

# INTRODUCTION TO THE SPECIAL ISSUE ON UNAWARENESS

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The B.E. Journal of Theoretical Economics is very proud to present the special issue on unawareness. The special issue collects invited unpublished “classics” on unawareness together with submitted new developments on games with unawareness, decision theory with unawareness, and applications.

The first article of the special issue on “*Game theory without Partitions, and Applications to Speculation and Consensus*” by John Geanakoplos is a true classic and has inspired the literature on unawareness since at least 1989. Geanakoplos’ example of Sherlock Holmes and Doctor Watson with the dog that didn’t bark surfaced again and again in many papers on unawareness that followed. It underlines the fact that a person who is unaware of an event cannot infer anything from the non-occurrence of that event. The article focuses on mistakes in information processing rather than unawareness per se. Yet, early on it was believed that mistakes in information processing as modelled with non-partitional information structures might be able to capture unawareness. Only subsequent work by Dekel, Lipman, and Rustichini (1998) claimed that no matter how you model information structures, natural properties of unawareness clash with merely assuming a state space. Nevertheless, Geanakoplos’ work on mistakes of information remains relevant to the unawareness literature in a perhaps less obvious way. Ely, Chen, and Luo (2012) showed that those natural properties of unawareness imply properties necessary for partitional information structures. So clearly you cannot implicitly assume these properties (by assuming natural properties on unawareness as in Dekel, Lipman, and Rustichini, 1998) and simultaneously give up these properties by weakening the information structures as in Geanakoplos’ article. This must lead to the contradiction observed by Dekel, Lipman, and Rustichini (1998). This contradiction was later resolved by allowing for a lattice of spaces in Heifetz et al. (2006) who showed that you can have a strong notion of knowledge (with essentially the same properties as in partitional information structures) and non-trivial unawareness. Unawareness is not due to boundedly rational information processing but rather due to bounded perception. However, as economics becomes more and more interested in studying biases, mistakes, and bounded rationality, Geanakoplos’ study of mistakes in information processing should experience a renaissance. He demonstrates how exactly speculation and consensus depends on properties of information processing. His simple event-based approach with a non-partitional possibility correspondence could be applied to other problems in economics. The paper is likely to inspire many more papers to come.

The second article “*Do I Know ? An Axiomatic Model of Awareness of Knowledge*” by Cesaltina Pires is in fact one of the first (if not the first) paper on unawareness in economics. It

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was part of Pires’s PhD dissertation in economics submitted in 1994 at MIT. Unfortunately, the paper was not submitted for publication and thus it remains largely unknown, when it could have influenced the economics literature alongside with Modica and Rustichini (1994, 1999) and Dekel, Lipman, and Rustichini (1998). Being a highly original paper, it introduced a logic of awareness for the single agent case. It clearly anticipates the notion of propositionally generated awareness that has become the standard notion of awareness in the economics literature. For instance, Pires weakens the negative introspection axiom to what is now known as weak negative introspection, i.e., if the agent does not know a proposition and is aware of the proposition, then she knows that she does not know the proposition. The paper also anticipates that the “subjective” state space of the agent must differ from a modeler’s “objective” state space. It even starts looking at dynamics of awareness. This paper was a remarkable achievement for its time and it deserves to become known.

We are extremely delighted to include into the special issue “*Games with Unawareness*” by Yossi Feinberg. This article is the latest of a number of versions that appeared over the years as working papers. I still remember when back in 2004 as a fresh PhD I received an email from Yossi Feinberg with the first version of his paper. I thought we just barely figured out how to tractably model unawareness in interactive settings but Yossi was already on his way of studying games with unawareness. Over the years, he refined his approach and presented an ambitious unified theory of games in normal-form with unawareness, repeated games with unawareness, games with incomplete information and unawareness, and extensive-form games with unawareness. The unifying modeling principle is explicit descriptions of how “player  $i$  perceives that player  $j$  perceives ... that player  $k$  perceives the game”, i.e., possibly infinite sequences of interactive views of the game. This complements nicely the perhaps more parsimonious but less explicit event-based approach to interactive unawareness and should prove to be useful when minute descriptions of sequences of perceptions are explicitly required. Alternative approaches to games with unawareness have been presented by Rêgo and Halpern (2012), Halpern and Rêgo (2014), Heifetz, Meier, and Schipper (2013a), and Meier and Schipper (2014a). Yossi’s examples throughout various iterations of his paper served as inspirations for many works on unawareness that followed.

Bayesian games with unawareness are the focus of “*Knowledge, Awareness, and Probabilistic Beliefs*” by Tomasz Sadzik. A first version of the paper was part of his dissertation at Stanford GSB in 2008. Tomasz extends awareness logics by allowing for a “probability of at least  $\alpha$ ” modal operator for each agent. He proves soundness and completeness. Soundness means that every theorem derivable from the axioms obtains in all states (where it is defined) of all these unawareness models. Completeness means that every formula that obtains in all states in every unawareness model is provable from axioms. Tomasz’s results complement earlier soundness and completeness results for awareness logics without the “probability of at least  $\alpha$ ” modality by for instance Fagin and Halpern (1988) and Heifetz, Meier, and Schipper (2008). Tomasz goes well beyond the foundations of awareness. Having a “probability of at least  $\alpha$ ” modality allows him to apply his work to Bayesian games with unawareness, where he studies subtleties of equilibria under unawareness. Tomasz assumes that players at lower awareness levels take “average” strategies of players at higher awareness levels into account. Thus, he interprets a player’s strategy as (generating an) objective (distributions of) actions rather than as objects of opponents’ subjective beliefs. This complements work by Meier and Schipper (2014a) who focus on the latter assumption in Bayesian games with unawareness.

*“Prudent rationalizability in Generalized Extensive-Form Games with Unawareness”* by Aviad Heifetz, Martin Meier and myself was originally conceived of as a companion paper to Heifetz, Meier, and Schipper (2013a). We define a version of extensive-form rationalizability with cautious belief and illustrate how it differs from extensive-form rationalizability even in standard games without unawareness. This solution concept is then applied to games of disclosure of verifiable information under unawareness. We show that these games can be solved with prudent rationalizability in an intuitive way that captures unraveling of information level-by-level. More importantly we show that the unraveling result breaks down under unawareness, which is in stark contrast to games without unawareness.

Two articles of the special issue focus on decision theory with unawareness. *“Reverse Bayesianism: A Generalization”* by Edi Karni, Quitzé Valenzuela-Stookey, and Marie-Louise Vierø generalizes Karni and Vierø (2013) on updating beliefs upon becoming aware. In particular, they allow new discoveries not only to siphon probability mass off previously aware states in a relative likelihood preserving way but may also nullify some previous non-null states. In *“Ambiguity and Awareness: A Coherent Multiple Priors Model”*, Simon Grant, Ani Guerdjikova, and John Quiggin study the interaction of ambiguity with awareness of unawareness. The decision maker is somehow aware that she is unaware of some relevant events and therefore faces ambiguity as captured by multiple priors over events that she is able to fully conceptualize. An increase of awareness leads not just to an expansion of the state space but also to a contraction of the set of priors as she now faces less ambiguity-creating awareness of unawareness.

The special issue concludes with two applications, both in financial trading environments. Spyros Galanis and Stelios Kontronis study updating awareness based on price observations and eventual agreement in *“Updating Awareness and Information Aggregation”*. They assume that each trader upon observing actions of other traders that she cannot rationalize somehow increases her awareness to the minimal extent necessary to rationalize others’ actions. They show that in the limit, traders will eventually agree on the price of the security. Moreover, under an additional assumption there is information aggregation and agreement on the correct price. This dynamic approach complements nicely static analyses of speculation under unawareness in the literature (Heifetz, Meier, and Schipper, 2006, 2013b, Galanis, 2013, 2018, Meier and Schipper, 2014). An application to contract theory is presented in *“Delegation and Information Disclosure with Unforeseen Contingencies”* by Lei Haoran and Zhao Xiaojian. The authors study delegation of an investor to a financial expert. The expert is aware of events that the investor is unaware of. The expert reveals all events to the investor only if the investor is already aware of a sufficiently large set of events. Otherwise, there is no or partial revelation. The delegation set of the investor is larger the less aware she is. The authors also show how their observations depend on the particular assumptions about equilibrium selection and the uniform prior. This paper complements related papers by Auster and Pavoni (2021a, b) who assume that the principal/investor is unaware of some actions/financial securities.

The last two papers are examples of an emerging literature on applications to unawareness in economics. Given that we now have the tools in place to study unawareness, we can revisit the entire literature on asymmetric information and study how conclusions change when assuming asymmetric awareness. We expect to see many more papers on unawareness to appear.

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