ECONOMICS 134 (NEHRING)

SOLUTION KEY #5

Question 1

i) We have:

$$\begin{aligned} NPV_A &= -20 + \frac{2.4}{0.08} = 10 \\ NPV_B &= -30 + \frac{3.3}{0.08} = 11.25 \\ IRR_A &= \frac{2.4}{20} = 0.12 \\ IRR_B &= \frac{3.3}{30} = 0.11 \quad . \end{aligned}$$

Thus, project A has higher the IRR while project B has the higher NPV. The firm should choose project B since its NPV is higher than that of project A.

ii) We have:

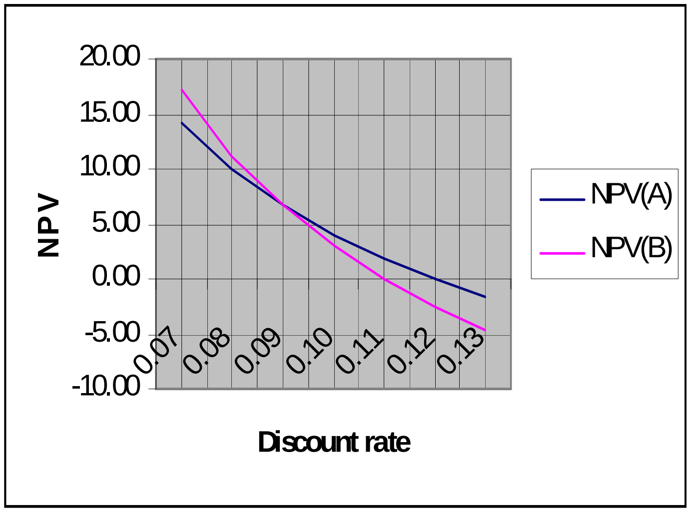
$$\begin{aligned} NPV_{B-A} &= -10 + \frac{0.9}{0.08} = 1.25 > 0 \\ IRR_{B-A} &= \frac{0.9}{10} = 0.09 > 0.08 \quad , \end{aligned}$$

which implies that the firm should choose project B.

iii) Now, IRR_{B-A} is less than the required rate of return. So, the firm should choose project A.

iv) The following table and graph show how the NPV's of project A and B change with the discount rate:

r	NPV(A)	NPV(B)
0.07	14.29	17.14
0.08	10.00	11.25
0.09	6.67	6.67
0.10	4.00	3.00
0.11	1.82	0.00
0.12	0.00	-2.50
0.13	-1.54	-4.62



v) It is consistent with our answer. When the discount rate is greater than 0.09, the NPV of project A is higher than that of project B.

Question 2

i) The bond will pay \$100 for each of the following 4 years and \$1100 at maturity. You can calculate the present value *without using spot rates* by using the respective zero-coupon bond's prices (respectively the corresponding discount factors equal to 1/1000 of the ZCB price):

$$100\times \frac{943}{1000} + 100 \times \frac{874}{1000} + 100\times \frac{794}{1000} + 100\times\frac{709}{1000} + 1100 \times \frac{622}{1000} = \boxed{1016.2}.$$

ii) If the YTM, i.e. the discount rate (!), were a constant ten percent, the present value of the bond would be exactly one thousand.

iii) Since the present value is greater than one thousand, that is, it takes more than one thousand dollars invested now to yield this fixed stream, the yield to maturity as the "average interest rate" must be lower than ten percent. The yield to maturity is in fact 9.6%.

Question 3

i) Your pre-tax wealth after 20 years will be

$$100000 \times 1.06^{20} = 320,710.$$

You need to pay 25% taxes on the capital gain of \$220,710, which is

$$0.25 \times 220,710 = 55,178.$$

ii) Your after-tax wealth is therefore

$$320710 - 55178 = 265,532;$$

this amounts to an annualized after-tax return is 5%, which is obtained by solving the following equation for r:

$$100,000(1+r)^{20} = 265,532,$$

ie.

$$r = (2.655)^{\frac{1}{20}} - 1 = \boxed{0.05}.$$

iii) Your pre-tax wealth in 10 years will be

$$100000 \times 1.06^{10} = 179,080.$$

You need to pay 25% taxes on the capital gain of \$79,080, which is

$$0.25 \times 79,080 = 19,770.$$

Your after-tax wealth in 10 years is therefore

$$179080 - 19770 = 159,310.$$

Hence (*why?*) your after-tax wealth in 20 years must be

$$100,000 \times 1.5931 \times 1.5931 = 253,800,$$

which is $\boxed{\textit{\$11,732 less}}$ than what you got in i).

Roughly speaking, you end up with more money in i) than in iii) because in i) you can make money in the second decade on the total result of your investment over the first decade, including unrealized *capital gains. By contrast, in iii), in the second decade you can only make many on the net result of your investment over the first decade after payment of capital gains taxes.*

iv) In this scenario, your post-tax annual return is

$$(1 - 0.75) \times 0.06 = 0.045,$$

i.e. 4.5%. In 20 years, this leads to an after-tax wealth of

$$100,000 \times 1.045^{20} = 241,170,$$

still less than in iii) but not that much.