None of the agent's types of which the principal is initially aware raises her awareness of further types:  $\Theta(\theta) = \Theta_P$  for all  $\theta \in \Theta_P$ .*
- (iii) *None of the agent's types of which the principal is initially unaware raises her awareness to a level that includes his own type:  $\min(\Theta(\theta)) > \theta$  for all  $\theta = 1, \dots, j - 1$ .*
- (iv) *The principal offers an optimal menu of contracts for all types of which she has been or became aware.*
- (v) *There is bunching at the top: Each type  $\theta = 1, \dots, j - 1$  chooses the contract designed for type  $\min(\Theta(\theta))$ . In words, low marginal cost types of whom the principal remains unaware select a contract for the lowest marginal cost type of which she becomes aware.*

The proof proceeds by induction on the levels of rationalizability and is contained in Appendix B. At the first level, any type of the agent chooses optimally among the menu of contracts offered by the principal. At the second level, the principal is certain that any type of the agent observes incentive and participation constraints. The third level implies crucial constraints on the disclosure of awareness by agents. The type with the highest marginal cost is indifferent between raising her awareness or not, because he is held to his outside option in any event. According to the tie-breaking assumption, he does not raise the principal's awareness. For the other types of which the principal is initially aware, raising the principal's awareness

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<sup>9</sup>We discuss the role of this assumption after stating the result.

of lower marginal costs would reduce the optimal quantities for them. Since information rents earned by these types are weighted by those quantities, it would decrease their payoffs.

Low-cost types of which the principal is initially unaware may raise the principal’s awareness, but only of low-cost types higher than their own. Such types may tell the principal that there could be more efficient procurers than she has considered, but not as efficient as they themselves actually are. Consequently, at the fourth level, if the principal’s awareness is raised, she realizes that no type she is now aware of would want to raise her awareness to that extent. She cannot rationalize the agent’s disclosure action any further. In line with the best-rationalizability principle embodied in our solution concept, the principal stays with her third-level rationalizable strategy of offering a menu of contracts for all types she is aware that is optimal w.r.t. some of her third-level belief systems. At that point, the procedure concludes.

We discuss the role of assumptions embodied in our solution concept. Assuming logconcavity of beliefs rules out (standard) bunching (see Appendix A). It ensures that the quantities offered to lower marginal cost types are larger, because it implies monotone inverse hazard rates in the principal’s first-order conditions. To some extent, it aligns incentives of the principal with incentives of the agent as the agent’s payoff function has decreasing differences in quantities and marginal cost types. It also ensures that only local incentive compatibility constraints need to be considered and thus simplifies the principal’s optimization problem. Without this assumption, the menu of contracts offered by the principal would be much more difficult to analyze.

Reverse Bayesianism facilitates the comparison of the agent’s payoff from making the principal aware or not. It says that the distribution over agent’s types upon becoming aware respects the same relative likelihoods for types the principal has been aware of initially, unless they are ruled out. Therefore, it has implications for the inverse hazard rates in the principal’s first-order conditions before and after becoming aware. Without this assumption, the agent’s disclosure decision would be much more difficult to analyze and would depend on additional assumptions on the value function  $v$ .

Assuming that the principal knows that the type with the highest marginal costs would not raise her awareness facilitates the analysis and allows us to derive a nonempty set of strategies with our solution concept. Without this assumption, upon seeing disclosure, we cannot rule out the principal being certain that only the type with the highest marginal cost made her aware. However, when offering a menu of a single contract that is tailor-made for this type and hence ignores all incentive compatibility constraints for other types, all other types will want to raise her awareness as well. But then, believing this and offering a menu of contracts for all those types, they would not all want to disclose. Thus, we are led to a cycle. In other words, common belief in cautious rationality, logconcavity, and reverse Bayesian may be empty without the assumption of common belief in the tie-breaking condition.<sup>10</sup>

While such a tie-breaking restriction is foreign to rationalizability concepts, which place restrictions on beliefs rather than directly on behavior, they are common for equilibrium notions. For instance, when constructing mixed equilibria, the players are typically indifferent among

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<sup>10</sup>It is well known that further restrictions may not shrink the set of strategies but may yield a different set of strategies. See Heifetz, Meier, and Schipper (2021, Section 4.2) for an example on extensive-form rationalizability versus prudent rationalizability. Battigalli and Prestipino (2013) and Battigalli and Friedenberg (2012) also discuss how restrictions affect  $\Delta$ -rationalizability or extensive-form best response sets.

an infinite number of mixtures and the game theorist picks the one that suits the equilibrium construction. With our tie-breaking assumption, we follow a similar convention just for one type of which the principal is initially aware.

## 4 Unawareness of High Marginal Cost Events Only

### 4.1 Three Marginal Cost Types

Recall the three-type example of Section 3,  $\bar{\Theta} = \{1, 2, 3\}$ , but now assume that  $\Theta_P = \{1, 2\}$ : The principal is unaware of the highest marginal-cost type.

We characterize  $\Delta$ -prudent rationalizable strategies level-by-level. The analysis of level 1 is analogous to the analysis of level 1 in Section 3.1. The principal offers expected profit maximizing menus of two contracts in the lower tree and of at most three contracts in the upper tree given some full-support belief system. The agent at level 1 may or may not make the principal aware of  $\theta = 3$  and self-selects a contract from the menu according to his incentives. Since the agent's payoff has decreasing differences in quantities and marginal costs (Lemma 2), optimal contract quantities are decreasing in marginal cost type modulo transfer (Lemma 3). If his participation constraint cannot be satisfied with any contract, he selects the outside option.

At level 2, the principal is certain of first-level  $\Delta$ -prudent rationalizable strategies of the agent. Thus, she is certain that the agent observes participation constraints and self-selects among contracts according to his incentives.

Since the first level imposes no restrictions on the agent's decision regarding disclosure, both actions of disclosing and not are rationalizable for every type of the agent at level 1. So, any belief system  $\beta_P \in B_P^2$  must put strict positive probability on any type making her aware when reaching her information set in the upper tree  $T_{\bar{\Theta}}$ . Our solution concept requires in this case that her marginal belief on  $\bar{\Theta}$  satisfy reverse Bayesianism. The principal's problem at this level in tree  $T_{\bar{\Theta}}$  is the same as in Section 3.1. Her menu of contracts in the lower tree  $T_{\Theta_P}$  solves:

$$\max_{(q_{\Theta_P}^\theta, t_{\Theta_P}^\theta)_{\theta=1,2}} p_{\Theta_P}(1) (v(q_{\Theta_P}^1) - t_{\Theta_P}^1) + p_{\Theta_P}(2) (v(q_{\Theta_P}^2) - t_{\Theta_P}^2)$$

subject to  $IC_{\Theta_P}^{(1)}$ :

$$t_{\Theta_P}^1 - q_{\Theta_P}^1 \geq t_{\Theta_P}^2 - q_{\Theta_P}^2$$

and  $PC_{\Theta_P}^{(2)}$ :

$$t_{\Theta_P}^2 - 2q_{\Theta_P}^2 \geq 0.$$

Again, in stating these problems, we make use of the fact that the principal at level 2 is certain that the agent is rational and that the principal's belief on agent types is logconcave.

The solutions to this problem are:

$$\begin{aligned} v'(\hat{q}_{\Theta_P}^1) &= 1, \\ v'(\hat{q}_{\Theta_P}^2) &= 2 + \frac{p_{\Theta_P}(1)}{p_{\Theta_P}(2)}; \end{aligned}$$



transfers are:

$$\begin{aligned}\hat{t}_{\Theta_P}^1 &= \hat{q}_{\Theta_P}^1 + \hat{q}_{\Theta_P}^2, \\ \hat{t}_{\Theta_P}^2 &= 2\hat{q}_{\Theta_P}^2.\end{aligned}$$

In this setting, reverse Bayesianism implies that, for all  $\theta = 1, 2$ ,

$$\hat{q}_{\Theta}^{\theta} = \hat{q}_{\Theta_P}^{\theta}.$$

Once again, this is a consequence of Lemma 12 (ii) in the appendix but can be easily shown directly. For  $\theta = 2$ , the equality is an (almost) immediate consequence of reverse Bayesianism:

$$\begin{aligned}\frac{p_{\bar{\Theta}}(1)}{p_{\bar{\Theta}}(2)} &= \frac{p_{\Theta_P}(1)}{p_{\Theta_P}(2)}, \\ 2 + \frac{p_{\bar{\Theta}}(1)}{p_{\bar{\Theta}}(2)} &= 2 + \frac{p_{\Theta_P}(1)}{p_{\Theta_P}(2)}, \\ v'(\hat{q}_{\bar{\Theta}}^2) &= v'(\hat{q}_{\Theta_P}^2).\end{aligned}$$

For the agent, any level-1 rationalizable strategy is also level-2 rationalizable. At level 3, the agent is certain of level-2  $\Delta$ -prudent rationalizable strategies of principal. It is obvious that the highest marginal cost type,  $\theta = 3$ , is indifferent between disclosing or not because after disclosing his existence he would remain the highest marginal cost type and be held to his outside option.

The intermediate marginal cost type,  $\theta = 2$ , strictly prefers to disclose the existence of  $\theta = 3$ . To see this, note that without disclosure, he is held to his outside option. With disclosure, he can earn positive information rents:

$$u_A(\hat{q}_{\bar{\Theta}}^2, \hat{t}_{\bar{\Theta}}^2, 2) \geq \hat{t}_{\bar{\Theta}}^3 - 2\hat{q}_{\bar{\Theta}}^3 \geq u_A(\hat{q}_{\bar{\Theta}}^3, \hat{t}_{\bar{\Theta}}^3, 3) = 0 = u_A(\hat{q}_{\Theta_P}^2, \hat{t}_{\Theta_P}^2, 2).$$

The lowest marginal cost type,  $\theta = 1$ , also has an incentive to raise the principal's awareness. From the incentives for type  $\theta = 2$ , (binding) incentive compatibility for type  $\theta = 1$ , and the result that  $\hat{q}_{\bar{\Theta}}^{\theta} = \hat{q}_{\Theta_P}^{\theta}$ ,

$$\begin{aligned}u_A(\hat{q}_{\bar{\Theta}}^2, \hat{t}_{\bar{\Theta}}^2, 2) &\geq u_A(\hat{q}_{\Theta_P}^2, \hat{t}_{\Theta_P}^2, 2) \\ u_A(\hat{q}_{\bar{\Theta}}^2, \hat{t}_{\bar{\Theta}}^2, 2) + \hat{q}_{\bar{\Theta}}^2 &\geq u_A(\hat{q}_{\Theta_P}^2, \hat{t}_{\Theta_P}^2, 2) + \hat{q}_{\Theta_P}^2 \\ \hat{t}_{\bar{\Theta}}^2 - \hat{q}_{\bar{\Theta}}^2 &\geq \hat{t}_{\Theta_P}^2 - \hat{q}_{\Theta_P}^2 \\ \hat{t}_{\bar{\Theta}}^1 - \hat{q}_{\bar{\Theta}}^1 &\geq \hat{t}_{\Theta_P}^1 - \hat{q}_{\Theta_P}^1 \\ u_A(\hat{q}_{\bar{\Theta}}^1, \hat{t}_{\bar{\Theta}}^1, 1) &\geq u_A(\hat{q}_{\Theta_P}^1, \hat{t}_{\Theta_P}^1, 1).\end{aligned}$$

We conclude that at level 3, all types of the agent (except for type  $\theta = 3$ , who is indifferent) have an incentive to raise the principal's awareness.

For the principal, there is no reduction of the set of strategies at level 3 since there were none for the agent at level 2. Consequently, at level 4 there is no reduction of the set of strategies of the agent.

At level 4, the principal is certain of level-3  $\Delta$ -prudent rationalizable strategies of the agent. Thus, when made aware, she knows any type could have made her aware. Thus, she continues to take all types into account. No further reduction of her strategies is possible.

This completes the analysis of the model with three types in which the principal is unaware of high marginal cost types only. We observe that in any  $\Delta$ -prudent rationalizable outcome, every type of the agent raises her awareness, except perhaps for the type with the highest marginal costs, who is indifferent. In any case, she offers a menu of contracts optimal for all types of which she has become aware.

## 4.2 Any Finite Number of Marginal Cost Types

In this section, we now assume that  $\Theta_P = \{1, \dots, j\}$  for some  $1 < j < n$ . The principal is initially unaware of the higher marginal cost types  $\{j+1, \dots, n\}$ , which is why we call this case the model with unawareness of high marginal costs only.

**Proposition 2** *Consider the model in which the principal is unaware of higher marginal cost events only:  $\Theta_P = \{1, \dots, j\}$  for some  $1 < j < n$ . In any  $\Delta$ -prudent rationalizable outcome, the following holds:*

- (i) *The highest marginal cost type,  $\theta = n$ , is indifferent between fully raising the principal's awareness and remaining silent:  $\Theta(n) \in \{\bar{\Theta}, \Theta_P\}$ .*
- (ii) *All other types of the agent prefer to fully raise the principal's awareness:  $\Theta(\theta) = \bar{\Theta}$  for all  $\theta < n$ .*
- (iii) *The principal offers an optimal menu of contracts of all types of which she is or becomes aware.*

*Thus, if it is common belief upon becoming aware that the highest marginal cost type would raise the principal's awareness, then any outcome involves full awareness.*

The proof is contained in the appendix. On one hand, this result sounds intuitive. If there are events that could potentially increase marginal costs, why not make the principal aware of it? The type with the highest marginal costs is indifferent since he receives the utility of his outside option anyway. Other types benefit via additional information rents. Yet, the result is not obvious because types of whom the principal has already been aware benefit from higher information rents but are also harmed by the lower transfers to the newly-discovered higher marginal cost types. Our result verifies that last effect is overcompensated by additional information rents.

The analytical roles of the assumptions of common belief in rationality, caution, logconcavity, and reverse Bayesianism are the same as in Proposition 1. We do not make a tie-breaking assumption for the type with the highest marginal cost except for the final conclusion. This is because it is enough for the principal to have full support beliefs over strategies of the agent such that she does not rule out this type having raised her awareness upon becoming aware. Adopting the tie-breaking assumption for the entire analysis would not lead to any changes. Now, this tie-breaking assumption is different from the one used in Section 3: When a type of the agent is indifferent between raising the principal's awareness or not, he raises the principal's awareness, while in Section 3 we assumed he would not. Although—once again—we do not model this explicitly, we may motivate the current assumption as type  $\theta = n$  being optimistic (even

infinitesimally so) that the principal may reward his being forthcoming with a bonus in the contract menu.

What would happen here if, instead, we made the tie-breaking assumption of Proposition 1? Suppose that the high cost type never raises the principal’s awareness. Then, the principal would be certain of this at the next level of the solution procedure. Consequently, she would offer a menu of contracts optimal for the second-highest marginal cost type being the new highest marginal cost type, who is now held to his outside option. At the next level of the solution procedure, the second-highest marginal cost type would have no incentive to raise the principal’s awareness. At first glance, we may conjecture that, by induction, not raising awareness unravels and no type of the agent wants to raise the principal’s awareness. A subtle issue, however, is that a menu with contracts for less types is obviously not an element of the set of strategies with menus of contracts for all types. So this would not lead to a reduction of strategy sets. It is not clear to us how this “silence outcome” could be consistent with common cautious belief in rationality and the other restrictions.

## 5 Discussion

The previous observations can be captured in a simple punch line: *The principal is happily made aware of inefficiencies but kept tacitly in the dark about some efficiencies.* Asymmetric awareness acts as an additional welfare friction on top of asymmetric information. Compared to the standard screening problem, communication arises naturally as some types of the agent may have an incentive to raise the principal’s awareness. Allowing for communication improves welfare.

To make this last observation more precise, we consider ex-post social surplus,  $W(q, \theta) := v(q) - \theta q$  for each  $\theta \in \bar{\Theta}$ . This function is strictly concave and maximized at the first-best allocation rule  $q^*(\theta) = (v')^{-1}(\theta)$ . For any solution to the principal’s problem  $\hat{q}_{\Theta_P}^\theta$  for  $\theta \in \Theta_P$ , information rents imply that  $\hat{q}_{\Theta_P}^\theta \leq q^*(\theta)$ .

In the model with unawareness of high marginal cost events only, under our tie-breaking assumption, any  $\Delta$ -prudent rationalizable outcome involves full awareness (Proposition 2). For any type  $\theta \in \Theta_P$ , we have that  $\hat{q}_{\bar{\Theta}}^\theta = \hat{q}_{\Theta_P}^\theta$  (Lemma 12(ii)). Hence, communication between the agent and the principal before contracting does not change the menu for these types. The types of which the principal is unaware in this scenario are higher-cost types who, in the absence of communication, would find all options in the menu unsuitable and reject the principal’s offer. Therefore, the agent raising the principal’s awareness before contracting does not affect the menu for types already under the principal’s radar and enables trade with all other types, leading to an ex-post welfare improvement:

$$W\left(\hat{q}_{\Theta_P}^\theta, \theta\right) = W\left(\hat{q}_{\bar{\Theta}}^\theta, \theta\right)$$

for all  $\theta \in \Theta_P$  and

$$W\left(\hat{q}_{\bar{\Theta}}^\theta, \theta\right) \geq 0 = W\left(\hat{q}_{\Theta_P}^\theta, \theta\right)$$

for all  $\theta \in \bar{\Theta} \setminus \Theta$ .

Now, in the model with unawareness of low marginal cost events only,  $\Delta$ -prudent rationalizable outcomes involve either no raising of awareness or only a partial awareness increase, with

bunching at the top. Types  $\theta \in \Theta_P$  do not exploit the opportunity to communicate with the principal. For each type  $\theta \in \bar{\Theta} \setminus \Theta_P$ , let  $\underline{\theta} = \min(\Theta_P)$  be the type with the lowest cost of which the principal is initially aware, and let  $\underline{\theta}'$  be the type with the lowest cost of which the principal is made aware by the agent:  $\underline{\theta}' = \min(\Theta(\theta))$ . It must be that  $\theta < \bar{\theta}'$ ; without communication, such a type would choose the option in the menu designed for  $\bar{\theta}$ , while after raising the principal's awareness they bunch on the option for  $\bar{\theta}'$ . In either case, the quantities awarded to these lowest-cost types are the first best—optimal contracts feature the *no distortion at the top* property. Thus, ex-post welfare is once again raised: Concavity (in  $q$ ) and the property of strictly increasing differences of  $W(q, \theta)$  in  $(q, \theta)$  imply that

$$q^*(\theta) > q^*(\bar{\theta}') > q^*(\bar{\theta})$$

and that

$$W(q^*(\bar{\theta}'), \theta) > W(q^*(\bar{\theta}), \theta).$$

It is important to point out that we are talking about ex-post welfare given the *possibility* of disclosure, not imposing the action of disclosure on all types. Under unawareness of low marginal cost types, the types of which the principal is initially aware suffer decreases in information rents if the principal's awareness is raised. The key is that, in any  $\Delta$ -prudent rationalizable outcome, they would choose not to raise the principal's awareness in the first place. Of course, this point is obvious if we only consider the agent; the value of this discussion is that the possibility of disclosure improves social welfare, not just the agent's own welfare.

We interpret our model as one of changing the awareness level of the principal. How is it different from the principal assigning zero probability to some marginal cost types? If the principal assigns zero probability to some marginal cost type  $\theta$ , then she assigns probability 1 to the complement of  $\theta$ ; that is, she is certain that the agent is not of type  $\theta$ . Hence, receiving a message like “Have you considered that the agent could have type  $\theta$ ?” (a question) or “The agent could be of type  $\theta$  or not of type  $\theta$ .” (a tautology) does not contain inherent information—unless it gets information attributed strategically in a cheap-talk game. It is not clear why such a message should change the principal's probabilistic assessment of  $\theta$ , especially if she is absolutely certain that the agent's type is not  $\theta$  (as implied by assigning zero probability to  $\theta$ ). Yet, if she were unaware of  $\theta$ , she realizes upon receiving such a message that she did not consider that the agent's type could be  $\theta$  and hence may reevaluate her probabilistic assessment of the other types. Thus, we find the interpretation of unawareness much more compelling for the implications of belief change we study in this paper.

Whether or not the principal is unaware of  $\theta$  or just assigns zero probability to it can be tested in a choice experiment (Schipper, 2013). Suppose that, besides contracting with the agent, the principal also contracts with some other party; e.g., an insurance contract, if she were risk averse. This contract could have a clause, say, in an addendum on p. 976 specifying that something bad happens if the agent is of type  $\theta$  and something good if he is not. There could be also a second contract that is identical to the first one except that in the addendum on p. 976 it is specified that no matter whether the agent is of type  $\theta$  something good happens. If the principal is indifferent between the two contracts, this is consistent with her both assigning zero probability to  $\theta$  or being unaware of  $\theta$  (presumably because she did not bother to read the addendum). Now, consider a third contract identical to the first two except that in said addendum it specifies something good when the agent is of type  $\theta$  and something bad when he

is not. If the principal is indifferent between the second and third contracts, she either assigns zero probability to “not  $\theta$ ” or is unaware of  $\theta$ . However, if the principal is indifferent between all three contracts, then unawareness of  $\theta$  is revealed because she cannot assign zero probability to both  $\theta$  and its complement.

Probability zero may also lead to different implications in our model in which the principal is unaware of low marginal cost types only. In the three-type example of this model, at level 4 of the rationalizability procedure, the principal cannot rationalize if she finds herself at an information set in which she has become aware of additional marginal cost types. Consequently, she resorts to her level 3 (which are equivalent to her level 2) rationalizable contract menus. If instead the principal had assigned zero probability to some types, she could alternatively now give up this belief restriction and, very much in the spirit of the best rationalization principle embodied in rationalizability notions (Battigalli, 1996), perfectly rationalize the disclosure move of the agent.

This discussion begs the question whether the principal should not be aware that she is unaware of some types; in principle, we can always imagine that there are more or less efficient types. However, recall that we interpret the unidimensional types just as a score of potentially complicated multidimensional types (like in Asker and Cantillon (2006) or Bajari, Houghton, and Tadelis, 2014). With such an interpretation, unawareness of cost types should not be interpreted as unawareness of for instance the next higher cost type but rather as unawareness of a particular cost-type dimension. In principle, while the principal could be aware that she is unaware of *some* cost-type dimension. Yet, she cannot be aware that she is unaware of a *particular* dimension—otherwise, she would be aware of it. That is, a feature of awareness of unawareness (e.g., see Schipper, 2022) is that the principal could be uncertain whether she considered all cost-dimensions but cannot know that she missed a particular dimension. In this first investigation of the screening problem with unawareness, we do not allow for awareness of unawareness. Without the principal explicitly allowing to do something about her awareness of unawareness, like investigating whether or not there could be further costs dimensions (e.g., consult with a specialist), it is unclear how awareness of unawareness would affect the conclusions. In many settings, principals such as regulators, CEOs etc. have to justify their actions, which sometimes requires them to testify what exactly they had taken or not taken into account. Requesting money for something unspecific usually does not go down well with legislators. Even if they face awareness of unawareness and do their own investigations on what they may miss, at the end of the day they can only take into account events that they are or became aware of. That is why we think that our focus on (un)awareness of specific types is relevant.

We believe that there are various further avenues for future research: First, we could consider the extension to multi-dimensional screening with unawareness of some dimensions. Second, it would be interesting to combine disclosure of awareness with disclosure of information, the latter having recently been studied by Pram (2021) and Ali, Lewis, and Vasserman (2022). Third, it is natural to think about the case of multiple agents, such as in an auction environment. Agents may now also compete by raising the awareness of the principal. Fourth, it would also be interesting to explore the consequences of giving up belief restrictions like logconcavity and reverse Bayesianism. Lemma 13 in the Appendix provides first-order conditions that apply as well when, instead of assuming preservation of relative likelihood ratios as in reverse Bayesianism, we only assume that awareness preserves the ordering of likelihood ratios. This seems to be

in the spirit of extended Bayesianism axiomatized in Piermont (2021). Fifth, what happens in the long run as a principal contracts repeatedly with agents drawn randomly each round from a large set and discovers over time more and more contingencies? Finally, the clear differences in the theoretical predictions of our two models lend themselves to experimental testing in a lab environment.

## A Preliminaries

In this appendix, we collect results that we repeatedly use throughout the analysis. Let  $\Theta$  be the set of types of which the principal is or has become aware; this set may be a proper subset of  $\bar{\Theta}$ . Let  $\kappa^{(1)} > \dots > \kappa^{(m)}$  be the order statistics of  $\Theta$ , where  $m = |\Theta|$ . Finally, denote by  $p^i$  the principal's probability assigned to marginal cost type  $\kappa^{(i)}$ . Her constrained optimization problem given her awareness of the types in  $\Theta$  is:

$$\max_{(q^i, t^i)_{i=1, \dots, m} \in ([0, b]^2)^m} \sum_{i=1}^m p^i (v(q^i) - t^i)$$

subject to, for all  $i, j = 1, \dots, m$ :

IC $_{i,j}$ :

$$t^i - \kappa^{(i)} q^i \geq t^j - \kappa^{(i)} q^j$$

PC $_i$ :

$$t^i - \kappa^{(i)} q^i \geq 0.$$

**Lemma 1** For all  $i = 2, \dots, m$ , PC $_1$  and IC $_{i,i-1}$  implies PC $_i$ .

PROOF. Observe that:

$$0 \leq t^1 - \kappa^{(1)} q^1 \stackrel{\kappa^{(1)} > \kappa^{(2)}}{\leq} t^1 - \kappa^{(2)} q^1 \stackrel{\text{IC}_{2,1}}{\leq} t^2 - \kappa^{(2)} q^2 \stackrel{\kappa^{(2)} > \kappa^{(3)}}{\leq} t^2 - \kappa^{(3)} q^2 \leq \dots \stackrel{\text{IC}_{m,m-1}}{\leq} t^m - \kappa^{(m)} q^m.$$

This establishes the result. □

Recall that  $u_A(t, q, \kappa) = t - \kappa q$ .

**Lemma 2** For every  $t \in \mathbb{R}$ ,  $u_A(t, q, \kappa)$  has strictly decreasing differences in  $(q, \kappa)$ : for any  $q'' > q'$  and  $\kappa'' > \kappa'$ ,

$$u_A(q'', t, \kappa'') - u_A(q'', t, \kappa') < u_A(q', t, \kappa'') - u_A(q', t, \kappa').$$

PROOF. For any  $t \in \mathbb{R}$  and  $q'' > q'$  and  $\kappa'' > \kappa'$ ,

$$u_A(q'', t, \kappa'') - u_A(q'', t, \kappa') < u_A(q', t, \kappa'') - u_A(q', t, \kappa')$$

$$\begin{aligned} t - \kappa'' q'' - t + \kappa' q'' &< t - \kappa'' q' - t + \kappa' q' \\ (\kappa' - \kappa'') q'' &< (\kappa' - \kappa'') q'. \end{aligned}$$

The result follows.  $\square$

The next observation follows from the previous lemmas and a generalization of Topkis' theorem by Edlin and Shannon (1998, Theorem 1).

**Lemma 3** *For every  $t \in \mathbb{R}$ , the agent's optimal  $q$  is strictly decreasing in  $\kappa$ .*

**Lemma 4** *If for all  $i = 2, \dots, m$ ,  $q^i \geq q^{i-1}$  ( $q^i > q^{i-1}$ ), then  $IC_{i,i-1}$  implies  $IC_{i,j}$  (with strict inequality) for all  $j < i$ .*

PROOF. We prove by induction on the order statistics of marginal cost types.

Base case: For  $i = 3, \dots, m$ ,

$$t^i - \kappa^{(i)} q^i \geq (>) t^{i-2} - \kappa^{(i)} q^{i-2}.$$

Rewrite  $IC_{i-1,i-2}$ :

$$\begin{aligned} t^{i-1} - \kappa^{(i-1)} q^{(i-1)} &\geq t^{i-2} - \kappa^{(i-1)} q^{i-2} \\ t^{i-1} - t^{i-2} &\geq \kappa^{(i-1)} (q^{i-1} - q^{i-2}). \end{aligned}$$

Since  $q^{i-1} - q^{i-2} \geq (>) 0$  and  $\kappa^{(i)} < \kappa^{(i-1)}$ ,

$$\begin{aligned} t^{i-1} - t^{i-2} &\geq (>) \kappa^{(i)} (q^{i-1} - q^{i-2}) \\ t^{i-1} - \kappa^{(i)} q^{i-1} &\geq (>) t^{i-2} - \kappa^{(i)} q^{i-2} \end{aligned}$$

implies now:

$$t^i - \kappa^{(i)} q^{(i)} \stackrel{IC_{i,i-1}}{\geq} t^{i-1} - \kappa^{(i)} q^{i-1} \geq (>) t^{i-2} - \kappa^{(i)} q^{i-2}.$$

This proves the base case.

Induction hypothesis: For  $i, j = 2, \dots, m$  with  $i > j$ ,

$$t^i - \kappa^{(i)} q^i \geq t^{i-j} - \kappa^{(i)} q^{i-j}.$$

Inductive step: For  $i, j = 2, \dots, m$  with  $i > j$ , rewrite  $IC_{i-j,i-j-1}$ :

$$\begin{aligned} t^{i-j} - \kappa^{(i-j)} q^{(i-j)} &\geq t^{i-j-1} - \kappa^{(i-j)} q^{i-j-1} \\ t^{i-j} - t^{i-j-1} &\geq \kappa^{(i-j)} (q^{i-j} - q^{i-j-1}). \end{aligned}$$

Since  $q^{i-j} - q^{i-j-1} \geq (>) 0$  and  $\kappa^{(i)} < \kappa^{(i-j)}$ ,

$$\begin{aligned} t^{i-j} - t^{i-j-1} &\geq (>) \kappa^{(i)} (q^{i-j} - q^{i-j-1}) \\ t^{i-j} - \kappa^{(i)} q^{(i-j)} &\geq (>) t^{i-j-1} - \kappa^{(i)} q^{i-j-1} \end{aligned}$$

implies now:

$$t^i - \kappa^{(i)}q^i \stackrel{\text{Ind. Hyp.}}{\geq} t^{i-j} - \kappa^{(i)}q^{i-j} \geq (>) t^{i-j-1} - \kappa^{(i)}q^{i-j-1}.$$

This completes the proof.  $\square$

Analogously, we can prove the following lemma:

**Lemma 5** *If for all  $i = 2, \dots, m$ ,  $q^i \geq q^{i-1}$  ( $q^i > q^{i-1}$ ), then  $IC_{i,i+1}$  implies  $IC_{i,j}$  (with strict inequality) for all  $j$  with  $m \geq j > i$ .*

**Lemma 6** *For all  $i = 2, \dots, m$ ,  $IC_{i,i-1}$  binds in the principal's optimum.*

PROOF. We prove by contradiction. Suppose that for some  $i = 2, \dots, m$ ,  $IC_{i,i-1}$  does not bind:

$$t^i - \kappa^{(i)}q^i > t^{i-1} - \kappa^{(i)}q^{i-1}.$$

Then, the principal can decrease the transfer to marginal cost type  $\kappa^{(i)}$  by  $t^i - t^{i-1} - \kappa^{(i)}(q^i - q^{i-1}) > 0$ . This increases her expected payoff while satisfying  $IC_{i,i-1}$  with equality. Observe that  $IC_{j,j-1}$  would be still satisfied for all  $j = 1, \dots, m$ ,  $j \neq i$ .  $\square$

**Lemma 7** *For all  $i = 1, \dots, m-1$ ,  $IC_{i,i+1}$  are satisfied in the principal's optimum.*

PROOF. For any  $i = 1, \dots, m-1$ , Lemma 6 implies:

$$\begin{aligned} t^{i+1} - \kappa^{(i+1)}q^{i+1} &= t^i - \kappa^{(i+1)}q^i \\ t^{i+1} - t^i &= \kappa^{(i+1)}(q^{i+1} - q^i). \end{aligned}$$

Lemma 3 implies  $q^{i+1} - q^i \geq 0$ . Since  $\kappa^{(i)} > \kappa^{(i+1)}$ , we have:

$$\begin{aligned} t^{i+1} - t^i &\leq \kappa^{(i)}(q^{i+1} - q^i) \\ t^{i+1} - \kappa^{(i)}q^{i+1} &\leq t^i - \kappa^{(i)}q^i, \end{aligned}$$

as desired.  $\square$

**Remark 1**  *$PC_1$  is binding in the principal's optimum.*

Thus, we can reduce the principal's optimization problem to:

$$\max_{(q^i, t^i)_{i=1, \dots, m} \in ([0, b]^2)^m} \sum_{i=1}^m p^i (v(q^i) - t^i)$$

subject to, for all  $i = 2, \dots, m$ ,

$IC_{i,i-1}$ :

$$t^i - \kappa^{(i)}q^i = t^{i-1} - \kappa^{(i)}q^{i-1}$$



PC<sub>1</sub>:

$$t^1 - \kappa^{(1)}q^1 = 0$$

M<sub>*i,i-1*</sub>:

$$q^i \geq q^{i-1}.$$

Note that, even though monotonicity is implied by decreasing differences of the agent's objective function (Lemmas 2 and 3), it is a constraint for the principal.

We can rewrite the first two classes of constraints recursively as follows. For  $i = 2, \dots, m$ ,

$$t^i = \sum_{j=1}^{i-1} (\kappa^{(j)} - \kappa^{(j+1)})q^j + \kappa^{(i)}q^i,$$

$$t^1 = \kappa^{(1)}q^1.$$

We omit momentarily the monotonicity constraints; they will be verified later. Substituting the remaining constraints into the principal's objective function yields the following unconstrained optimization problem:

$$\max_{(q^i)_{i=1, \dots, m} \in [0, b]^m} p^1 \left( v(q^1) - \kappa^{(1)}q^1 \right) + \sum_{i=2}^m p^i \left( v(q^i) - \sum_{j=1}^{i-1} (\kappa^{(j)} - \kappa^{(j+1)})q^j - \kappa^{(i)}q^i \right).$$

Deriving first-order conditions yields: For  $i = 1, \dots, m$ ,

$$v'(\hat{q}^i) = \kappa^{(i)} + \frac{\sum_{j>i}^m p^j}{p^i} (\kappa^{(i)} - \kappa^{(i+1)}) \quad (1)$$

$$\hat{t}^i = \sum_{j=1}^{i-1} (\kappa^{(j)} - \kappa^{(j+1)})\hat{q}^j + \kappa^{(i)}\hat{q}^i, \quad (2)$$

where  $\frac{\sum_{j>i}^m p^j}{p^i}$  is the inverse hazard rate. Transfers can be rewritten recursively as:

$$\hat{t}^i = \hat{t}^1 + \sum_{1 < j \leq i} \kappa^{(j)}(\hat{q}^j - q^{j-1}),$$

which separates them into the transfer to the highest marginal-cost type and the information rents.

The lowest marginal cost type provides efficient output (“no distortion at the top”), i.e.,

$$v'(\hat{q}^m) = \kappa^{(m)},$$

while all other types underprovide in the principal's optimum. Monotonicity implies that lower marginal cost types receive higher transfers. However, monotonicity is not implied by the first-order conditions; we need an additional condition. Since  $v''(q) < 0$ , we have for all  $i = 1, \dots, m - 1$ ,

$$\hat{q}^i \leq \hat{q}^{i+1}$$

$$\begin{aligned}
v'(\hat{q}^i) &\geq v'(\hat{q}^{i+1}) \\
\kappa^{(i)} + \frac{\sum_{j>i}^m p^j}{p^i} (\kappa^{(i)} - \kappa^{(i+1)}) &\geq \kappa^{(i+1)} + \frac{\sum_{j>i+1}^m p^j}{p^{i+1}} (\kappa^{(i+1)} - \kappa^{(i+2)}) \\
\kappa^{(i)} - \kappa^{(i+1)} + \frac{\sum_{j>i}^m p^j}{p^i} (\kappa^{(i)} - \kappa^{(i+1)}) &\geq \frac{\sum_{j>i+1}^m p^j}{p^{i+1}} (\kappa^{(i+1)} - \kappa^{(i+2)}).
\end{aligned}$$

Our cost structures satisfies the following assumption of equidistant marginal cost types:

**Assumption 1** For all  $i, j = 1, \dots, m-1$  such that  $i+j+1 \leq m$ , we have  $\kappa^{(i)} - \kappa^{(i+1)} = \kappa^{(i+j)} - \kappa^{(i+j+1)}$ .

Then, for all  $i = 1, \dots, m-1$ , the previous inequality becomes:

$$1 + \frac{\sum_{j>i}^m p^j}{p^i} \geq \frac{\sum_{j>i+1}^m p^j}{p^{i+1}}.$$

A sufficient condition for this inequality is:

$$\frac{\sum_{j>i}^m p^j}{p^i} \geq \frac{\sum_{j>i+1}^m p^j}{p^{i+1}},$$

that the inverse hazard rate is non-increasing in  $i$  (or the hazard rate is non-decreasing in  $i$ ); this can be obtained as a consequence of logconcavity.

**Definition 2** Probability distribution  $p$  is logconcave if for all  $i = 2, \dots, m-1$ ,

$$(p^i)^2 \geq p^{i+1} p^{i-1}.$$

**Lemma 8** If  $p$  is logconcave, then:

(i) Relative likelihoods are non-increasing in  $i$ : For any  $i, j = 1, \dots, m$  and  $k$  such that  $j > i$  and  $j+k \leq m$ ,

$$\frac{p^{i+k}}{p^i} \geq \frac{p^{j+k}}{p^j}.$$

(ii) Inverse hazards rates are non-increasing in  $i$ : For any  $i, j = 1, \dots, m$  with  $j > i$ ,

$$\frac{\sum_{k>i}^m p^k}{p^i} \geq \frac{\sum_{k>j}^m p^k}{p^j}.$$

PROOF. Let  $p$  be logconcave and  $p^i > 0$  for all  $i = 1, \dots, m$ . Note that for  $i = 1, \dots, m-2$ ,

$$\begin{aligned}
(p^{i+1})^2 &\geq p^i p^{i+2} \\
\frac{p^{i+1}}{p^i} &\geq \frac{p^{i+2}}{p^{i+1}} \\
\log p^{i+1} - \log p^i &\geq \log p^{i+2} - \log p^{i+1}.
\end{aligned}$$

Inductively, we have for  $j > i$ ,  $j + 1 \leq n$ ,

$$\log p^{i+1} - \log p^i \geq \log p^{j+1} - \log p^{j+1}.$$

(i) For any  $i, j = 1, \dots, m$  and  $k$  such that  $j + k \leq m$ ,

$$\begin{aligned} \frac{p^{i+k}}{p^i} &= \frac{p^{i+1}}{p^i} \frac{p^{i+2}}{p^{i+1}} \cdots \frac{p^{i+k}}{p^{i+k-1}} \\ \log \left( \frac{p^{i+k}}{p^i} \right) &= \log \left( \frac{p^{i+1}}{p^i} \frac{p^{i+2}}{p^{i+1}} \cdots \frac{p^{i+k}}{p^{i+k-1}} \right) \\ &= (\log p^{i+1} - \log p^i) + (\log p^{i+2} - \log p^{i+1}) + \dots + (\log p^{i+k} - \log p^{i+k-1}) \\ &\geq (\log p^{j+1} - \log p^j) + (\log p^{j+2} - \log p^{j+1}) + \dots + (\log p^{j+k} - \log p^{j+k-1}) \\ &= \log \left( \frac{p^{j+1}}{p^j} \frac{p^{j+2}}{p^{j+1}} \cdots \frac{p^{j+k}}{p^{j+k-1}} \right) = \log \left( \frac{p^{j+k}}{p^j} \right) \\ &= \frac{p^{i+1}}{p^i} \frac{p^{i+2}}{p^{i+1}} \cdots \frac{p^{i+k}}{p^{i+k-1}} = \frac{p^{i+k}}{p^i}, \end{aligned}$$

where the inequality follows from log concavity applied to each term of the sum.

(ii) Rewrite (i) as:

$$p^j p^{i+k} - p^i p^{j+k} \geq 0.$$

Then,

$$\begin{aligned} \sum_{k=1}^{m-j} \left( p^j p^{i+k} - p^i p^{j+k} \right) &\geq 0 \\ p^j \left( \sum_{k>i}^{m-j+i} p^k \right) - p^i \left( \sum_{k>j}^m p^k \right) &\geq 0 \\ p^j \left( \sum_{k>i}^m p^k \right) - p^i \left( \sum_{k>j}^m p^k \right) &\geq 0. \end{aligned}$$

This establishes the lemma. □

We conclude from Lemma 8:

**Lemma 9** *If  $p$  is logconcave, the first-order conditions (1) and (2) are valid for the solution to principal's optimization problem.*

While the previous observations should be well-known, we were unable to locate a complete treatment of the finite case in the literature.

The next lemma establishes that reverse Bayesian updating is consistent with logconcavity if the update is a truncation.

**Lemma 10** Recall  $\bar{\Theta} = \{1, \dots, n\}$ . Suppose  $\Theta'$  is a truncation of  $\bar{\Theta}$ :  $\Theta' \subseteq \bar{\Theta}$  and for any  $j \in \bar{\Theta}$  with  $\min \Theta' \leq j \leq \max \Theta'$ , we have  $j \in \Theta'$ . Let  $p_{\bar{\Theta}}$  and  $p_{\Theta'}$  be distributions on  $\bar{\Theta}$  and  $\Theta'$  respectively such that  $p_{\Theta'}$  is the conditional distribution of  $p_{\bar{\Theta}}$ . If  $p_{\bar{\Theta}}$  is logconcave, then  $p_{\Theta'}$  is also logconcave.

PROOF. Since  $p_{\bar{\Theta}}$  is logconcave, for all  $i = \min \Theta' + 1, \dots, \max \Theta' - 1$ ,

$$\frac{p_{\Theta'}^i}{p_{\Theta'}^{i-1}} = \frac{\frac{p_{\bar{\Theta}}^i}{\sum_{j=\min \Theta'}^{\max \Theta'} p_{\bar{\Theta}}^j}}{\frac{p_{\bar{\Theta}}^{i-1}}{\sum_{j=\min \Theta'}^{\max \Theta'} p_{\bar{\Theta}}^j}} = \frac{p_{\bar{\Theta}}^i}{p_{\bar{\Theta}}^{i-1}} \geq \frac{p_{\bar{\Theta}}^{i+1}}{p_{\bar{\Theta}}^i} = \frac{\frac{p_{\bar{\Theta}}^{i+1}}{\sum_{j=\min \Theta'}^{\max \Theta'} p_{\bar{\Theta}}^j}}{\frac{p_{\bar{\Theta}}^i}{\sum_{j=\min \Theta'}^{\max \Theta'} p_{\bar{\Theta}}^j}} = \frac{p_{\Theta'}^{i+1}}{p_{\Theta'}^i},$$

where the first and last equality follows from  $p_{\Theta'}$  being the conditional distribution of  $p_{\bar{\Theta}}$ .  $\square$

Reverse Bayesian updating is essentially equality of relative likelihoods. More generally, we can consider ordering in relative likelihoods. The proof of the following lemma is similar to the proof of Lemma 8 but makes no use of logconcavity.

**Lemma 11** Consider two distributions  $p$  and  $q$  on  $\bar{\Theta} = \{1, \dots, n\}$  such that  $p$  dominates  $q$  in relative likelihoods: For  $i = 1, \dots, n - 1$ ,

$$\frac{p^{i+1}}{p^i} \geq (>) \frac{q^{i+1}}{q^i}.$$

Then:

(i) For all  $i, m$  with  $i + m \leq n$ ,

$$\frac{p^{i+m}}{p^i} \geq (>) \frac{q^{i+m}}{q^i};$$

(ii)  $p$  dominates  $q$  in inverse hazard rates:

$$\frac{\sum_{m>i}^n p^m}{p^i} \geq (>) \frac{\sum_{m>i}^n q^m}{q^i} \text{ for } i = 1, \dots, n - 1.$$

PROOF. We have

$$\log p^{i+1} - \log p^i \geq (>) \log q^{i+1} - \log q^i \text{ for all } i = 1, \dots, n - 1.$$

(i) For  $i, m = 1, \dots, n$ , with  $i + m \leq n$ ,

$$\begin{aligned} \frac{p^{i+m}}{p^i} &= \frac{p^{i+1}}{p^i} \frac{p^{i+2}}{p^{i+1}} \cdots \frac{p^{i+m}}{p^{i+m-1}} \\ \log \left( \frac{p^{i+m}}{p^i} \right) &= \log \left( \frac{p^{i+1}}{p^i} \frac{p^{i+2}}{p^{i+1}} \cdots \frac{p^{i+m}}{p^{i+m-1}} \right) \\ &= (\log p^{i+1} - \log p^i) + (\log p^{i+2} - \log p^{i+1}) + \dots + (\log p^{i+m} - \log p^{i+m-1}) \\ &\geq (>) (\log q^{i+1} - \log q^i) + (\log q^{i+2} - \log q^{i+1}) + \dots + (\log q^{i+m} - \log q^{i+m-1}) \end{aligned}$$

$$\begin{aligned}
&= \log \left( \frac{q^{i+1}}{q^i} \frac{q^{i+2}}{q^{i+1}} \cdots \frac{q^{i+m}}{q^{i+m-1}} \right) = \log \left( \frac{q^{i+m}}{q^i} \right) \\
&\frac{q^{i+1}}{q^i} \frac{q^{i+2}}{q^{i+1}} \cdots \frac{q^{i+m}}{q^{i+m-1}} = \frac{q^{i+m}}{q^i},
\end{aligned}$$

as desired.

(ii) Rewrite (i) as

$$p^i q^{i+m} - q^i p^{i+m} \geq (>) 0.$$

Then

$$\begin{aligned}
&\sum_{m=1}^{n-i} (p^i q^{i+m} - q^i p^{i+m}) \geq (>) 0 \\
&p^i \left( \sum_{m>i}^n q^m \right) - q^i \left( \sum_{m>i}^n p^m \right) \geq (>) 0.
\end{aligned}$$

This completes the proof.  $\square$

Combining the previous lemma and reverse Bayesianism leads to the following corollary.

**Corollary 1** *Suppose  $\Theta'$  is a truncation of  $\bar{\Theta}$ :  $\Theta' \subseteq \bar{\Theta}$  and for any  $j \in \bar{\Theta}$  with  $\min \Theta' \leq j \leq \max \Theta'$ , we have  $j \in \Theta'$ . Let  $p_{\bar{\Theta}}$  and  $p_{\Theta'}$  be distributions on  $\bar{\Theta}$  and  $\Theta'$  respectively such that  $p_{\Theta'}$  is the conditional distribution of full-support distribution  $p_{\bar{\Theta}}$ . If  $\max \Theta = \max \Theta'$ , then for all  $p_{\bar{\Theta}}$  and  $p_{\Theta'}$  we have:*

(i) *For all  $i \in \Theta'$  and  $m$  with  $i + m \leq \max \Theta'$ ,*

$$\frac{p_{\bar{\Theta}}^{i+m}}{p_{\bar{\Theta}}^i} = \frac{p_{\Theta'}^{i+m}}{p_{\Theta'}^i};$$

(ii) *Inverse hazard rates are equal:*

$$\frac{\sum_{m>i}^{\max \bar{\Theta}} p_{\bar{\Theta}}^m}{p_{\bar{\Theta}}^i} = \frac{\sum_{m>i}^{\max \Theta'} p_{\Theta'}^m}{p_{\Theta'}^i} \text{ for } i = 1, \dots, \Theta' - 1.$$

Reverse Bayesianism allows us to compare the optimal menus across principal's awareness levels.

**Lemma 12** *Suppose  $\Theta'$  is a truncation of  $\bar{\Theta}$ :  $\Theta' \subseteq \bar{\Theta}$  and for any  $j \in \bar{\Theta}$  with  $\min \Theta' \leq j \leq \max \Theta'$ , we have  $j \in \Theta'$ . Let  $p_{\bar{\Theta}}$  and  $p_{\Theta'}$  be distributions on  $\bar{\Theta}$  and  $\Theta'$  respectively such that  $p_{\Theta'}$  is the conditional distribution of logconcave full-support distribution  $p_{\bar{\Theta}}$ . For  $i \in \Theta'$ , let  $\hat{q}_{\bar{\Theta}}^i$  and  $\hat{q}_{\Theta'}^i$  denote the solutions for agent  $i$  of the principal's optimization problems w.r.t.  $p_{\bar{\Theta}}$  and  $p_{\Theta'}$ , respectively.*

(i) *If  $\max \bar{\Theta} > \max \Theta'$ , then for all  $i \in \Theta'$ ,  $\hat{q}_{\bar{\Theta}}^i < \hat{q}_{\Theta'}^i$ .*

(ii) If  $\max \bar{\Theta} = \max \Theta'$ , then for all  $i \in \Theta'$ ,  $\hat{q}_{\bar{\Theta}}^i = \hat{q}_{\Theta'}^i$ .

PROOF. (i): Consider any  $i \in \Theta'$  with  $i < \max \Theta'$ . Since  $v''(q) < 0$  and, by Lemma 10,  $p_{\Theta'}$  is logconcave, we use first-order conditions to show:

$$\begin{aligned}
v'(\hat{q}_{\bar{\Theta}}^i) &> v'(\hat{q}_{\Theta'}^i) \\
\kappa^{(i)} + \frac{\sum_{j=i+1}^n p_{\bar{\Theta}}^j}{p_{\bar{\Theta}}^i} (\kappa^{(i)} - \kappa^{(i+1)}) &> \kappa^{(i)} + \frac{\sum_{j=i+1}^{\max \Theta'} p_{\Theta'}^{(j)}}{p_{\Theta'}^{(i)}} (\kappa^{(i)} - \kappa^{(i+1)}) \\
\frac{\sum_{j=i+1}^n p_{\bar{\Theta}}^{(j)}}{p_{\bar{\Theta}}^{(i)}} &> \frac{\sum_{j=i+1}^{\max \Theta'} p_{\Theta'}^{(j)}}{p_{\Theta'}^{(i)}} \\
\frac{\sum_{j=i+1}^{\max \Theta'} p_{\bar{\Theta}}^{(j)}}{p_{\bar{\Theta}}^{(i)}} + \frac{\sum_{j=\max \Theta'+1}^n p_{\bar{\Theta}}^{(j)}}{p_{\bar{\Theta}}^{(i)}} &> \frac{\sum_{j=i+1}^{\max \Theta'} p_{\Theta'}^{(j)}}{p_{\Theta'}^{(i)}} \\
\frac{\sum_{j=i+1}^{\max \Theta'} p_{\Theta'}^{(j)}}{p_{\Theta'}^{(i)}} + \frac{\sum_{j=\max \Theta'+1}^{|\bar{\Theta}|} p_{\bar{\Theta}}^{(j)}}{p_{\bar{\Theta}}^{(i)}} &> \frac{\sum_{j=i+1}^{\max \Theta'} p_{\Theta'}^{(j)}}{p_{\Theta'}^{(i)}} \\
\frac{\sum_{j=|\bar{\Theta}|+1}^{|\Theta|} p_{\bar{\Theta}}^{(j)}}{p_{\bar{\Theta}}^{(i)}} &> 0,
\end{aligned}$$

where the second-to-last line follows from reverse Bayesianism.

Now consider  $i = \max \Theta'$ :

$$\begin{aligned}
v'(\hat{q}_{\bar{\Theta}}^{(\max \Theta')}) &> v'(\hat{q}_{\Theta'}^{(\max \Theta')}) \\
\kappa^{(\max \Theta')} + \frac{\sum_{j=\max \Theta'}^n p_{\bar{\Theta}}^{(j)}}{p_{\bar{\Theta}}^{(\max \Theta')}} (\kappa^{(\max \Theta')} - \kappa^{(\max \Theta'+1)}) &> \kappa^{(\max \Theta')} \\
\frac{\sum_{j=\max \Theta'}^n p_{\bar{\Theta}}^{(j)}}{p_{\bar{\Theta}}^{(\max \Theta')}} (\kappa^{(\max \Theta')} - \kappa^{(\max \Theta'+1)}) &> 0.
\end{aligned}$$

(ii) For  $i = \min \Theta', \dots, n$ ,

$$\begin{aligned}
v'(\hat{q}_{\bar{\Theta}}^i) &= v'(\hat{q}_{\Theta'}^i) \\
\kappa^{(i)} + \frac{\sum_{j=i+1}^n p_{\bar{\Theta}}^j}{p_{\bar{\Theta}}^i} (\kappa^{(i)} - \kappa^{(i+1)}) &= \kappa^{(i)} + \frac{\sum_{j=i+1}^n p_{\Theta'}^{(j)}}{p_{\Theta'}^{(i)}} (\kappa^{(i)} - \kappa^{(i+1)}) \\
\frac{\sum_{j=i+1}^n p_{\bar{\Theta}}^j}{p_{\bar{\Theta}}^i} (\kappa^{(i)} - \kappa^{(i+1)}) &= \frac{\sum_{j=i+1}^n p_{\Theta'}^{(j)}}{p_{\Theta'}^{(i)}} (\kappa^{(i)} - \kappa^{(i+1)}),
\end{aligned}$$

where the last line follows now from reverse Bayesianism.  $\square$

More generally, dominance of relative likelihoods allows us to compare solutions to the principal's optimization problems.

**Lemma 13** Suppose  $\Theta'$  is a truncation of  $\bar{\Theta}$ :  $\Theta' \subseteq \bar{\Theta}$  and for any  $j \in \bar{\Theta}$  with  $\min \Theta' \leq j \leq \max \Theta'$ , we have  $j \in \Theta'$ . Let  $p_{\bar{\Theta}}$  and  $p_{\Theta'}$  be logconcave full-support distributions on  $\bar{\Theta}$  and  $\Theta'$ . For  $i \in \Theta'$ , let  $\hat{q}_{\bar{\Theta}}^i$  and  $\hat{q}_{\Theta'}^i$  denote the solutions for agent  $i$  of the principal's optimization problems w.r.t.  $p_{\bar{\Theta}}$  and  $p_{\Theta'}$ , respectively.

- (i) If  $p_{\bar{\Theta}}$  relative likelihood dominates  $p_{\Theta'}$  on the latter's support and  $\max \bar{\Theta} \geq \max \Theta'$ , then for all  $i \in \Theta'$ ,  $\hat{q}_{\bar{\Theta}}^i < \hat{q}_{\Theta'}^i$ .
- (ii) If  $p_{\Theta'}$  relative likelihood dominates  $p_{\bar{\Theta}}$  on the former's support and  $\max \bar{\Theta} = \max \Theta'$ , then for all  $i \in \Theta'$ ,  $\hat{q}_{\bar{\Theta}}^i > \hat{q}_{\Theta'}^i$ .

PROOF. (i): Consider first the case  $\max \bar{\Theta} > \max \Theta'$  and take  $i \in \Theta'$  such that  $i < \max \Theta'$ . By  $v''(q) < 0$  and logconcavity of  $p_{\Theta'}$  (Lemma 10), we use first-order conditions to show:

$$\begin{aligned} v'(\hat{q}_{\bar{\Theta}}^i) &> v'(\hat{q}_{\Theta'}^i) \\ \kappa^{(i)} + \frac{\sum_{j=i+1}^n p_{\bar{\Theta}}^j}{p_{\bar{\Theta}}^i} (\kappa^{(i)} - \kappa^{(i+1)}) &> \kappa^{(i)} + \frac{\sum_{j=i+1}^{\max \Theta'} p_{\Theta'}^j}{p_{\Theta'}^i} (\kappa^{(i)} - \kappa^{(i+1)}) \\ \frac{\sum_{j=i+1}^n p_{\bar{\Theta}}^j}{p_{\bar{\Theta}}^i} &> \frac{\sum_{j=i+1}^{\max \Theta'} p_{\Theta'}^j}{p_{\Theta'}^i} \\ \frac{\sum_{j=i+1}^{\max \Theta'} p_{\bar{\Theta}}^j}{p_{\bar{\Theta}}^i} + \frac{\sum_{j=\max \Theta'+1}^n p_{\bar{\Theta}}^j}{p_{\bar{\Theta}}^i} &> \frac{\sum_{j=i+1}^{\max \Theta'} p_{\Theta'}^j}{p_{\Theta'}^i}. \end{aligned}$$

To see the last inequality, note that since  $p_{\bar{\Theta}}$  relative likelihood dominates  $p_{\Theta'}$  on latter's support,

$$\frac{\sum_{j=i+1}^{\max \Theta'} p_{\bar{\Theta}}^j}{p_{\bar{\Theta}}^i} > \frac{\sum_{j=i+1}^{\max \Theta'} p_{\Theta'}^j}{p_{\Theta'}^i}$$

follows from Lemma 11. For  $i = \max \Theta'$ , the same arguments as in the proof of Lemma 12 apply.

Now take the case  $\max \bar{\Theta} = \max \Theta'$ . For  $i = \min \Theta', \dots, n$ , the term  $\frac{\sum_{j=\max \Theta'+1}^n p_{\bar{\Theta}}^j}{p_{\bar{\Theta}}^i}$  does not exist in above inequality. This completes the proof of (i).

- (ii) For  $i = \min \Theta', \dots, n$ ,

$$\begin{aligned} v'(\hat{q}_{\bar{\Theta}}^i) &< v'(\hat{q}_{\Theta'}^i) \\ \kappa^{(i)} + \frac{\sum_{j=i+1}^n p_{\bar{\Theta}}^j}{p_{\bar{\Theta}}^i} (\kappa^{(i)} - \kappa^{(i+1)}) &< \kappa^{(i)} + \frac{\sum_{j=i+1}^n p_{\Theta'}^{(j)}}{p_{\Theta'}^{(i)}} (\kappa^{(i)} - \kappa^{(i+1)}) \\ \frac{\sum_{j=i+1}^n p_{\bar{\Theta}}^j}{p_{\bar{\Theta}}^i} &< \frac{\sum_{j=i+1}^n p_{\Theta'}^{(j)}}{p_{\Theta'}^{(i)}}, \end{aligned}$$

where the last line follows now from Lemma 11.  $\square$

**Remark 2** The proof of Lemma 12 reveals that  $p_{\bar{\Theta}}$  relative likelihood dominating  $p_{\Theta'}$  or the latter being the conditional of the former is not required for  $\hat{q}_{\bar{\Theta}}^{\max \Theta'} < \hat{q}_{\Theta'}^{\max \Theta'}$  if  $\max \bar{\Theta} > \max \Theta'$ .

The last part of Lemma 13 does not treat the case  $\max \bar{\Theta} > \max \Theta'$  when  $p_{\Theta'}$  relative likelihood dominates in  $p_{\bar{\Theta}}$  on the former's support. The reason is that the difference in upper bounds of the support makes it difficult to compare the distributions without further assumptions on inverse hazard rates.

## B Proofs

### Proof of Proposition 1

As in the previous section of the appendix, let  $\kappa^{(i)}$  denote the order statistics of marginal costs in  $\Theta \subseteq \bar{\Theta}$ , where  $\kappa^{(1)}$  refers to the highest marginal cost.

**Level 1. Principal:** At her information set  $h_P$ , any first-level  $\Delta$ -prudent rationalizable strategy  $s_P$  of the principal assigns a menu of contracts

$$\left\{ \left( q_{\Theta_{h_P}}^i, t_{\Theta_{h_P}}^i \right) \right\}_{i=1, \dots, |\Theta_{h_P}|} \in ([0, b]^2)^{|\Theta_{h_P}|}$$

that maximize expected profits w.r.t. some full support belief on types  $\kappa^{(1)}, \dots, \kappa^{(|\Theta_{h_P}|)}$ . (Since the principal's beliefs are full support over strategies and marginal cost types, her marginal on marginal cost types must be full support at this level.)

*Agent:* Consider any last information set of the agent in tree  $T_{\Theta}$ . Note that these information sets are singleton; i.e., the agent is certain of the menu offered by the principal. If  $s_A$  is a first-level rationalizable strategy of the agent, it selects at this information set a contract  $(q, t) \in s_P(h_P)$  that maximizes expected payoff given the incentive and participation constraints. If none of the contracts in  $s_P(h_P)$  satisfies the participation constraint, he selects the outside option. Since the agent's payoff function satisfies decreasing differences (Lemma 2), the selected contract quantity is decreasing in the marginal cost type (Lemma 3).

At any first information sets of the agent in tree  $T_{\Theta}$ , any strategy is first-level prudent rationalizable for any marginal cost type  $\kappa^{(i)}$  with  $i = 1, \dots, |\Theta|$ . Disclosure is rational if he believes with sufficiently large probability that the principal will offer a better deal after disclosure. Non-disclosure is optimal if he believes with sufficiently high probability that disclosure will not lead to a better deal.

**Level 2. Principal:** The principal is certain of first-level  $\Delta$ -prudent rationalizable strategies of the agent. Thus, she is certain that the agent observes participation constraints and self-selects among contracts according to his incentives. Moreover, she is certain that the agent's chosen quantities are monotone in the agent's marginal cost type.

First-level  $\Delta$ -prudent rationalizability imposes no restrictions on the agent's strategies w.r.t. to raising the principal's awareness. Since her belief system is cautious (i.e., full support beliefs), she believes at any of her information sets at which she became aware that any type that she is now aware of could have raised her awareness. If  $\Theta$  is the set of types that the principal is aware of at her information set, denote by  $p_{\Theta}^i$  the principal's marginal probability that the agent's type has marginal cost  $\kappa^{(i)}$  with  $i = 1, \dots, |\Theta|$ . Since  $p_{\Theta}$  logconcave and full support,



the principal's optimization problem and its solution is given as in Appendix A. Moreover, if  $\Theta' \subseteq \Theta''$ , then  $p_{\Theta'}$  and  $p_{\Theta''}$  satisfy reverse Bayesianism since the principal believes at any of her information sets that any type could have raised her awareness. Thus, quantities for each type are monotone across trees in which he exists (Lemma 12 (i)).

*Agent:* No additional strategies are eliminated at level 2.

**Level 3.** *Principal:* No additional strategies are eliminated at level 3 since no further strategies of the agent have been eliminated at level 2.

*Agent:* The agent is certain of level-2  $\Delta$ -prudent rationalizable strategies of the principal. Type  $\kappa^{(1)}$  is indifferent between disclosing any  $\Theta$  and not disclosing, since he is always held to his outside option with any level-2  $\Delta$ -rationalizable strategy of the principal. Thus, by the tie-breaking assumption, he does not disclose. We show by induction on order statistics of types that all  $i = 2, \dots, |\Theta'|$  prefer not to disclose  $\Theta'' \supsetneq \Theta'$ .

Base case  $i = 2$ :

$$\begin{aligned} u_A \left( \hat{q}_{\Theta''}^2, \hat{t}_{\Theta''}^2, \kappa^{(2)} \right) &< u_A \left( \hat{q}_{\Theta'}^2, \hat{t}_{\Theta'}^2, \kappa^{(2)} \right) \\ \hat{t}_{\Theta''}^2 - \kappa^{(2)} \hat{q}_{\Theta''}^2 &< \hat{t}_{\Theta'}^2 - \kappa^{(2)} \hat{q}_{\Theta'}^2 \\ \hat{t}_{\Theta''}^1 - \kappa^{(2)} \hat{q}_{\Theta''}^1 &< \hat{t}_{\Theta'}^1 - \kappa^{(2)} \hat{q}_{\Theta'}^1 \\ \kappa^{(1)} \hat{q}_{\Theta''}^1 - \kappa^{(2)} \hat{q}_{\Theta''}^1 &< \kappa^{(1)} \hat{q}_{\Theta'}^1 - \kappa^{(2)} \hat{q}_{\Theta'}^1 \\ \left( \kappa^{(1)} - \kappa^{(2)} \right) \hat{q}_{\Theta''}^1 &< \left( \kappa^{(1)} - \kappa^{(2)} \right) \hat{q}_{\Theta'}^1 \end{aligned}$$

follows from  $\hat{q}_{\Theta''}^1 < \hat{q}_{\Theta'}^1$  (Lemma 12 (i)). (The third line follows from the incentive compatibility constraints. The fourth line follows from the participation constraint of marginal cost type  $\kappa^{(1)}$ .)

Induction hypothesis:

$$u_A \left( \hat{q}_{\Theta''}^i, \hat{t}_{\Theta''}^i, \kappa^{(i)} \right) < u_A \left( \hat{q}_{\Theta'}^i, \hat{t}_{\Theta'}^i, \kappa^{(i)} \right).$$

Inductive step: For  $i$  with  $1 < i < |\Theta'|$ ,

$$\begin{aligned} u_A \left( \hat{q}_{\Theta''}^{i+1}, \hat{t}_{\Theta''}^{i+1}, \kappa^{(i+1)} \right) &< u_A \left( \hat{q}_{\Theta'}^{i+1}, \hat{t}_{\Theta'}^{i+1}, \kappa^{(i+1)} \right) \\ \hat{t}_{\Theta''}^{i+1} - \kappa^{(i+1)} \hat{q}_{\Theta''}^{i+1} &< \hat{t}_{\Theta'}^{i+1} - \kappa^{(i+1)} \hat{q}_{\Theta'}^{i+1} \\ \hat{t}_{\Theta''}^i - \kappa^{(i+1)} \hat{q}_{\Theta''}^i &< \hat{t}_{\Theta'}^i - \kappa^{(i+1)} \hat{q}_{\Theta'}^i \\ \hat{t}_{\Theta''}^i - \kappa^{(i)} \hat{q}_{\Theta''}^i + \kappa^{(i)} \hat{q}_{\Theta''}^i - \kappa^{(i+1)} \hat{q}_{\Theta''}^i &< \hat{t}_{\Theta'}^i - \kappa^{(i)} \hat{q}_{\Theta'}^i + \kappa^{(i)} \hat{q}_{\Theta'}^i - \kappa^{(i+1)} \hat{q}_{\Theta'}^i \\ u_A \left( \hat{q}_{\Theta''}^i, \hat{t}_{\Theta''}^i, \kappa^{(i)} \right) + \left( \kappa^{(i)} - \kappa^{(i+1)} \right) \hat{q}_{\Theta''}^i &< u_A \left( \hat{q}_{\Theta'}^i, \hat{t}_{\Theta'}^i, \kappa^{(i)} \right) + \left( \kappa^{(i)} - \kappa^{(i+1)} \right) \hat{q}_{\Theta'}^i \end{aligned}$$

follows now from the induction hypothesis and  $\hat{q}_{\Theta''}^i < \hat{q}_{\Theta'}^i$  (Lemma 12 (i)).

Next, we show that marginal cost type  $|\Theta'| + 1$  prefers not to disclose:

$$\begin{aligned} u_A \left( \hat{q}_{\Theta''}^{|\Theta'|+1}, \hat{t}_{\Theta''}^{|\Theta'|+1}, \kappa^{(|\Theta'|+1)} \right) &< u_A \left( \hat{q}_{\Theta'}^{|\Theta'|}, \hat{t}_{\Theta'}^{|\Theta'|}, \kappa^{(|\Theta'|+1)} \right) \\ \hat{t}_{\Theta''}^{|\Theta'|+1} - \kappa^{(|\Theta'|+1)} \hat{q}_{\Theta''}^{|\Theta'|+1} &< \hat{t}_{\Theta'}^{|\Theta'|} - \kappa^{(|\Theta'|+1)} \hat{q}_{\Theta'}^{|\Theta'|} \end{aligned}$$

$$\begin{aligned}
& \hat{t}_{\Theta''}^{|\Theta'|} - \kappa^{(|\Theta'|+1)} \hat{q}_{\Theta''}^{|\Theta'|} < \hat{t}_{\Theta'}^{|\Theta'|} - \kappa^{(|\Theta'|+1)} \hat{q}_{\Theta'}^{|\Theta'|} \\
& \hat{t}_{\Theta''}^{|\Theta'|} - \kappa^{(|\Theta'|)} \hat{q}_{\Theta''}^{|\Theta'|} + \kappa^{(|\Theta'|)} \hat{q}_{\Theta''}^{|\Theta'|} - \kappa^{(|\Theta'|+1)} \hat{q}_{\Theta''}^{|\Theta'|} < \\
& \hat{t}_{\Theta'}^{|\Theta'|} - \kappa^{(|\Theta'|)} \hat{q}_{\Theta'}^{|\Theta'|} + \kappa^{(|\Theta'|)} \hat{q}_{\Theta'}^i - \kappa^{(|\Theta'|+1)} \hat{q}_{\Theta'}^{|\Theta'|} \\
u_A \left( \hat{q}_{\Theta''}^{|\Theta'|}, \hat{t}_{\Theta''}^{|\Theta'|}, \kappa^{(|\Theta'|)} \right) + \left( \kappa^{(|\Theta'|)} - \kappa^{(|\Theta'|+1)} \right) \hat{q}_{\Theta''}^{|\Theta'|} < \\
u_A \left( \hat{q}_{\Theta'}^{|\Theta'|}, \hat{t}_{\Theta'}^{|\Theta'|}, \kappa^{(|\Theta'|)} \right) + \left( \kappa^{(|\Theta'|)} - \kappa^{(|\Theta'|+1)} \right) \hat{q}_{\Theta'}^{|\Theta'|},
\end{aligned}$$

which follows from the previous inductive proof and  $\hat{q}_{\Theta''}^{|\Theta'|} < \hat{q}_{\Theta'}^{|\Theta'|}$  (Lemma 12 (i)). It now follows immediately that any  $i$  with  $|\Theta'| + 1 \leq i < |\Theta''|$  prefers to raise the principal's awareness at most to  $\Theta_{(i-1)}$ , where  $\Theta_{(i-1)}$  denotes the space of moves of nature in which  $i - 1$  is the lowest marginal cost type.

**Level 4. Principal:** If the principal is not made aware of additional marginal cost types (i.e., her information set in the lowest tree), then no further restrictions are imposed by fourth-level  $\Delta$ -prudent rationalizability. That is, she offers a menu of contracts that maximize expected utility w.r.t. a full support belief over marginal cost types she has been aware of subject to the participation constraint for marginal cost type  $\kappa^{(1)}$  and incentive compatibility constraints for all others.

If the principal's awareness is raised to  $\Theta_{(i)}$ , then since the principal is now certain of level-3  $\Delta$ -prudent rationalizable strategies of the agent, she realizes that no type in  $\Theta_{(i)}$  has had an incentive to make her aware of  $\Theta_{(i)}$ . (Any type in  $i \in \Theta_{(i)} \setminus \Theta_P$  would have raised her awareness at most only to  $\Theta_{(i-1)}$ .) Thus, she cannot further rationalize the agent's action; she is allowed to believe anything according to some belief system in  $B_P$  and her set of strategies is not refined further:  $R_P^4 = R_P^3$ .

*Agents:* No additional strategies can be eliminated at level 4 since there were none eliminated for the principal at level 3.

Since none of the players' sets of strategies are refined at level 4, none are refined at further levels. This completes the proof.  $\square$

## Proof of Proposition 2

**Level 1.** This part of the proof is analogous to that of Proposition 1.

**Level 2.** This part of the proof is also analogous to that of Proposition 1. Compared to the proof of Proposition 1, however, it is convenient to use the dual order on the marginal cost types. That is, we now denote by  $\kappa^{(1)}$  the type with the lowest marginal costs and by  $\kappa^{(|\Theta_P|)}$  the type with the highest marginal costs of which the principal is initially aware. This makes the order statistics of marginal cost types in lower spaces invariant to raising awareness of additional types with higher marginal costs. The first-order conditions can be obtained from Appendix A with the appropriate adjustments in notation for the dual order statistics.

Unlike in the proof of Proposition 1, reverse Bayesianism implies now (Lemma 12 (ii)) that

for  $i = 1, \dots, |\Theta'|$  and any  $\Theta'' \supseteq \Theta'$ ,

$$\hat{q}_{\Theta''}^i = \hat{q}_{\Theta'}^i.$$

**Level 3. Principal:** No additional strategies are eliminated at level 3 since no further strategies of the agents have been eliminated at level 2.

*Agent:* Any level-3  $\Delta$  prudent rationalizable strategy of the agent must satisfy the following:

(a) Type  $\kappa^{(\bar{\Theta})}$  is indifferent between disclosing or not disclosing  $\bar{\Theta}$  to the principal since he is always held to his outside option. When not disclosing, his participation constraint is violated and he selects the outside option. When disclosing, he receives the same payoff as from taking the outside option.

(b) We show by induction on the dual order statistics of agent types that all  $i = 1, \dots, |\bar{\Theta}| - 1$  prefer to disclose  $\bar{\Theta}$  over any other  $\Theta$ .

Base case  $i = |\bar{\Theta}| - 1$ : For any  $\Theta_j = \{1, \dots, j\}$  (i.e., all marginal cost types from the lowest marginal cost type to marginal cost type  $\kappa^{(j)}$ ) with  $j \leq |\bar{\Theta}| - 1$ ,

$$\begin{aligned} u_A \left( \hat{q}_{\bar{\Theta}}^{|\bar{\Theta}|-1}, \hat{t}_{\bar{\Theta}}^{|\bar{\Theta}|-1}, \kappa^{(|\bar{\Theta}|-1)} \right) &\geq u_A \left( \text{raise awareness to } \Theta_j, \kappa^{(\bar{\Theta}-1)} \right) \\ \hat{t}_{\bar{\Theta}}^{|\bar{\Theta}|-1} - \kappa^{(|\bar{\Theta}|-1)} \hat{q}_{\bar{\Theta}}^{|\bar{\Theta}|-1} &\geq 0 \\ \hat{t}_{\bar{\Theta}}^{|\bar{\Theta}|} - \kappa^{(|\bar{\Theta}|-1)} \hat{q}_{\bar{\Theta}}^{|\bar{\Theta}|} &\geq 0, \end{aligned}$$

where the r.h.s. of the second line follows from the fact that type  $\kappa^{(|\bar{\Theta}|-1)}$  would select the outside option when the principal is aware only of  $\Theta_j$  if  $j < |\bar{\Theta}| - 1$  or be held to the payoff of the outside option if  $j = |\bar{\Theta}| - 1$ . The last line follows from incentive compatibility as well as the highest-marginal cost type's participation constraint.

Induction hypothesis: For any  $i$  with  $1 < i < |\bar{\Theta}|$ ,

$$u_A \left( \hat{q}_{\bar{\Theta}}^{i+1}, \hat{t}_{\bar{\Theta}}^{i+1}, \kappa^{(i+1)} \right) \geq u_A \left( \hat{q}_{\Theta_j}^{i+1}, \hat{t}_{\Theta_j}^{i+1}, \kappa^{(i+1)} \right).$$

Inductive step: We prove for  $i$  with  $1 \leq i < |\bar{\Theta}|$  that:

$$u_A \left( \hat{q}_{\bar{\Theta}}^i, \hat{t}_{\bar{\Theta}}^i, \kappa^{(i)} \right) \geq u_A \left( \hat{q}_{\Theta_j}^i, \hat{t}_{\Theta_j}^i, \kappa^{(i)} \right).$$

If  $j < i$ , then disclosing just  $\Theta_j$  results in  $i$  taking the outside option and there is nothing to prove. If  $j = i$ , then disclosing just  $\Theta_j$  results in  $i$  being held to his outside option and the payoff is zero. Again, there is nothing to prove. Next, consider the case  $j > i$ :

$$\begin{aligned} u_A \left( \hat{q}_{\bar{\Theta}}^i, \hat{t}_{\bar{\Theta}}^i, \kappa^{(i)} \right) &\geq u_A \left( \hat{q}_{\Theta_j}^i, \hat{t}_{\Theta_j}^i, \kappa^{(i)} \right) \\ \hat{t}_{\bar{\Theta}}^i - \kappa^{(i)} \hat{q}_{\bar{\Theta}}^i &\geq \hat{t}_{\Theta_j}^i - \kappa^{(i)} \hat{q}_{\Theta_j}^i \\ \hat{t}_{\bar{\Theta}}^{i+1} - \kappa^{(i)} \hat{q}_{\bar{\Theta}}^{i+1} &\geq \hat{t}_{\Theta_j}^{i+1} - \kappa^{(i)} \hat{q}_{\Theta_j}^{i+1} \\ \hat{t}_{\bar{\Theta}}^{i+1} - \kappa^{(i+1)} \hat{q}_{\bar{\Theta}}^{i+1} + \kappa^{(i+1)} \hat{q}_{\bar{\Theta}}^{i+1} - \kappa^{(i)} \hat{q}_{\bar{\Theta}}^{i+1} &\geq \hat{t}_{\Theta_j}^{i+1} - \kappa^{(i+1)} \hat{q}_{\Theta_j}^{i+1} + \kappa^{(i+1)} \hat{q}_{\Theta_j}^{i+1} - \kappa^{(i)} \hat{q}_{\Theta_j}^{i+1} \\ u_A \left( \hat{q}_{\bar{\Theta}}^{i+1}, \hat{t}_{\bar{\Theta}}^{i+1}, \kappa^{(i+1)} \right) + \left( \kappa^{(i+1)} - \kappa^{(i)} \right) \hat{q}_{\bar{\Theta}}^{i+1} &\geq u_A \left( \hat{q}_{\Theta_j}^{i+1}, \hat{t}_{\Theta_j}^{i+1}, \kappa^{(i+1)} \right) + \left( \kappa^{(i+1)} - \kappa^{(i)} \right) \hat{q}_{\Theta_j}^{i+1}, \end{aligned}$$

which follows from the induction hypothesis,  $\kappa^{(i+1)} > \kappa^{(i)}$ , and from the fact that  $\hat{q}_{\Theta}^{i+1} = \hat{q}_{\Theta_j}^{i+1}$  (Lemma 12 (ii)).

**Level 4. *Principal:*** At level 4, the principal is now certain of level-3  $\Delta$ -prudent rationalizable strategies of the agent. Thus, upon becoming aware of more types, she is certain that all agents could have raised her awareness. (Full support beliefs become now crucial for not ruling out that the high cost type could have made her aware.) She cannot exclude any type and there is no further reduction of her strategy set.

*Agent:* Since the principal's set of strategies was not reduced at level 3, no strategies of the agent are eliminated at level 4.

Since none of the player's sets of strategies were refined at level 4, none are refined at further levels. This completes the proof.  $\square$

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