ECONOMICS 134 (NEHRING)

SOLUTION KEY #7

Question 1

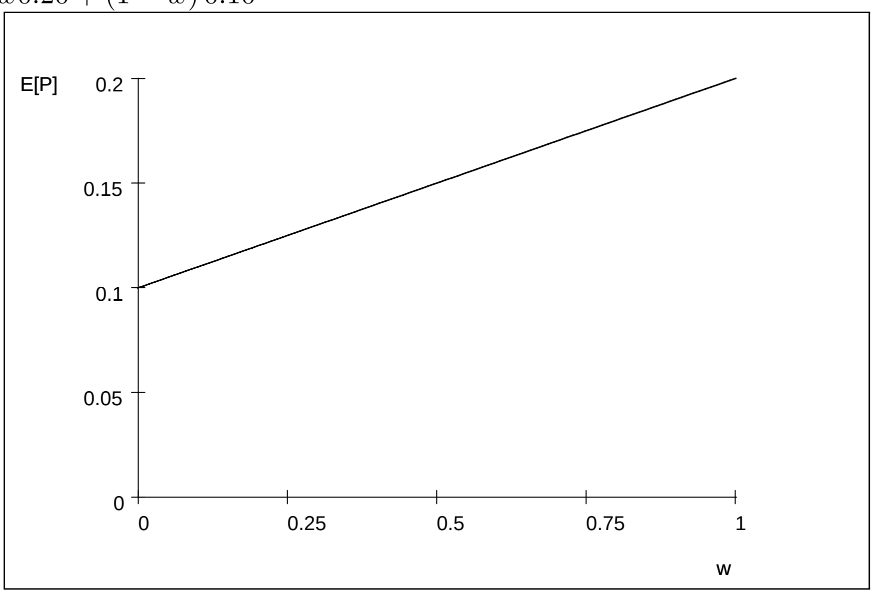
We are given $\mu_x = 0.20$, $\mu_y = 0.10$, $\sigma_x = 0.40$, $\sigma_y = 0.30$, and $\rho_{xy} = 0$.

Assuming the share of the portfolio held in Supertech (w) is bounded (i.e. $0 \leq w \leq 1$) then we have

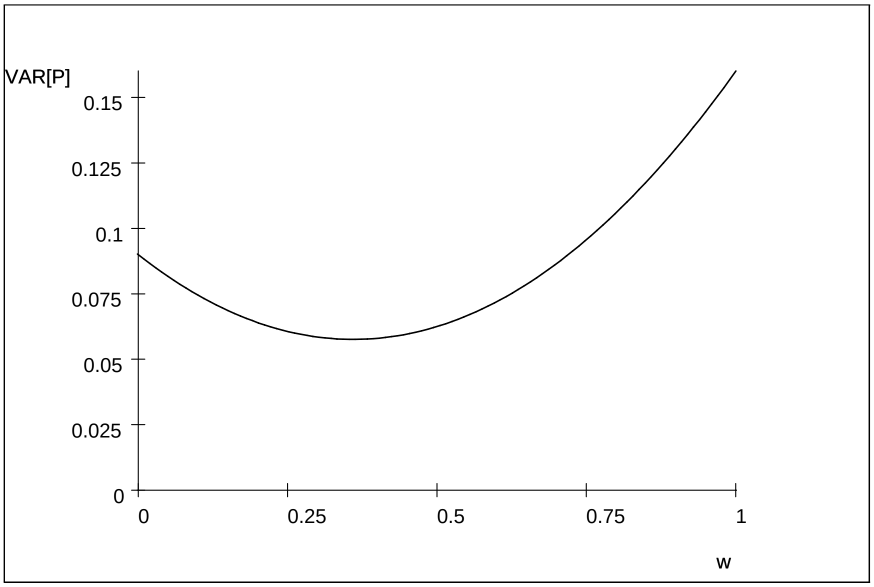
$$\begin{aligned} E[P] &= w\mu_x + (1-w)\mu_y \\ Var[P] &= w^2\sigma_x^2 + (1-w)^2\sigma_y^2 + 2(1-w)w\rho_{xy}\sigma_x\sigma_y \\ \sigma_P &= \sqrt{Var[P]} \end{aligned}$$

i) Graph of expected value:

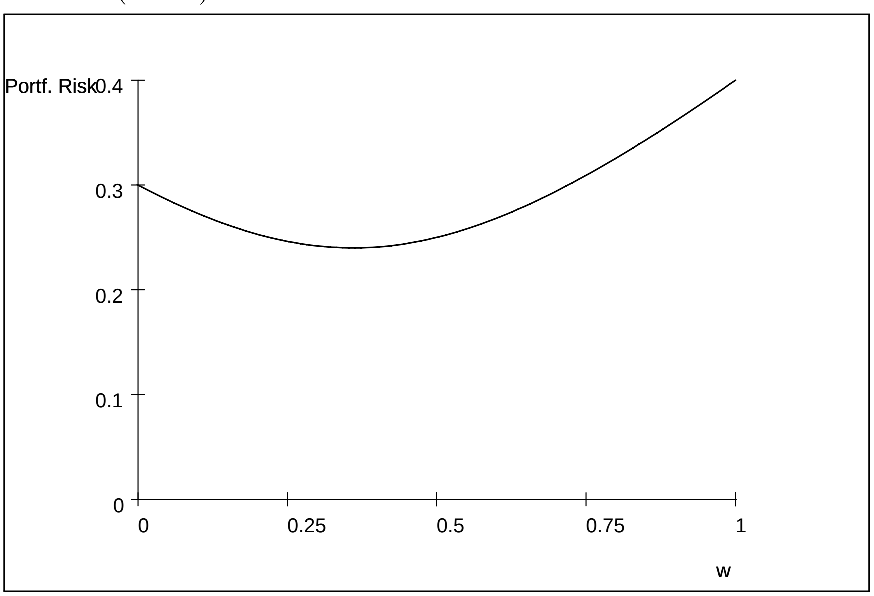
$E[P] = w0.20 + (1-w)0.10$



$Var[P] = w^2 0.16 + (1-w)^2 0.09$



$\sigma_P = \sqrt{w^2 0.16 + (1-w)^2\, 0.09}$



iv) Would a rational, risk averse investor ever put all his money into Supertech? $\boxed{\text{Yes}}$ it just depends on how risk averse they are.

v) Would a rational, risk averse investor ever put all this money into Slowpoke? $\boxed{\text{No}}$. Since the expected value is increasing in w and the variance actually decreases when w is small, the portfolio with $w = 0$ is strictly dominated by , for example, the portfolio with $w = 0.3$.

Question 2

i) Notice we have

$$Var\,(P) = w^2\sigma_x^2 + (1-w)^2\,\sigma_y^2 + 2\,(1-w)\,w\rho_{xy}\sigma_x\sigma_y$$

This is just a convex function. To find the minimum take the derivative with respect to w and then set this equal to zero to find the minimum variance portfolio. That is

$$\frac{d}{dw}Var\,[P] = 2w(.16) - 2\,(1-w)\,(.09) - 2w\rho_{xy}(.12) + 2\,(1-w)\,\rho_{xy}\sigma_x\sigma_y$$

Now if we set this expression equal to zero and solving for w, we get some value w^* in each case. If $0 \leq w^* \leq 1$, then we will have found the weight of X in the minimum variance portfolio w_{MVP} .

$$2w(.16) - 2\,(1-w)\,(.09) - 2w\rho_{xy}(.12) + 2\,(1-w)\,\rho_{xy}(.12) = 0$$

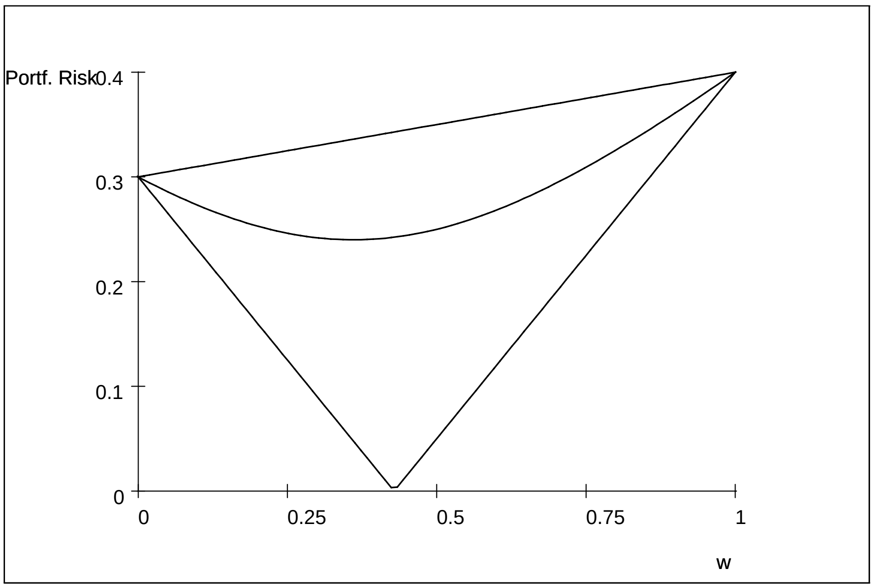
$$w^* = -1.0\frac{-.18 + .24\rho_{xy}}{.5 - .48\rho_{xy}}$$

Now plugging in our known values of $\rho_{xy}, \sigma_x, \sigma_y$ we find that when $\sigma_x = .4, \sigma_y = .3$ we have

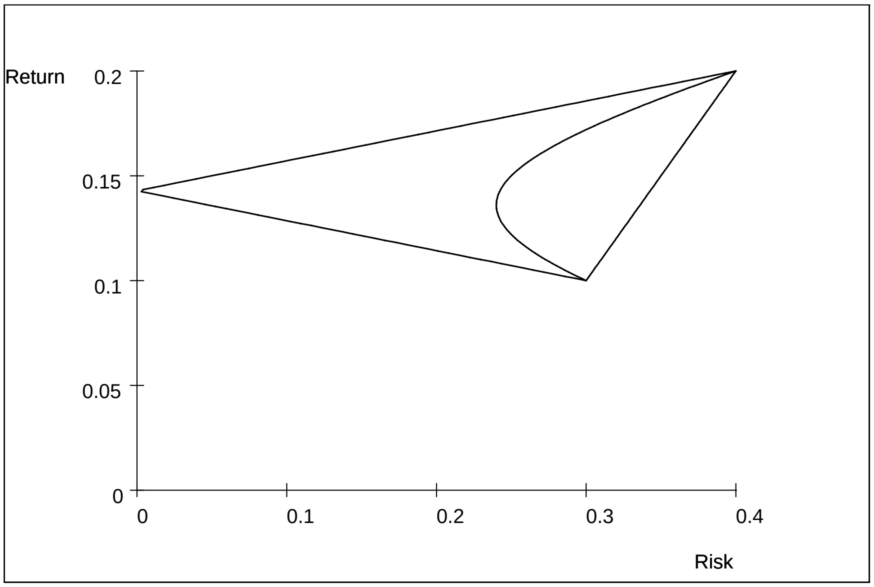
$$\begin{aligned}
\text{For } \rho_{xy} &= 0\text{ , we find } w^* = .36 \\
\text{For } \rho_{xy} &= .9\text{ , we find } w^* = .-.529,\ \text{hence } w_{MVP} = 0 \\
\text{For } \rho_{xy} &= 1\text{ , clearly } w_{MVP} = 0. \\
\text{For } \rho_{xy} &= -1\text{ , we find } w^* = .429
\end{aligned}$$

ii) We find that as the correlation between the two assets decreases, the weight of the more risky asset (X) increases (or remains at 0). This makes intuitive sense, since it says that the potential for reducing total portfolio risk by investing in the more risky asset is larger when that risk is more independent from (or even hedges) the risk of the less risky asset Y.

iii) The three curves depict the portfolio risk for $\rho_{xy} = 1, = 0,$ and $= -1$ (from top to bottom).



v) The three curves depict the risk-return tradeoff for $\rho_{xy} = 1, = 0,$ and $= -1$ (from right to bottom).



iv) The points on each of these curves with the expected return $0.15 = 0.5 \times 0.2 + 0.5 \times 0.1$ represent the RRC associated with equal weights in X and Y.

Question 3

(i) Yes. Since $\mu_x > \mu_y$ and $\sigma_x < \sigma_y$, if $\rho_{xy} = 1$, then w =1.

(ii) No. The investor could increase μ and decrease σ by increasing w. (w=1 is a trivial example.)

(iii) Yes. Particularly if the assets are negatively correlated.

(iv) No. w=1 maximizes μ regardless of risk.

Question 4

$\mu_m = 0.22$, $\mu_{Rf} = 0.1$.

(i) $\beta_p = (\mu_p - \mu_{R_f})/(\mu_m - \mu_{R_f}) = (0.34 - 0.1)/(0.22 - 0.1) = 0.24/0.12 = \boxed{2}$.

(ii) $\beta_x = 0 \Rightarrow \mu_p = \mu_{R_f} \Rightarrow \boxed{\mu_x = 0.1}$.

(iii) $\mathrm{P}_0 = \mathrm{P}_1/(1+\mu_x) \Rightarrow \mathrm{P}_1 = \mathrm{P}_0 \times(1+\mu_x)$ and $\mu_x = \beta_x \times (\mu_x - \mu_{R_f}) + \mu_{R_f} = -1/2$

$\times(0.12) + 0.1 = 0.04$. $\mu_x = 0.04$, $P_1 = P_0 \times (1+\mu_x) = 28 \times (1.04) = \boxed{29.12}$.

(iv) $\beta_z = (\mu_z - \mu_{R_f})/(\mu_m - \mu_{R_f}) = (0.16\text{-}0.1)/(0.22\text{-}0.1) = \boxed{0.5}$,

and $\beta_z = \rho_{zm} \times \sigma_z \,/\, \sigma_m = \rho_{zm} \times 1 = \rho_{zm} = 0.5$. 's' is irrelevant.

(v) Risks are rewarded in the CAPM model in order to make the representative investor hold the market portfolio M. The risk associated with asset X that needs to be rewarded is the "systematic" risk" i.e., the part of the risk that is correlated with the return on M. The representative investor doesn't worry about the part of the risk of X that is uncorrelated to the protfolio he holds in equilibrium, i.e. M.

Question 5

(i) $E(R_p) = 6/(6+9) \times \mu_x + 9/(6+9) \times \mu_y = 2/5 \times 0.08 + 3/5 \times 0.2 = 0.67/5 = 0.152$.

$Var(R_p) = (2/5)^2 \times 0.4^2 + 2 \times (2/5) \times (3/5) \times 0 + (3/5)^2 \times 0.5^2 = 0.1156 \Rightarrow \sigma_{R_p} = 0.34$.

{The equation for the CML is : $\mu=0.02 + 0.5\times\sigma$ OR $\sigma= -0.04+ 2\times\mu$.}

(ii) Keep $\mu=0.152$ and solve for σ from the market like equation : $\sigma = -0.04+ 2\times(0.152)$ $=0.264$. Thus $\sigma_{R_p} - \sigma = 0.34 - 0.264 = \boxed{0.076}$ reduction.

(iii) You could suggest the above portfolio, but since Ray would expect the same return (μ_p unchanged), you cannot expect a fee (GET it!!!). Instead, keep the same risk, and maximize μ: solve for μ from the market line equation

$$\sigma= -0.04 + 2\times\mu \Rightarrow 0.34 = -0.04 + 2 \times \mu \Rightarrow \mu = 0.19.$$

You have increased Ray's expected return by $(0.19 - 0.152) \times 150{,}000 = \boxed{\$5700}$. That is the most you could charge .