ECN 134

SOLUTION KEY #2

1. i) The firm must invest today the amount x which solves the equation

$$x \times (1.08)^{27} = 1,500,000,$$

i.e. x = \$ $\boxed{187,780}$.

ii) PV of the Smiths' offer: \$ 115,000.

PV of the Joneses offer: $\frac{1}{1.1^3}150000 = \$112,700$. You should choose the Smiths' offer.

iii) a) $PV = \frac{1000}{0.1} = \boxed{10,000}$.

b) $PV = \frac{1}{1.1}\frac{500}{0.1} = \boxed{4,545}$.

c) $PV = \frac{1}{1.1^2}\frac{2420}{0.1} = \boxed{20,000}$.

2. r = 0%:

Add the stream of payments: -150 + 8×(-10) + 12×(50 × 0.6) = $\boxed{130}$.

Perfect competition implies free entry and exit. Therefore, as long as positive profits exist in nut trees, the price of trees will be bid up (or the price of the final product will be bid down through increasing supply). This drives the net present value of the nut tree to zero.

$$-150 + 8\times(-10) + 12\times(50\times p) = 0 \text{ has solution } p = 23/60 = \boxed{0.383}.$$

r = 4%:

Use the annuity formula for each of the three streams:

$$-150 + [-10/0.04\times(1-1/1.04^{8})] + 1/1.04^{8}\times[50\times0.6/0.04\times(1 - 1/1.04^{12})] = \boxed{-11.6}.$$

The equilibrium price has to be higher since the NPV is currently less than zero. To find the equilibrium price, set NPV equal zero and solve for p:

$$-150 + [-10/0.04\times(1 - 1/1.04^{8})] + 1/1.04^{8} \times [50\times p/0.04\times(1-1/1.04^{12})] = 0.$$

Then the equilibrium price is: $\boxed{0.63}$.

$$-150 + [-10/0.04\times(1 - 1/1.04^{8})] + 1/1.04^{8} \times [50\times p/0.04\times(1-1/1.04^{120})] = 0.$$

Thus, if the trees bear fruit for 120 years, the equilibrium price is: $\boxed{0.24}$.

Because we value future payments less than current payments - the higher the interest rate or the further into the future, the greater the discount.

Ten times as productive does not mean ten times the "value". An imperfect analogy might be to diminishing marginal returns: since each increase in the tree's productivity comes further off, each increase must be worth less and less.

3. i) Use the annuity formula, solving for the annual cash flow:

x$\cdot\frac{1}{0.14}$(1-$\frac{1}{1.14^{10}}$) =10,000 .

$$\text{Hence } x=10{,}000\times(0.14/(1-\tfrac{1}{1.14^{10}}))= \boxed{1917.1}.$$

ii) Of the first payment of \$ 1917, \$ 1400 are interest, and the difference of \$ 517 goes towards repaying the debt, which will thus have been reduced to $\boxed{\$\ 9483}$. Note that this is equal to the future value in year 1 of the remaining 9 annual payments; note also that in the first year, you have paid back less than 10% of your debt.

iii) After the fifth payment, you still need to pay back five installments whose value exactly equals what you owe (since you will exactly repay your debt after 10 years); the value of the remaining five payments is thus simply $\frac{1}{0.14}(1-\frac{1}{1.14^5})\times 1917.1 = \boxed{6581.7}$.

The amount should be greater than one-half of ten-thousand since in the early years, a large fraction of the annual payment covers interest in addition to the repayment of the principal.

iv) Use the annuity formula: $10000 = 1600/0.14 \times (1\text{-}1/1.14^t)$ and solve for t:

$$14/16 = 1 - 1/1.14^t \rightarrow 1/1.14^t = 1/8 \text{ and } 1.14^t = 8.$$

Use natural log: t = log(8)/log(1.14) yielding t=$\boxed{15.87}$.

Alternately, use the double approximation given in class for r=14%:

$$(1+r)^t = 2 \text{ but our right hand side is eight, so } (1+r)^t = 2^3 \text{ and thus } (1+r)^{t/3} = 2.$$

Then apply the rule: t=(0.7/0.14) $\times$ 3 = 15 years (approximately).

v) Thus 15.87 $\times$ 1600 = 25392. But 10$\times$ 1917.135 = 19171.35. So case four takes longer. Since you pay off less in each period, you need to pay off the debt over a longer period and ultimately pay more.