Econ 134 - Financial Economics

Nehring

Problem Set # 6

Two out of four problems will be graded for serious effort, for a maximum of 80 points. In addition, parts of two problem will be graded on success, for an additional 20 points. Maximal total score: 80+20=100 points.

1. (20) You consider investing in one of three portfolios X, Y, or Z, for one year. The means and standard deviations of annual returns in % for the three portfolios are given by the following matrix; annual returns are distributed normally:

	X	Y	Z
mean	5	7	5
std.dev.	20	20	10

Rank the three portfolios in order of the probability of

i) the one-year return being negative,

ii) the one-year return being less than 5 %,

iii) the one-year return being less than 10 %.

Hint: you do not *need a table for the normal distribution to arrive at the correct answers to i) through iii).*

iv) Could you imagine a rational investor preferring X to Y?

v) Could you imagine a rational investor preferring X to Z?

vi) Could you imagine a risk-averse investor preferring X to Z?

2. (40) Cindy allocates her wealth between a risk-free asset F and a risky asset M; think of F as "bonds" and M as "stocks". She cares about the expected return and standard deviation of the chosen portfolio. Specifically, Cindy has a quadratic utility-function $U(\mu, \sigma) = \mu - \frac{\alpha}{2}\sigma^2$. You have the following information about expected risks and returns: $(R_F =)\ \mu_F = 0.02,\ \sigma_F = 0,\ \mu_M = 0.08,\ \sigma_M = 0.2.$

Cindy can freely borrow and lend at the risk-free rate.

i) As a result, the return $\mu(\sigma)$ for bearing risk is given by some linear function ("the Capital Market Line") $\mu(\sigma) = \pi\sigma + R_F$, with π denoting the reward for risk-bearing or "price of risk". What is π, given the above data? This value for π is similar to the historic value 0.37 for the period 1889 to 1978.

ii) Show that the demand for risk σ^D as a function of its price π and the degree of risk-aversion α is given by the function $\boxed{\sigma^D = \frac{\pi}{\alpha}}$. (*Hint: the optimal amount of risk borne is found by maximizing* $U(\mu(\sigma), \sigma) = \pi\sigma + R_F - \frac{\alpha}{2}\sigma^2$.) Interpret the formula economically, in particular how the demand for risk depends on its price and on the coefficient α.

iii) Cindy chooses to hold 60% of her wealth in stocks, and 40% in bonds. What must Cindy's α be? Illustrate Cindy's choice graphically.

iv) Suppose now that the risk of stocks goes down to $\sigma_M = 0.16$. Making use of Cindy's inferred risk attitude in part iii), what is Cindy's demand for risk now? How much of her wealth does she hold in stocks now? Illustrate Cindy's adjusted choice graphically. (If you failed to come up with an answer to iii), assume that Cindy's α is equal to 4).

v) Assume again that $\sigma_M = 0.2$, and that α is as in iii).

Suppose the government wanted to induce Cindy "not to gamble on the stock-market" by offering a subsidized pension-plan which yields the risk-less rate R_Q and is available to Cindy only if she puts *all* her money into the pension plan. How large must the subsidy R_Q -R_F be for Cindy to subscribe to the plan?

3. (40) Consider a world with one *representative investor* Sammy allocating his wealth between a risk-free asset F and a risky asset M ("stocks"). Sammy can freely borrow and lend at the risk-free rate, and has a quadratic utility-function $U(\mu, \sigma) = \mu - \frac{\alpha}{2}\sigma^2$ with $\alpha = 2$. You have the following information about expected risks and returns:

$(R_F =)\ \mu_F = 0.04,\ \sigma_F = 0,\ \mu_M = 0.1,\ \sigma_M = 0.3$. The portfolio of existing assets consist of 50% bonds and 50% stocks.

i) Is there an excess demand for or an excess supply of risk? Illustrate your answer graphically using indifference curves.

ii) Is there an excess demand for or an excess supply stocks (M)?

iii) How would the price and expected return of stocks adjust (qualitatively) to reach an equilibrium?

iv) Compute the equilibrium price of risk π and the equilibrium expected return μ_M.

v) Suppose that investors become less risk-averse compared to iv), i.e. that the representative investor's $\alpha = 1$. Compute the effect of this on the expected return μ_M, and on his bond-holdings *in equilibrium*. Explain briefly.

vi) Alternatively, with α again =2, assume that the total portfolio consists now of 70% stocks and only 30% bonds. Compare the resulting equilibrium expected return μ_M with that computed in iv).Explain briefly why it changes the way it does.

4. (30) Let R_X be the rate of return on "Supertech" , and R_Y be the rate of return on "Slowpoke". The following table describes the probability distribution of returns corresponding to the "representative investor's" expectations:

$State$	$\Pr ob$	R_X	R_Y
I	0.2	-0.10	$-0.20\ (= a)$
II	0.4	-0.10	$0.15\ (= b)$
III	0.3	0.20	0.15
IV	0.1	0.50	0.10

i) Compute for both R_X and R_Y : expected value, variance , standard deviation.

ii) Compute also the covariance and correlation coefficient ρ_{XY} between X and Y; estimate the latter before performing the calculation. Which states contribute positively to the covariance, which negatively?

iii) In which direction would the covariance change if a were -0.15 instead of -0.20 / if b were 0.20 instead of 0.15 ?

iv) For each state, compute the rate of return of a portfolio P consisting of 40% Supertech and 60% Slowpoke; call this random variable R_P and compute its expected value and variance.

v) Verify the last computation by obtaining $E(R_P)$ and $VAR(R_P)$ directly from the parameters of the return distribution of the underlying assets X and Y.