# Delegation and Information Disclosure with Unforeseen Contingencies[*]

Albert Haoran Lei[†] Xiaojian Zhao[‡]

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**Abstract**

We incorporate unawareness into the delegation problem between a financial expert and an investor, and study their pre-delegation communication. The expert has superior awareness of the possible states of the world, and decides whether to reveal some of them to the investor. We find that the expert reveals all the possible states to the investor if the investor is initially aware of a large set of possible states, but reveals partially or nothing otherwise. An investor with a higher degree of unawareness tends to delegate a larger set of projects to the expert, giving rise to a higher incentive for the expert to keep her unaware.

KEYWORDS: Delegation, Unawareness, Unforeseen Contingencies, Financial Advice
JEL CLASSIFICATION: D82, D83, D86

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[†]School of Economics and Trade, Hunan University. Email: hleiaa@connect.ust.hk.
[‡]Monash University and Chinese University of Hong Kong (Shenzhen). Email: xjzhao81@gmail.com.

# 1 Introduction

Financial advice service plays an important role for retail investors. While experienced retail investors may have the ability to choose self-directed investments, most retail investors rely on professional financial advice to make investment decisions. In the United States, the revenue of financial planning and advice industry has been rapidly growing, estimated at $55 billion in 2018.[1] In the United Kingdom, it has been found that people with limited wealth would pay on average £258 for advice on investing an inheritance of £60,000.[2] Retail investors demand financial advice, as they may be unfamiliar with, or even *unaware* of the potential risk of the available investment choices. However, conflicts of interest between retail investors and financial advisors might make services of the latter curses for the investors. The combination of information asymmetry and conflicts of interest may turn financial consultancy to detriments of investors.

This paper considers a situation in which investors have limited understanding of factors determining the returns of available investment choices and attempts to study how the degree of an investor's financial illiteracy affects the quality of an expert's financial service. Specifically, when can experts benefit from withholding superior awareness regarding potential risks from the clients? How will the conflicts of interest and the investor's limited cognition jointly determine the quality of the expert's financial advice, and the investor's reliance on the expert's discretion for making his financial decisions?

To answer these questions, we incorporate the concept of unawareness to capture an investor's unforeseen contingencies in the economic and financial environment, and investigate how the investor's degree of unawareness affects

[1]https://www.ibisworld.com/industry-trends/market-research-reports/finance-insurance/securities-commodity-contracts-other-financial-investments-related-activities/financial-planning-advice.html

[2]See page 21 of the final report of Financial Advice Market Review in 2016: https://www.fca.org.uk/publication/corporate/famr-final-report.pdf

the quality of the expert's financial services. We model the investor-expert interaction by adopting the framework of delegation problems, and add a pre-delegation awareness-revelation stage. Prior to delegation, the expert might be willing to reveal eye-opening information to the investor to induce a more favorable delegation set of investment decisions. Both the investor's and the expert's payoffs depend on the implemented investment decision and the realized state of the world.

In the benchmark case in which the investor is aware of all the contingencies in the state space, standard delegation theory suggests that the optimal delegation set for the expert is an interval, under some regularity conditions on the distribution of states.[3] In the presence of investor unawareness, we obtain several findings. First, in terms of the revelation of possible states contained in the expert's advice, the investor tends to reveal less states as the investor's degree of unawareness increases. Second, full revelation, partial revelation and no revelation may all appear depending on the states of which the investor is initially aware. Third, as for the delegation outcome, an investor of a higher degree of unawareness tends to delegate a larger set of projects.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces the setting of the model. Section 4 analyzes the model and summarizes our key results. The last section concludes. All proofs and some technical discussions are relegated to the Appendix.

## 2 Literature

This paper contributes to a growing literature on contracts with unawareness. Contracting parties may be unaware of available *actions* (von Thadden and Zhao, 2012, 2014; Auster and Pavoni, 2020, 2021) or unaware of possible *states* (Zhao, 2011; Filiz-Ozbay, 2012; Auster, 2013; Ma and Schipper, 2014).

[3]See Martimort and Semenov (2006).

Our paper belongs to the latter as the investor in our model may be unaware of states of the world. Specifically, our study is related to the literature on information disclosure with unawareness (Heifetz, Meier, and Schipper, 2011; Li, Peitz, and Zhao, 2014, 2016; Schipper and Woo, 2019) as awareness may expand in dynamic games.

Closest to our model is Auster and Pavoni (2021) that also considers a delegation problem yet with the investor being unaware of possible actions rather than states. This difference makes the equilibrium selection play a vital role in our results, as will be discussed below. Moreover, our paper focuses on the uniform-quadratic setting while Auster and Pavoni (2021) is more general in the sense that they allow for a large scope of utility functions and a class of state distributions satisfying certain regularity conditions. In another work, Auster and Pavoni (2020) extend their framework by introducing multiple agents competing for investors via a menu of options.

This paper also belongs to the extensive literature on optimal delegation starting from Holmstrom (1977, 1980). Alonso and Matouschek (2008) characterize the optimal delegation set when the feasible delegation sets are compact and the players' preferences take a generalized quadratic form. Kováč and Mylovanov (2009) show that the optimal mechanism can be stochastic in a quadratic preferences setting, and provide a sufficient condition for the optimal mechanism to be deterministic. Variants of the delegation model have been used in political economy (Bendor, Glazer, and Hammond, 2001; Krishna and Morgan, 2001), organization and regulation (Aghion and Tirole, 1997; Baron and Myerson, 1982), and trade (Amador and Bagwell, 2013), while the application in financial advice to unaware investors is still in its infancy.

Lastly, this paper is part of the literature on financial advice. These works, including Loewenstein, Cain, and Sah (2011), Inderst and Ottaviani (2012), Lusardi and Mitchell (2014) and Gui, Huang, and Zhao (2020a) among others, focus on how conflicts of interest and investors' limited financial knowledge lead to welfare loss and the related regulatory issues. This research stream may provide guidance for public policy to deal with financial illiteracy. For example, Gui, Huang, and Zhao (2020b) conduct experiments and surveys to investigate financial literacy associated with investor awareness, and find that financial education program significantly reduces the participants' tendency to invest in high-risk products, especially for those who are risk-averse.

# 3 Model

An investor (she) seeks financial advice from an expert (he) and then delegates the investment choice to him. Factors that affect the return of available investment options are summarized by a one-dimensional random variable $\theta$. We refer to $\theta$ as a state of the world. The state space $\Theta$ is assumed to be $[0, 1]$. The set of available investment options is $Y = [\underline{y}, \bar{y}]$.

The investor cannot observe the state of the world. Moreover, she is only aware of a subset, $[\theta_1, \theta_2]$, of the original state space. We refer to $[\theta_1, \theta_2]$ as the investor's initial *awareness set*. In contrast, the expert is aware of the complete state space. The expert decides how to expand the investor's awareness set (*revelation phase*), followed by the investor delegating a set of investment decisions to the expert (*delegation phase*). The expert then privately observes the realized state of the world and implements his most desirable investment decision in the delegation set (*investment phase*). Finally, both investor and expert receive the payoffs depending on the realized state and the chosen investment decision. More formally, the timing is follows, also depicted in Figure 1.

1. Revelation Phase: the expert strategically expands the investor's awareness set from $[\theta_1, \theta_2]$ to some compact set $\hat{\Theta} \subseteq [0, 1]$.

2. Delegation Phase: Given her updated awareness set $\hat{\Theta}$, the investor chooses a compact delegation set $D \subseteq Y$.

3. Investment Phase: The expert privately observes the realized state $\theta$ and then chooses some investment option $y$ from $D$.

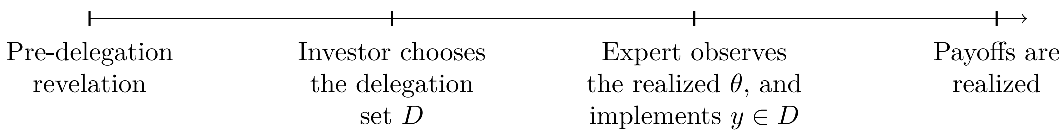


Figure 1: Timeline

The investor's utility function $u^I$ and the expert's utility function $u^E$ are given by

$$u^I(y, \theta) = -\frac{1}{2}(y - \theta - b)^2,$$
$$u^E(y, \theta) = -\frac{1}{2}(y - \theta)^2.$$

While it appears to suffice to describe the model with the timing above, we additionally provide a detailed formalization of the strategic interaction with unawareness in Appendix B.

As in the standard delegation model, we rule out contingent monetary transfers, and let $b > 0$, representing the conflicting interests between the expert and the investor. Given a realized state $\theta$, the expert's most preferred option is $y^E(\theta) = \theta$ while the investor's most preferred option is $y^I(\theta) = \theta + b$. We interpret a higher value of $\theta$ as a riskier economic environment, and a higher level of $y$ as a more defensive investment strategy involving a conservative plan of portfolio allocation aimed at minimizing the risk of losing principal. Both the investor and the expert prefer a more defensive investment decision in a riskier state. Moreover, the optimal investment option of the investor is more defensive than that of the expert in that $y^I(\theta) - y^E(\theta) = b > 0$ for all $\theta$. The parameter $b$ thus captures the difference between the investor's and the expert's risk tolerances.

Our model departs from the earlier works in that the investor is unaware of some possible states. Moreover, the expert can make the investor aware of additional states by, for example, providing a professional report on the possibility of an economic boom driven by technological innovations. This setting is consistent with the empirical facts[4] that retail investors with limited financial literacy tend to be more dependent on the experts' advice in making investments.

We impose the following assumptions on the players' difference in the attitudes toward risks, the available investment decisions and the players' beliefs throughout the paper.

**A1** $b < 1/2$.

**A2** $\underline{y} < -\frac{1}{2} - b$ and $\bar{y} > 1 + b$.

**A3** $\theta$ is uniformly distributed, and the investor's belief about $\theta$ is a uniform distribution on her awareness set, both of which are known by the expert.

Assumption A1 guarantees that delegation is valuable for the investor even in the benchmark case in which the investor is aware of all possible states. We also impose a relatively larger action set in Assumption A2 to avoid tedious calculations in discussing many corner solutions. Assumption A3 makes our analysis much easier. Our restriction of the investor's evolution of beliefs with growing awareness is aligned with the "reverse Bayesianism" as in Karni and Vierø (2013, 2015). Assumption A3 also suggests that the expert can perfectly identify the support of the investor's belief distribution, while the investor is unaware of her unawareness.[5]

[4]Hackethal, Inderst, and Meyer (2012) use data from a large bank and demonstrate that less-educated investors were more likely to report relying on the advisor's investment advice. Georgarakos and Inderst (2014) find that financial advice significantly affects the likelihood that less-educated households will hold risky assets.

[5]Relatedly, Galperti (2019) studies a persuasion model in which the support of the agent's belief distribution is larger than that of the principal's belief distribution. In contrast to our model, players can agree to disagree, implying for people's opinions on "controversial" topics such as political issues. The assumption of an unaware principal fits more naturally in the context of professional consultancy such as financial advice due to his potential lack of professional knowledge.

# 4 Analysis

## 4.1 Solution concept

The widely-used concept of Nash equilibrium does not apply in the presence of an unaware player. Halpern and Rêgo (2014) define a general solution concept for extensive games with unaware players. Intuitively, it requires that each agent chooses their best move given their local beliefs of the whole game at their respective node. Our solution concept follows the generalized Nash equilibrium proposed by Halpern and Rêgo (2014) with the modification that we solve for the generalized Nash equilibrium (and henceforth, *equilibrium*) by backward induction to eliminate the inappropriate solutions.

Generally, the investor's optimal delegation set and the expert's revelation choice are not unique. This paper focuses on the expert's choice of the largest awareness set and the investor's choice of the largest delegation set. Specifically, we assume that (i) if the expert is indifferent between two enlarged awareness sets $\hat{\Theta}_1$ and $\hat{\Theta}_2$ with $\hat{\Theta}_1 \subseteq \hat{\Theta}_2$, he will choose $\hat{\Theta}_2$ in the revelation phase and (ii) if the investor is indifferent between two delegation sets $D_1$ and $D_2$ with $D_1 \subseteq D_2$, she will choose $D_2$ in the delegation phase. Our equilibrium selection here echoes some experimental studies of delegation. Falk and Kosfeld (2006) show that the principal's control demotivates the agent's productive activity. Similarly, Charness, Cobo-Reyes, Jiménez, Lacomba, and Lagos (2012) suggest that given more authority, agents perform better due to the nonstrategic motivation caused by a sense of enhanced responsibility.

It is worth noticing that our focus on the maximal delegation set is relevant for the expert's strategic awareness choice in equilibrium. While Auster and Pavoni (2021) also consider the largest delegation set, in their paper, however, this equilibrium selection does not affect the outcome. To examine possible variations of results in our model, we discuss the situations in which the investor does not necessarily choose the maximal delegation set in

Section 4.6.

For a set $S$, denote by $\mathcal{C}(S)$ the set of all closed intervals in $S$ and denote by $\mathcal{T}(S)$ the set of all compact subsets of $S$. Let the expert's revelation strategy be $\sigma : \mathcal{C}(\Theta) \to \mathcal{T}(\Theta)$, the investor's delegation choice $D^* : \mathcal{T}(\Theta) \to \mathcal{T}(Y)$ and the expert's investment choice $y^* : \mathcal{T}(Y) \times \Theta \to Y$. We call $(\sigma, D^*, y^*)$ a *generalized strategy profile* as in Halpern and Rêgo (2014).

**Definition 1.** A generalized strategy profile $(\sigma, D^*, y^*)$ is an *equilibrium* if

1. The expert chooses the investment option that maximizes his utility given the delegation set $D \in \mathcal{T}(Y)$ and the state of the world $\theta \in \Theta$:

$$y^*(D, \theta) \in \arg\max_{y \in D} u^E(y, \theta)$$

2. Given the expert's investment strategy $y^*$ and the investor's updated awareness set $\hat{\Theta}$, the investor chooses the delegation set that maximizes her expected utility:

$$D^*(\hat{\Theta}) \in \arg\max_{D \in \mathcal{T}(Y)} \mathbb{E}u^I\Big(y^*(D, \theta), \theta \,\Big|\, \theta \in \hat{\Theta}\Big)$$

Specifically, if the investor is indifferent between $D_1^*$ and $D_2^*$ and $D_1^*(\hat{\Theta}) \subseteq D_2^*(\hat{\Theta})$ for all updated awareness set $\hat{\Theta}$, then she will choose $D_2^*$.

3. Given the expert's investment strategy $y^*$, the investor's delegation strategy $D^*$ and the investor's initial awareness set $\Theta_0$, the expert chooses the awareness set $\sigma(\Theta_0) \in \mathcal{T}(\Theta)$ to maximize his expected utility:

$$\sigma(\Theta_0) \in \arg\max_{\hat{\Theta} \in \mathcal{T}(\Theta)} \mathbb{E}u^E\Big[y^*\Big(D^*(\hat{\Theta}), \theta\Big), \theta \,\Big|\, \theta \in \Theta\Big]$$

subject to $\Theta_0 \subseteq \sigma(\Theta_0) \subseteq \Theta$. Specifically, if the expert is indifferent between $\sigma_1$ and $\sigma_2$ with $\sigma_1(\Theta_0) \subseteq \sigma_2(\Theta_0)$, then he will choose $\sigma_2$.

Given our solution concept, we solve the model backwards.

## 4.2 Investment choice

The expert's optimal investment strategy is given by

$$y^*(D,\theta) \in \arg\min_{y\in D} |y-\theta| \,. \tag{1}$$

The choice function $y^*$ above is well-defined due to the compactness of $D$.

## 4.3 Delegation choice

Let the investor's updated awareness set be an interval, $[\underline{\theta},\bar{\theta}]$. Intuitively, the investor would delegate those options above some threshold to the expert because she is upwardly biased. In the uniform-quadratic setting, the threshold is $\underline{\theta}+\min\left\{2b, \frac{\bar{\theta}-\underline{\theta}}{2}+b\right\}$. Moreover, the investor would also delegate those options close to $\underline{y}$ because she is unaware of states in $[0,\underline{\theta}]$.

By standard delegation theory, when $b < \left(\bar{\theta}-\underline{\theta}\right)\big/2$, the delegation is *valuable* for the investor and the minimal optimal delegation set would be $\left[\underline{\theta}+2b, \bar{\theta}+b\right]$; when $b \geq (\bar{\theta}-\underline{\theta})/2$, the delegation is not valuable and the delegation set would be a singleton; that is, $\left\{\frac{\bar{\theta}+\underline{\theta}}{2}+b\right\}$. By adding options that the investor believes would never be chosen by the expert, we get the maximal optimal delegation set in both cases.

**Proposition 1.** *If the investor's awareness set is $[\underline{\theta},\bar{\theta}]$ in the delegation phase, then the investor's optimal delegation strategy is:*

$$D^*([\underline{\theta},\bar{\theta}]) = Y \setminus \left(\underline{\theta}-\Delta, \underline{\theta}+\Delta\right), \ \textit{where}\ \Delta = \min\left\{2b, \frac{\bar{\theta}-\underline{\theta}}{2}+b\right\}.$$

By Proposition 1, the delegation choice of the investor is characterized by a gap $\Delta$ given her awareness set $\hat{\Theta}(=[\underline{\theta},\bar{\theta}])$. The delegation choice depends on both the conflict of interests, characterized by $b$ and the investor's awareness set, characterized by $\underline{\theta}$ and $\bar{\theta}$. An investor of a higher degree of unawareness (or equivalently, a smaller awareness set) tends to delegate more in equilibrium. Figure 2 depicts the optimal delegation choice of an unaware investor with the awareness set $[\underline{\theta}, \bar{\theta}]$. The axis represents all possible investment options. The undelegated options, $(\underline{\theta} - \Delta, \underline{\theta} + \Delta)$, are represented by the dashed line, the delegation set the solid lines.

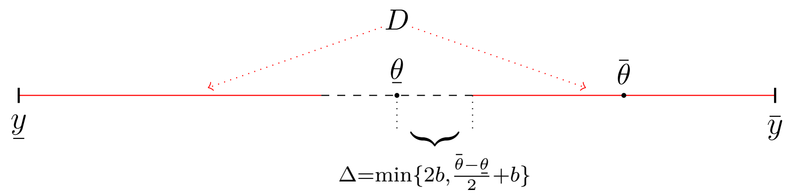


Figure 2: Delegation choice when the awareness set is $[\underline{\theta}, \bar{\theta}]$

It is because the investor is unaware of the states $[0, \underline{\theta}]$ that she is willing to delegate options below $\underline{\theta} - \Delta$. From her perspective the expert would never implement those lower options. In other words, the investor is naively too pessimistic about the economic environment and finds no reason for the expert to choose these extremely aggressive investment options. Thus, delegating these aggressive options to the expert involves no loss from the investor's perspective.

## 4.4 Revelation choice

To solve for the expert's revelation choice in equilibrium, we firstly focus on the simplest case that the expert could manipulate the investor's awareness set arbitrarily; that is, we ignore the constraint that $\sigma(\Theta_0) \supseteq \Theta_0$ at this moment but still require the awareness set to be a closed interval or a singleton in $\Theta$. Note that from Proposition 1, the expert always prefers $(\bar{\theta} - \underline{\theta})/2 \leq b$ if possible.

**Lemma 2.** *If the expert could choose any closed interval or singleton in $\Theta$ as the investor's awareness set, then the expert would make the investor only aware of either of the two extreme states; that is, $\hat{\Theta} = \{0\}$ or $\hat{\Theta} = \{1\}$.*

By Proposition 1, the gap $\Delta$ is minimized if and only if $\underline{\theta} = \bar{\theta}$. Therefore, the expert would make the investor only aware of a singleton. We illustrate

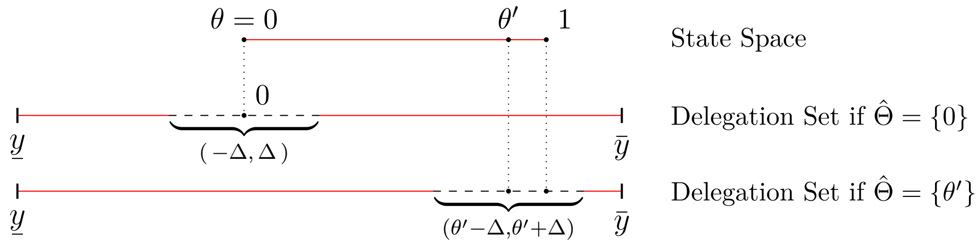


Figure 3: Illutration of Lemma 2

the proof of Lemma 2 in Figure 3. The upper line represents the whole state space $[0, 1]$. Specifically, we focus on the two awareness singleton sets, $\{0\}$ and $\{\theta'\}$ with $0 < \theta' < 1$. The middle dashed line represents the corresponding undelegated projects when $\hat{\Theta} = \{0\}$, the lower dashed line when $\hat{\Theta} = \{\theta'\}$. Compare the two revelation choices of $\theta = 0$ and $\theta' \in (0, 1)$. To maximize his expected utility, the expert wants to induce as many projects in $[0, 1]$ to be delegated as possible. If the investor is aware of either of the extreme states, say $\theta = 0$, the measure of the undelegated projects in $[0, 1]$ would be only $\Delta$. By contrast, if the investor is aware of some $\theta'$ between the two extreme states, the measure of the undelegated projects in $[0, 1]$ will be greater than $\Delta$. Thus the expert prefers to reveal $\theta = 0$ to induce a more favorable delegation set. Therefore, the expert prefers to interact with either the most optimistic investor with $\theta = 0$ believing that the economic environment is certainly the safest, or the most pessimistic investor with $\theta = 1$.

Now we turn to the expert's revelation strategy with the constraint that $\sigma(\Theta_0) \supseteq \Theta_0$. A direct corollary of Lemma 2 is that if the investor finds the delegation valuable before revelation, the expert would make the investor aware of the whole state space.

**Proposition 3.** *If the investor's awareness set $\Theta_0 (= [\theta_1, \theta_2])$ satisfies $\theta_2 - \theta_1 \geq 2b$, then the expert's optimal revelation strategy is full revelation; that is, $\sigma(\Theta_0) = [0, 1]$.*

Here we briefly sketch the proof of Proposition 3. When delegation is valuable for the investor, the delegation gap would be $2b$. Similarly to the argument in Lemma 2, the expert would expand the investor's awareness set to the lower bound $\underline{\theta} = 0$. Note that the expert is indifferent between whether to expand the upper bound of the investor's awareness set or not. Therefore, the expert's revelation strategy would be full revelation.

It is interesting to consider whether the expert is willing to expand awareness when delegation is not valuable. In the case of $\bar{\theta} - \underline{\theta} < 2b$, the set of undelegated projects take the form of $(\underline{\theta} - \Delta^*, \underline{\theta} + \Delta^*)$ where $\Delta^* = \frac{\bar{\theta}-\underline{\theta}}{2} + b$. The expert would not increase the upper bound of the awareness set because it strictly shrinks the delegation set. Nevertheless, the expert faces trade-offs between whether informing the investor of those low states or not. On the one hand, revealing lower states would increase the measure of the undelegated projects in $[0, 1]$, which might harm the expert's welfare. On the other hand, decreasing the lower bound of the awareness set would also decrease the upper bound of the set of undelegated projects. The latter strictly benefits the expert when the lower bound of the undelegated projects $(\underline{\theta} - \Delta^*)$ is much lower than 0.

**Proposition 4.** *If the investor's initial awareness set $[\theta_1, \theta_2]$ satisfies $\theta_2 - \theta_1 < 2b$, then the expert's optimal revelation strategy is:*

$$\sigma(\Theta_0) = \begin{cases} [\theta_1, \theta_2] & \textit{if } b < \frac{3}{2}\theta_1 - \frac{\theta_2}{2}; \\ [0, \theta_2] & \textit{if } b \geq \frac{3}{2}\theta_1 - \frac{\theta_2}{2} \textit{ and } \theta_2 < 2b; \\ [0, 1] & \textit{otherwise.} \end{cases}$$

By Proposition 4, full revelation, partial revelation and no revelation may all appear in equilibrium. Note that in partial revelation, the expert would reveal all the lower states $(\theta < \theta_1)$ while keeping the investor unaware of the riskier states $(\theta > \theta_2)$.

Combining Propositions 3 and 4, we conclude that when the investor is aware of a large set of possible states $(\theta_2 - \theta_1 \geq 2b)$, the expert would choose full revelation; when the investor is aware of a small set of possible states

($\theta_2 - \theta_1 < 2b$), the expert could choose partial revelation or no revelation at all.

**Example 1.** Let $b = 0.1$. Figure 4 shows the revelation strategy of the expert characterized in Propositions 3 and 4. The investor's initial awareness set is $[\theta_1, \theta_2]$. The horizontal line represents all possible values of $\theta_1$, the vertical line possible values of $\theta_2$. Therefore, each point in the upper triangle determines a specific initial awareness set. Given any awareness set, the corresponding revelation outcome is depicted, where *FR*, *NR*, and *PR* stand for full revelation, no revelation, and partial revelation respectively.[6]

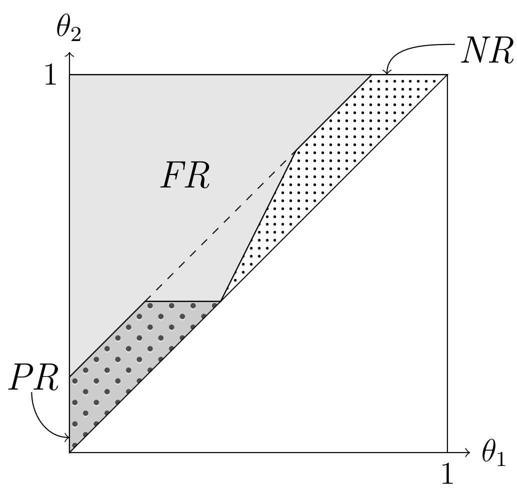


Figure 4: Revelation choices for all possible awareness sets

## 4.5 Summary

We have fully characterized the equilibrium results in the analysis above. The possible delegation and revelation outcomes are summarized in Figure 5,

[6]The small triangle, which lies in *FR* and is between *PR* and *NR* areas, exists due to our focus on the maximal awareness set. If the awareness set lies in the small triangle, the expert is indifferent between full revelation and partial revelation.

where the dashed lines represent the undelegated projects in the corresponding revelation outcome (full revelation, partial revelation, and no revelation).

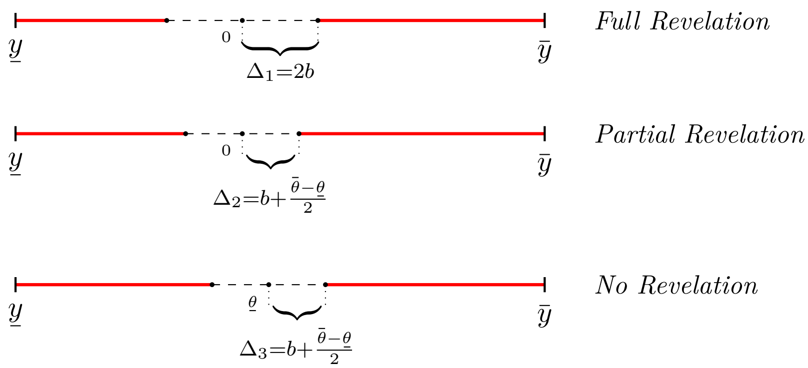


Figure 5: Delegation and revelation outcomes

By Proposition 4, an investor of a higher degree of unawareness tends to delegate more in equilibrium. Specifically, if the investor has a larger awareness set and the conflicts of interest is relatively small (that is, $b < (\theta_2 - \theta_1)/2$), the expert would choose full revelation and hence the investor's welfare would be the same as in the benchmark in which the investor is aware of the whole state space. In this case, financial advice *benefits* the investor. However, when the investor has a small awareness set and the unforeseen states are lower, the expert would choose no revelation and financial consultancy would not benefit the investor. Our result is in line with the financial literacy literature (Bucher-Koenen and Koenen, 2015; Calcagno and Monticone, 2015) that investors with higher financial literacy receive better advice and benefit more from advice.

## 4.6 (Non-)Robustness

Our main results above depend on the equilibrium selection of the maximal delegation set. Without that assumption, full revelation and no revelation of the expert may not exist in equilibrium. To wit, consider the simple case of a suspicious investor who does not delegate those lower options; that is, her delegation strategy is $D^*([\underline{\theta}, \bar{\theta}]) = [\underline{\theta} + \Delta, \bar{y}]$ where $\Delta = \min\{2b, \frac{\bar{\theta}-\underline{\theta}}{2} + b\}$. When the expert chooses the minimal awareness set[7], his optimal revelation strategy will be $\sigma(\Theta_0) = [0, \theta_2]$ where $\Theta_0 \equiv [\theta_1, \theta_1]$ is the investor's initial awareness set. The expert has a higher incentive to reveal lower states to a suspicious investor because an investor who is not suspicious also delegates those options below $(\underline{\theta} - \Delta)$, and hence lowering $\underline{\theta}$ may harm the expert.

We close our discussion by illustrating the case when both players choose the minimal sets whenever they are indifferent.

**Proposition 5.** *Suppose the expert always chooses the minimal awareness set and the investor always chooses the minimal delegation set. Let* $[\theta_1, \theta_2]$ *be the investor's initial awareness set and* $\hat{y} \equiv \frac{\theta_1+\theta_2}{2} + b$ *be the investor's initial most preferred option.*

1. *When* $b \leq \max\{\frac{\theta_2-\theta_1}{2}, \frac{1}{\sqrt[3]{32}}\}$*, the expert will choose full revelation in equilibrium.*

2. *When* $b > \max\{\frac{\theta_2-\theta_1}{2}, \frac{1}{\sqrt[3]{32}}\}$*, the expert's optimal revelation strategy is characterized below.*

   *(a) If* $\hat{y} = 1/2$*, the expert will choose no revelation;*

   *(b) If* $b$ *is relatively large and* $\hat{y} < 1/2$*, the expert will partially reveal states higher than* $\bar{\theta}$*;*

   *(c) If* $b$ *is relatively large and* $\hat{y} > 1/2$*, the expert will partially reveal states lower than* $\underline{\theta}$*.*

[7] In this situation, our focusing on the minimal awareness set does not affect the equilibrium outcome.

# 5 Conclusion

This paper analyzes the delegation problem in which the expert has a superior awareness of the states of the world under the uniform-quadratic setting, and shows to what extent it is not in the expert's interests to voluntarily reveal additional contingencies that are unforeseen by the investor, thereby possibly leading to an adverse delegation outcome for the investor.

In our model, a more literate investor foresees more contingencies and is more likely to find the delegation valuable. In contrast, a less literate investor has a subjective world perceiving fewer possibilities, which reduces her incentive to visit a financial advisor. This echoes the existing studies showing that investors with higher financial literacy are more likely to seek additional financial advice (Hackethal, Haliassos, and Jappelli, 2012; van Rooij, Lusardi, and Alessie, 2011).

On the other hand, financial advice from the experts may not be a sufficient instrument to cure the problem of financial illiteracy. In our model, the expert may not provide any awareness in equilibrium at all, or the expert may only make the investor aware of those safer states to induce the delegation of those more aggressive investments that might be suboptimal for the investor. In the European financial market, many consumers frequently receive advice from agents, but do not understand the potential impact of inducements and other incentives of those experts (Chater, Huck, and Inderst, 2010). Our analysis illustrates how investors' unawareness can be exploited through the expert's strategic awareness revelation, suggesting the demand of policy interventions such as imposing mandatory disclosure in financial advice.

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# Appendix

## Appendix A

We provide proofs of all the lemmas and propositions in Appendix A.

### A.1. Proof of Proposition 1

Following Alonso and Matouschek (2008), when $b < (\bar{\theta} - \underline{\theta})/2$ the delegation is valuable and the (minimal) optimal delegation set is $[\underline{\theta} + 2b, \bar{\theta} + b]$; when $b > (\bar{\theta} - \underline{\theta})/2$, the investor chooses her optimal project $y^* = \frac{\bar{\theta}+\underline{\theta}}{2} + b$ in the (minimal) optimal delegation set. By Assumption A1, the delegation can be valuable if the investor is aware of a relatively large set of states.

As we focus on the maximal delegation set, in equilibrium the investor delegates all projects above the threshold $y_H = \underline{\theta} + \min\{2b, \frac{\bar{\theta}-\underline{\theta}}{2} + b\}$. Moreover, she also delegates those projects below the threshold $y_L$ satisfying

$$-(y_H - \underline{\theta} - b)^2 = -(y_L - \underline{\theta} - b)^2.$$

The reason is that she believes these lower states will never be implemented by the expert. The lower cutoff is $y_L = \underline{\theta} - \min\{2b, \frac{\bar{\theta}-\underline{\theta}}{2} + b\}$. To sum up, the investor's optimal delegation choice is:

$$D^*([\underline{\theta}, \bar{\theta}]) = Y \setminus \left(\underline{\theta} - \Delta, \underline{\theta} + \Delta\right)$$

where $\Delta = \min\left\{2b, \frac{\bar{\theta}-\underline{\theta}}{2} + b\right\}$.

### A.2. Proof of Lemma 2

By Proposition 1, the delegation set is characterized by the gap $\Delta = \min\{2b, \frac{\bar{\theta}-\underline{\theta}}{2} + b\}$. Therefore, the expert has incentives to make the investor aware of one singleton to minimize the measure of the undelegated projects; that is, $\underline{\theta} = \bar{\theta}$.

The expert's expected utility depends on the interception of the set of undelegated options and his preferred options, $[0, 1]$. Suppose that the expert's revelation choice is $\theta' = 0$. Then the expert's expected utility would be

$$\mathbb{E}_\theta u^E(D', \theta) = -2\int_0^{b/2} x^2\,\mathrm{d}x + 0 = -b^3/12.$$

Suppose that the revelation choice is $\theta'' \in (0, 1)$. Then the intersection would be $(\theta'' - b, \theta'' + b) \cap [0, 1]$. When $b \leq \theta'' \leq 1 - b$, the expert's expected utility would be

$$\mathbb{E}_\theta u^E(D'', \theta) = -2\int_0^{b} x^2\,\mathrm{d}x + 0 = -2b^3/3 < -b^3/12.$$

When $\theta'' < b$, the expert's expected utility would be

$$\mathbb{E}_\theta u^E(D'', \theta) = -2\int_0^{\frac{\theta''+b}{2}} x^2\,\mathrm{d}x + 0 = -(\theta'' + b)^3/12 < -b^3/12.$$

The same case holds for $\theta'' > 1 - b$. Therefore, the expert's optimal revelation choice would be $\hat{\Theta} = \{0\}$ or symmetrically $\hat{\Theta} = \{1\}$.

## A.3. Proof of Proposition 3

If $\theta_2 - \theta_1 \geq 2b$, the gap $\Delta^* = \min\left\{2b, \frac{\bar{\theta}-\underline{\theta}}{2} + b\right\} = 2b$ as $\frac{\bar{\theta}-\underline{\theta}}{2} \geq \frac{\theta_2-\theta_1}{2} = b$. Let the new awareness set be $[\underline{\theta}, \bar{\theta}]$. The set of undelegated options is $(\underline{\theta} - 2b, \underline{\theta} + 2b)$.

First, the expert would choose $\bar{\theta} = 1$ as we focus on the maximal awareness set. Second, a lower $\underline{\theta}$ benefits the expert as more of his preferred options are delegated. The optimal choice is hence $\underline{\theta} = 0$.

## A.4. Proof of Proposition 4

Let the awareness set before revelation be $[\theta_1, \theta_2]$. The expert expands the awareness set to $\hat{\Theta} = [\underline{\theta}, \bar{\theta}]$ with $\underline{\theta} \leq \theta_1 \leq \theta_2 \leq \bar{\theta}$. Denote the expert's choice by $[\underline{\theta}^*, \bar{\theta}^*]$.

By Proposition 3, if in the solution $\bar{\theta}^* - \underline{\theta}^* \geq 2b$ then the expert must choose full revelation: $[\underline{\theta}^*, \bar{\theta}^*] = [0,1]$. Then we focus on $\bar{\theta} - \underline{\theta} < 2b$. In this case, the set of undelegated options take the form of $(\underline{\theta} - \Delta^*, \underline{\theta} + \Delta^*)$ where $\Delta^* = \frac{\bar{\theta}-\underline{\theta}}{2} + b$. Clearly, the set of undelegated options expands when $\bar{\theta}$ increases. Therefore, in the solution $\bar{\theta}^* = \theta_2$ must hold when the expert does not choose full revelation.

On the other hand, decreasing $\underline{\theta}$ by one unit, the agent decreases the lower bound and upper bound of the set $(\underline{\theta} - \Delta^*, \underline{\theta} + \Delta^*)$ by 3/2 and 1/2 units respectively.[8] As a result, lowering $\underline{\theta}$ strictly benefits the expert when $\underline{\theta}$ is close to 0, and he would choose $\underline{\theta}^* = 0$ in that situation. When revealing more awareness is beneficial to the expert, he would choose $\hat{\Theta} = [0, \theta_2]$.

We identify the conditions under which the expert is willing to reveal extra awareness by comparing his expected utility in the two cases: $[\underline{\theta}, \bar{\theta}] = [0, \theta_2]$ and $[\underline{\theta}, \bar{\theta}] = [\theta_1, \theta_2]$. Denote by $\ell$ the length of $(\underline{\theta} - \Delta^*, \underline{\theta} + \Delta^*) \cap [0,1]$. Due to the symmetrical form of the utility function, the expert is better off if and only if $\ell$ is smaller.

1. Suppose $\theta_1 > \frac{\theta_2-\theta_1}{2} + b$. Without awareness revelation, $\ell = \theta_2 - \theta_1 + 2b$. With partial revelation, $\ell = \theta_2/2 + b$. The expert would reveal nothing if and only if $\theta_2/2 + b > \theta_2 - \theta_1 + 2b \Leftrightarrow \theta_1 + \theta_2/2 < 3b$.

2. Suppose $\theta_1 < \frac{\theta_2-\theta_1}{2} + b$. Without awareness revelation, $\ell = \theta_1 + \frac{\theta_2-\theta_1}{2} + b > \frac{\theta_2}{2} + b$. The expert is always willing to expand awareness in this case.

Last, note that the expert would choose full revelation ($\hat{\Theta} = [0,1]$) instead of partial revelation ($\hat{\Theta} = [0, \theta_2]$) if $\theta_2 \geq 2b$. To sum up, if $\theta_2 - \theta_1 < 2b$, the expert's optimal revelation strategy is:

$$\sigma(\Theta_0) = \begin{cases} [\theta_1, \theta_2] & \text{if } b < \frac{3}{2}\theta_1 - \frac{\theta_2}{2}; \\ [0, \theta_2] & \text{if } b \geq \frac{3}{2}\theta_1 - \frac{\theta_2}{2} \text{ and } \theta_2 < 2b; \\ [0,1] & \text{otherwise.} \end{cases}$$

[8] We rule out the discussions about corner solutions by Assumption A2.

## A.5. Proof of Proposition 5

In this case, the investor's delegation choice, given her interim awareness set $[\underline{\theta}, \bar{\theta}]$, is

$$D^* = \begin{cases} [\underline{\theta} + 2b, \bar{\theta}] & \text{if } b < (\bar{\theta} - \underline{\theta})/2, \\ \{y^*\} & \text{otherwise,} \end{cases}$$

where $y^* = \frac{\underline{\theta}+\bar{\theta}}{2} + b$ is the investor's most preferred option. It follows that if $b < \frac{\theta_2 - \theta_1}{2}$, the expert will reveal all states to induce the largest delegation set. Next we focus on the case of interest when $b > \frac{\theta_2 - \theta_1}{2}$.

When $b > \frac{\theta_2 - \theta_1}{2}$, the expert might choose full, partial, or no revelation. Moreover, when the expert's optimal choice is not full revelation, the delegation set must contain only one point. Otherwise, the investor would find the delegation valuable and the expert should have chosen full revelation as argued above. Therefore, the delegation set can take two forms:

- The delegation set with *partial* or *no revelation* is $\{y^*\}$, and the expert's expected utility is

$$U_{NR} = \int_0^1 -\frac{1}{2}(y^* - \theta)^2 \, d\theta = -\frac{1}{6}(1 - 3y^* + 3(y^*)^2).$$

- The delegation set with *full revelation* is $[2b, 1]$, and the expert's expected utility is

$$U_{FR} = \int_0^{2b} -\frac{1}{2}(2b - \theta)^2 \, d\theta = -\frac{4}{3}b^3.$$

Note that $U_{NR} \leq -\frac{1}{24}$ with the equality at $y^* = 1/2$. Therefore, the inequality $U_{FR} > U_{NR}$ always holds as long as $b < \frac{1}{\sqrt[3]{32}} \approx \frac{1}{3.17}$. So the expert always chooses full revelation with a relatively small $b$. When $b > \frac{1}{\sqrt[3]{32}}$, the expert has incentives to reveal lower states if $y^* > 1/2$ while has incentives to reveal higher states if $y^* < 1/2$. Two observations are followed. First, when $y^* = 1/2$, the expert's optimal choice is no revelation. Second, when

$y^* \neq 1/2$ and $b$ is relatively large, the expert will reveal partially to make the induced action more close to $1/2$.[9] The revealed states can be high or low, depending on the sign of $y^*$ minus $1/2$.

# Appendix B

Heifetz, Meier, and Schipper (2011) pointed out that a more expressive framework than the standard extensive form game is needed to model strategic reasoning with unawareness. To capture the possibility that different players have different views of the game, Heifetz, Meier, and Schipper (2013) propose the notion of *generalized extensive-form game*. In this Appendix, we describe how our model can be formalized using their framework. The key is to use subtrees to characterize different views of players.

There are three players: the investor, the expert, and Nature. Let $\mathbf{T}$ denote the set of all subtrees, each subtree induced by a specific revelation choice of the expert. A subtree characterizes a subjective game from the perspective of the investor. At the beginning of the game, the investor's awareness set is $\Theta_0$. If the expert reveals nothing, the subjective game of the investor is depicted as the left subtree in Figure 6. If the expert expands the investor's awareness set to $\widetilde{\Theta}$, then the corresponding subtree $\widetilde{T}$ is depicted at the right in Figure 6. In the first stage, the expert's chosen awareness set as depicted at the right must contain the initial awareness set as depicted at the left. In all stages, the player's action is shown by the point on the arc. In the second stage, the investor chooses the delegation set. In the third stage, Nature draws the realized state. The investor's awareness set is shown by the shorter arc between the two solid lines. Note that the realized state might not be in the awareness set of the investor, as depicted in the right subtree. In the fourth and last stage, the expert chooses some investment option from the respective delegation set.

[9] The expert might still choose full revelation in this case when $b$ is slightly higher than $\frac{1}{\sqrt[3]{32}}$ and $y^*$ is far from $1/2$.

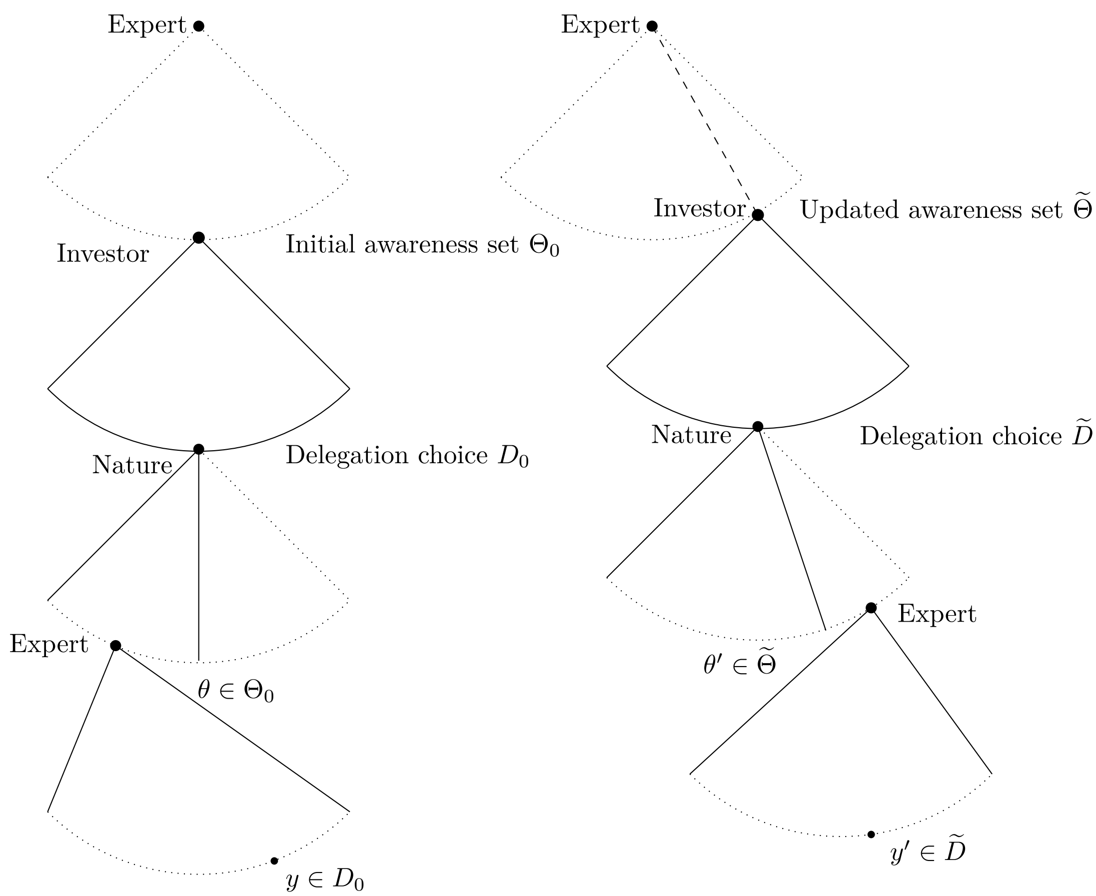


Figure 6: Subtrees $T_0$ and $\widetilde{T}$.