ECONOMICS 134   (NEHRING)

SOLUTION KEY #6

Question 1

Let R_x, R_y, R_z be returns of portfolios X,Y, and Z.

(i) $P(R_x < 0) = P(z<(0\text{-}5)/20) = P(z< \text{-}0.25)$ is greater than

$P(R_y < 0) = P(z<(0\text{-}7)/20) = P(z< \text{-}0.35)$, which in turn is greater than

$P(R_z < 0) = P(z<(0\text{-}5)/10) = P(z< \text{-}0.5)$.

You can determine the rank order of the probabilities by the rank order of the z scores without looking at the table for the normal distributions since $P(z<\text{-}0.25) > P(z<\text{-}0.35) > P(z > \text{-}0.5)$.

(ii) Similarly, $P(R_x < 5) = P(z<0)$; $P(R_y < 5) = P(z<\text{-}0.1)$; $P(R_z < 5) = P(z<0)$ and $P(z<0) > P(z<\text{-}0.1)$.

(iii) $P(R_x < 10) = P(z<0.25)$; $P(R_y < 10) = P(z<0.15)$; $P(R_z < 10) = P(z<0.5)$. Clearly, $P(z<0.5) > P(z<0.25) > P(z<0.15)$.

(iv) No, since X has lower mean than Y with the same risk.

(You can draw a graph in the (risk=μ,mean=σ) plane.)

(v) Yes, if he is risk-lover.

(vi) No, Z offers the same expected return but lower risk.

Question 2

$U(\mu, \sigma) = \mu - \alpha/2 \times \sigma^2$.

$\mu_F = 0.02$, $\sigma_F = 0$, $\mu_M = 0.08$, $\sigma_M = 0.2$.

(i) Note that π is the slope of CML. $\Rightarrow \pi = (\mu_M - \mu_F)/\sigma_M = (0.08 - 0.02)/0.2 = \boxed{0.3}$.

(ii) Find σ to max $U = (\pi\sigma + \mu_F) - \alpha/2 \times \sigma^2$.

$$\text{FOC: } \pi - \alpha\sigma = 0 \Rightarrow \sigma^D = \pi/\alpha.$$

(iii) Cindy's demand for risk is $\sigma^D = 0.6\sigma_M = 0.12$; hence, solve $0.12 = 0.3 / \alpha$, which yields $\boxed{\alpha = 2.5}$.

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(iv) π increases to (0.08 - 0.02)/0.16 = 0.375. Her demand for risk increases to 0.375/2.5=0.15. To bear that much risk, Cindy must invest 0.15/0.16= $\boxed{93.75\%}$ of her wealth in stocks.

(v) Her pre-subsidy utility U^* is determined by her presubsidy ex-pected return μ^* and risk σ^*. In iii), we found that $\sigma^* = 0.12$. μ^* is then obtained from the CML, $\mu^* = 0.02 + 0.3 \cdot 0.12 = 0.056$. It follows that $U^* = 0.056 - \frac{2.5}{2} \times (0.12)^2 = 0.038$.

To find R_Q, solve $U(R_Q, 0) = U^*$. Since $U(R_Q, 0) = R_Q$,

R_Q - R_F = $\boxed{1.8\%}$.

R_Q - R_F can be viewed as a measure of the gains from trade due to participation in the stock-market, much like the consumer surplus measure the gains from trade in the market for goods.

Question 3

$U(\mu, \sigma) = \mu - \alpha/2 \times \sigma^2$.

$\mu_F = 0.04$, $\sigma_F = 0$, $\mu_M = 0.1$, $\sigma_M = 0.3$, $\alpha = 2$.

(i) $\pi = (\mu_M - \mu_F)/\sigma_M = (0.1 - 0.04)/0.3 = 0.2$.

Hence $\sigma^D = \pi/\alpha = 0.1$. By comparison, $\sigma_T = 0.5 \cdot 0.3 = 0.15$.

Hence there is an excess supply of risk.

(ii) This means that there is an excess supply of stocks.

(iii) Stock prices would go down, thereby raising the expected return.

(iv) Solve $\sigma_T = \sigma^D = \pi/\alpha$ for π; this yields $\pi = 2 \cdot 0.15 = \boxed{0.3}$.

(v) If α were to decrease to 1, π would go down to 0.15, and μ_M to 0.085. The net effect on the representative investor's portfolio allocation between bonds and stocks would be zero; after all, *in equilibrium,* the representative investor must be holding the portfolio of existing assets which did not change.

(vi) Now the supply of stocks goes up, and hence that of risk. The return on stocks needs to rise for investors to hold more of them (which involves bearing more risk than before. In the new equilibrium, $\pi = 2 \cdot 0.21 = 0.42$, hence $\mu_M = 0.04 + 0.42 \cdot 0.3 = 0.166$.

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Question 4

(i) $E(R_X) = 0.2 \times (-0.1) + 0.4\times(-0.1) + 0.3 \times(0.2) + 0.1\times0.5 = \boxed{0.05}$.

$Var(R_X) = E(R_X^2) - [E(R_X)]^2 = 0.2\times(-0.1)^2+0.4\times(-0.1)^2+0.3\times(0.2)^2+0.1\times(0.5)^2 - (0.05)^2 = 0.0405 \Rightarrow \sigma_X = \sqrt{0.0405} = \boxed{0.2013}$.

$E(R_Y) = 0.2\times(-0.2) + 0.4\times(0.15) + 0.3 \times 0.15 + 0.1 \times 0.1 = \boxed{0.075}$.

$Var(R_Y) = E(R_Y^2) - [E(R_Y)]^2 = 0.2\times(-0.2)^2+0.4\times0.15^2+0.3\times0.15^2+ 0.1\times0.1^2 - 0.057^2 = 0.01912 \Rightarrow \sigma_Y = \sqrt{0.01912} = \boxed{0.1383}$.

(ii) $Cov(R_X,R_Y) = E[(R_X - E(R_X))(R_Y - E(R_Y))] = 0.2\times(-0.1-0.05)(-0.2-0.075) + 0.4\times(-0.1-0.05)(0.15-0.075) + 0.3\times(0.2-0.05)(0.15-0.075)+0.1\times(0.5-0.05)(0.1-0.075) = \boxed{0.00825}$.

States I, III, and IV contribute positively to the covariance, state II contributes negatively.

$\underline{\rho_{xy}} = Cov(R_X,R_Y)/(\sigma_X \times \sigma_Y) = \boxed{0.2963}$.

(iii) a) COV would decrease, since a positive contribution term becomes smaller; b) COV would decrease, since a negative contribution term becomes larger in absolute value, hence smaller.

(iv) In each state, $R_P = 0.4\times R_X + 0.6\times R_Y$.

State	R_P
I	$0.4\times(-0.1) + 0.6\times(-0.2) = -0.16$
II	$0.4\times(-0.1) + 0.6\times(0.15) = 0.05$
III	$0.4\times(0.2) + 0.6\times(0.15) = 0.17$
IV	$0.4\times(0.5) + 0.6\times(0.1) = 0.26$

$\Rightarrow E(R_P) = 0.2\times(-0.16) + 0.4\times(0.05) + 0.3\times(0.17) + 0.1\times(0.26) = 0.065$.

$Var(R_P)= 0.4^2\times Var(R_X) + 0.6^2\times Var(R_Y) + 2\times0.4\times0.6\times Cov(R_X,R_Y) = 0.0173$.

v) Verification: $E(R_P) =0.4\times0.05+0.6\times0.75=0.065$.

Verification of $VAR(R_P)$ dto.