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A THEORY OF RATIONAL CHOICE UNDER IGNORANCE

ABSTRACT. This paper contributes to a theory of rational choice for decision-makers with incomplete preferences due to partial ignorance, whose beliefs are representable as sets of acceptable priors. We focus on the limiting case of 'Complete Ignorance' which can be viewed as reduced form of the general case of partial ignorance. Rationality is conceptualized in terms of a 'Principle of Preference-Basedness', according to which rational choice should be isomorphic to asserted preference. The main result characterizes axiomatically a new choice-rule called 'Simultaneous Expected Utility Maximization'. It can be interpreted as agreement in a bargaining game (Kalai-Smorodinsky solution) whose players correspond to the (extremal) 'acceptable priors' among which the decision maker has suspended judgment. An essential but non-standard feature of Simultaneous Expected Utility choices is their dependence on the entire choice set. This is justified by the conception of optimality as compromise rather than as superiority in pairwise comparisons.

KEY WORDS: Ignorance, Ambiguity, Multiple priors, Rational choice, Incomplete preference, Robustness, Independence, Sure-thing principle, Context-dependence, Choice consistency

1. INTRODUCTION

Decisions often have to be made on the basis of limited information. Sometimes, this does not present any special difficulties to the decision maker; he may still be willing to rank all alternatives in a complete order and simply choose the best alternative. In other cases, he will take this informational limitation as a lack of adequate grounds for constructing such a ranking unambiguously; rather than arbitrarily declaring one of two alternatives superior, or both indifferent, he will find it more natural to acknowledge this lack and suspend judgment by asserting the *non-comparability*[1] of the two alternatives.

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In this paper, we deal with situations in which non-comparability arises from limited information about the likelihood of uncertain events. In formal terms, we will consider partial orders *R* that satisfy all of the standard consistency conditions characteristic of Subjective Expected Utility (SEU) preferences, with the exception of the completeness axiom. Such partial orders can be represented as unanimity-relations (intersections) of the SEU-orders associated with convex sets of probability measures ("belief sets" of "acceptable priors").[2] For instance, the extreme case of "complete ignorance" is represented by a maximally incomplete partial order in which the decision-maker weakly prefers one act over another if and only if the act generates a weakly better consequence in every state; this corresponds to an all-inclusive belief set. For another example, a classical statistician may be prepared to assume qualitative knowledge about the stochastic process generating the observations, but may not be willing to make any probabilistic assumptions about parameter values. Such qualitative knowledge can be described by a partial order R, for instance in terms of conditions of "exchangeability"[3]; the corresponding belief set would include all priors consistent with the assumed qualitative knowledge.

The goal of the paper is to develop a theory of rational choice for "decision-problems under uncertainty" (d.p.u.s) which are defined by a set of acts *X* and a partial order *R* on some universe of acts. While optimality for partial orders has been traditionally identified with "admissibility", i.e. the absence of feasible superior alternatives, we will argue that optimality is not exhausted by it: *some admissible acts may be superior to others (in a context-dependent way) as compromise choices*. The choice rule proposed, "Simultaneous Expected-Utility Maximization" (SIMEU), makes this intuition of optimal choice as a best compromise formally precise and provides an axiomatic justification for it. SIMEU can be interpreted as Kalai-Smorodinsky bargaining solution representing a fair compromise among "alter egos" corresponding to the different extremal priors.

Of the full axiomatic theory underlying SIMEU, the present paper presents half, namely the limiting case of "maximally non-comparable" preferences characterized by all-inclusive belief sets, which turn out to correspond to the classical notion of "Complete Ignorance" (CI); for a still valuable introduction to the classical literature, which culminated in the early 1950s prior to Savage's

"Foundations of Statistics" (1954), see Luce and Raiffa (1957, ch. 13). It has been shown in Nehring (1991, ch. 2) and Nehring (1992) how choice rules defined on CI problems can be extended to the class of general d.p.u.s.; a brief sketch is given in Section 5.4. The nature of the extension entails that CI problems can be viewed as reduced forms of general d.p.u.s.; the study of CI problems is thus of much greater applicability than is apparent at first.

The main result of the paper characterizes SIMEU in complete ignorance problems as equivalent to the conjunction of four axioms, Admissibility, Symmetry, Consequence Isomorphism and a context-dependent choice-consistency condition WAREP. Admissibility rules out the choice of ex-post dominated acts. Symmetry says that since CI preferences are symmetric with respect to arbitrary event-permutations, CI choices must be symmetric in the same way; due to the extreme richness in such symmetries, Symmetry precludes as-if expected utility maximization. The final axiom called "Consequence-Isomorphism" (CISO) has no precedent in the classical literature. It requires invariance of the choice rule with respect to positive affine transformations of consequence utilities state-by-state. CISO captures an understanding of optimal choice as compromise, and is a natural consequence of the bargaining metaphor. At a deeper level, CISO is motivated by the requirement that the choice-function take full account of the fact that asserting complete ignorance preferences is tantamount to denying the comparability of any two acts that are mutually ex-post non-dominated, however large the utility differences between them may be in particular states (Section 5.3). In contrast to the literature, we do not rely on an axiom that appeals to a principle of description invariance.

To put the contribution of SIMEU theory into relief, it is helpful to relate it to Savage's (1951) "minimax loss solution" (MML), its closest relative in the literature.[4] The MML solution is often preferred over straight maximin for its more plausible performance. For example, Radner-Marschak (1954) have shown for a class of problems of acquiring information under complete ignorance that rules which evaluate acts in terms of minimal and maximal possible utilities[5] often entail no information acquisition at all; by comparison, under MML the optimal amount of information is positive and decreases with the information cost, in accordance with intuition. Just like SIMEU, MML violates context-independent choice-consistency,[6] a feature which presumably has played a significant role in limiting its acceptance.[7] Arrow (1960, p. 72), for example, concluded, probably representatively for the classical literature, that a rational solution to complete ignorance problems is impossible: “Perhaps the most nearly definite statement is that of Milnor (1954) who showed in effect that every proposed ordering principle contradicts at least one reasonable axiom.”

In the literature, the context-dependence of MML has remained essentially ad hoc and without systematic theoretical justification. SIMEU theory remedies this fundamental deficit by explaining the context-dependence as inherent in the compromise character of optimal choice under ignorance. This is worked out more fully in Section 5, where we also introduce a “Principle of Preference-Basedness” to justify the key axioms of the theory, Symmetry and Consequence Isomorphism, in terms of the idea that a satisfactory choice-rule must make full use of the information embodied in the asserted preferences, including the asserted non-comparabilities. This Principle is of particular importance in providing a more cogent justification of the Consequence Isomorphism axiom. SIMEU differs from MML also in content; we argue that, by satisfying CISO, SIMEU reflects the extreme agnosticism inherent in Complete Ignorance preferences more faithfully than MML does.

The paper is structured as follows: Section 2 presents and interprets the SIMEU choice rule for general d.p.u.s in the two-event case. In Section 3, the formal framework is introduced, the SIMEU solution is formally defined, and its basic mathematical properties are established. Section 4 presents the rationality postulates of the theory and axiomatizes the SIMEU solution. A side result characterizes the lexicographic maximin-rule which is also shown to coincide with Barbera-Jackson’s (1988) “protective criterion”. Section 5 provides a more detailed account of the context-dependence of the solution, justifies the key axioms in terms of the Principle of Preference-Basedness, and briefly sketches the extension of SIMEU to general d.p.u.s. The appendix contains bits of extra material and the proofs.

There are four natural stopping points, intermediate or terminal, for reading this paper: at the end of this sentence, after Section 2 (the main idea), after Section 4 (the main result), and after the final Section 5 (the conceptual underpinnings); taking a deep breath is especially recommended after Section 4.

2. PRELIMINARY EXPOSITION OF SIMEU

This section explains the SIMEU choice rule for general partial orders in the two-state case. An act $x \in \mathbf{R}^2$ maps consequences to cardinal utilities.[8] A belief set Π is a closed convex subset of Δ^2, the unit simplex of $\mathbf{R}^2$; its elements are called "acceptable", its extreme points π' and π'' "extremal" priors. A "consistent" partial order R on $\mathbf{R}^2$ is one that can be represented as the unanimity relation R_Π induced by a belief set Π:

$$x \; R_\Pi \; y \text{ if and only if } \pi \cdot x \geqslant \pi \cdot y \text{ for all } \pi \in \Pi.$$

Note that unanimity with respect to all extremal priors is equivalent to unanimity with respect to all acceptable ones. A two-state decision-problem under uncertainty can then be specified as a pair (X, Π), where X denotes the choice-set, a convex and compact subset of $\mathbf{R}^2$; if $\Pi = \Delta^2$, the d.p.u. is one under complete ignorance.

An undisputed necessary condition of the optimality of an act x is its "admissibility," i.e., the absence of any feasible alternative that is strictly preferred to it. In the two-dimensional case, the set of admissible acts $\mathcal{A}(X, \Pi) = \mathcal{A}(X, R_\Pi) = \{x \in X \mid \text{for no } y \in X\text{: } y R_\Pi x \text{ and not } x R_\Pi y\}$ traces out the boundary of X between x' and x'', the optimal acts under π' and π'' in $\mathcal{A}(X, \Pi)$ respectively; see Figure 1 below. $\mathcal{A}(X, \Pi)$ may be understood as the set of acts that compete for enactment. –

While clearly necessary, we submit that admissibility is not *sufficient* as a criterion of optimality for partial orders. In particular, it seems natural to discriminate among admissible acts based on considerations of *robustness*. Intuitively speaking, an alternative lacks robustness if it is an especially poor choice under some prior. In Figure 1, choices of x' or x'' exemplify failures of even minimal robustness: while each act performs best against some prior (π' respectively π''), it performs worst against its opposite (i.e., π'' respectively π') compared to any other admissible act. Robustness requires at a minimum choosing an act somewhere in between x' and x''. An alternative is "optimally robust" if it minimizes the risk of being a poor choice by simultaneously taking into account all acceptable priors to the greatest extent possible. In other words, an optimal choice represents the best possible compromise among the different acceptable priors. This conception of optimal choice under non-comparability will be formalized axiomatically in terms a choice rule called "Simultaneous Expected-Utility Maximization" (SIMEU). It should be noted, however, that while the robustness interpretation helps to make intuitive sense of the proposed rule, the axioms themselves do not rely on this intuitively rather vague notion, but on the sharper concept of "structural isomorphism".

The SIMEU rule σ incorporates robustness by "implementing" each extremal prior π' and π'' "to the same degree". It is based on a cardinal measure λ of the "degree of implementation" defined as follows.

$$\lambda(x, \pi; X, \Pi) = \frac{\pi \cdot x - \min\{\pi \cdot y \,|\, y \in \mathcal{A}(X, \Pi)\}}{\max\{\pi \cdot y \,|\, y \in \mathcal{A}(X, \Pi)\} - \min\{\pi \cdot y \,|\, y \in \mathcal{A}(X, \Pi)\}},$$

with 0/0=1 by definition.

We will often suppress the arguments X and Π. In effect, $\lambda(\cdot, \pi)$ is the von Neumann-Morgenstern representation of the EU preferences induced by π such that $\max\{\lambda(y, \pi) \,|\, y \in \mathcal{A}(X, \Pi)\} = 1$ and $\min\{\lambda(y, \pi) \,|\, y \in \mathcal{A}(X, \Pi)\} = 0$. For example $\lambda(x'', \pi'') = 1$ and $\lambda(x'', \pi') = 0$.

The SIMEU choice rule σ is defined as the unique act that is admissible and implements both extremal priors to the same degree:

$$x \in \sigma(X, \Pi) \iff x \in \mathcal{A}(X, \Pi) \text{ and } \lambda(x, \pi'') = \lambda(x, \pi').$$

It is easily verified that $\sigma(X, \Pi)$ can equivalently be defined as the unique maximin in degrees of implementation, i.e.,

$$\sigma(X, \Pi) = \arg\max_{x \in X} \ \min(\lambda(x, \pi'), \lambda(x, \pi'')).$$

Geometrically, σ can be constructed as follows:[9]

Define two reference points y^1 and y^0 where π'' and π' simultaneously achieve their maximal respectively minimal expected utilities. y^1 is thus defined by the conditions $\pi'' \cdot y^1 = \pi'' \cdot x''$ and $\pi' \cdot y^1 = \pi' \cdot x'$, i.e., as intersection of the indifference-line for π'' through x'' with that for π' through x'. Similarly, y^0 is defined by $\pi'' \cdot y^0 = \pi'' \cdot x'$ and $\pi' \cdot y^0 = \pi' \cdot x''$. By construction, $\lambda(y^1, \pi'') = \lambda(y^1, \pi') = 1$ and $\lambda(y^0, \pi'') = \lambda(y^0, \pi') = 0$.

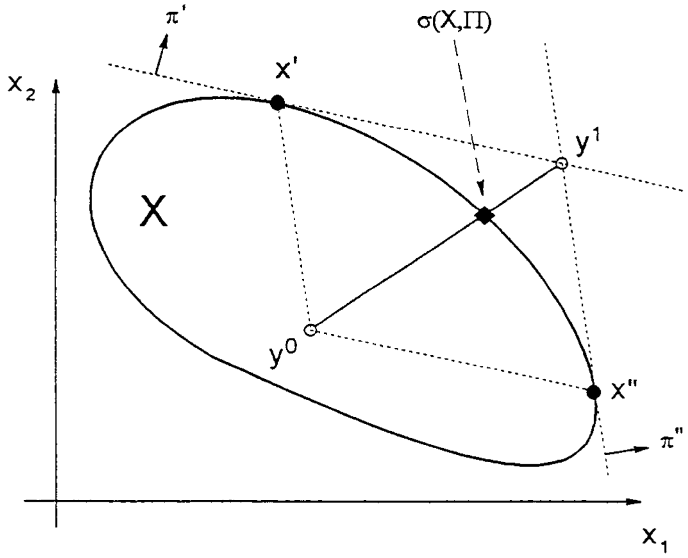

Figure 1. The SIMEU choice rule

By the affine definition of λ, setting $y^\gamma = \gamma y^0 + (1-\gamma)y^1$, $\lambda(y^\gamma, \pi'') = \gamma = \lambda(y^\gamma, \pi')$; the straight line through y^1 and y^0 describes therefore the locus of acts that implement π'' and π' to the same degree. $\sigma(X, \Pi)$ is given as the intersection of this line and the admissible set $\mathcal{A}(X, \Pi)$.

It is easy to see from this construction that σ is formally identical to the Kalai-Smorodinsky (1975) solution to a bargaining problem with two players whose preferences are the EU preferences with respect to π' and to π''. Technically speaking, define a mapping $\Psi : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ such that $\Psi(x) = (\pi' \cdot x, \pi'' \cdot x)$; Ψ maps into vectors of expected utilities and is one-to-one. If $\xi(Y, d)$ is defined as the Kalai-Smorodinsky solution for a feasible set of utilities Y and a "threat-point" d, σ can be characterized by

$$\xi(\Psi(X), \Psi(y^0)) = \Psi(\sigma(X, \Pi)).$$

Note that while y^0 is the threat-point (in act space), y^1 is the "ideal point" in the terminology of Kalai and Smorodinsky. To establish comparability to the definition of σ, we shall also write ξ in terms of the primitives as $\overline{\xi}$, with $\overline{\xi}(X, \Pi) = \Psi^{-1}(\xi(\Psi(X), \Psi(y^0)))$. The equivalence can then be restated as

$$\sigma(X, \Pi) = \overline{\xi}(X, \Pi).$$

One can use this purely formal equivalence to interpret σ as the fair outcome of a bargaining between the different fictitious "alter egos" of the decision maker given by his extremal priors, his different virtual Bayesian selves, as it were. This interpretation of σ as a fair bargaining solution extends to the general (finite) case: one can define $\sigma(X, \Pi) = \overline{\xi}(X, \Pi)$, where $\overline{\xi}$ refers to the lexicographic variant of the Kalai-Smorodinsky solution which has been defined and axiomatized by Imai (1983).[10]

An essential feature of SIMEU is its context-dependence. Consider, for example, in Figure 1 the subset X' of all acts in X above the straight line through y^0 and y^1. While $\sigma(X, \Pi)$ is still feasible in X', it is now worst against π' within the shrunken set of admissible acts $\mathcal{A}(X', \Pi) = \mathcal{A}(X, \Pi) \cap X'$; as a result, to preserve even minimal robustness, $\sigma(X', \Pi)$ must be to the left of $\sigma(X, \Pi)$, with lower payoff in state one and higher payoff in state two, thus violating context-independent choice-consistency conditions such as WARP.[11]

Due to the convexity of X and the smoothness of $\mathcal{A}(X, \Pi)$, $\sigma(X, \Pi)$ has a unique supporting "compromise prior" $\mu(X, \Pi) \in \Delta^2$ such that $\mu(X, \Pi) \cdot \sigma(X, \Pi) \geqslant \mu(X, \Pi) \cdot x$ for all $x \in X$. Clearly, the analogously defined compromise prior $\mu(X', \Pi)$ supporting $\sigma(X', \Pi)$ puts more weight on state two and less on state one. The context-dependence of SIMEU choices is thus reflected in context-dependent supporting priors, which, *by virtue of their context-dependence*, cannot be interpreted as the decision maker's subjective probabilities. Indeed, it is easily seen that *any* non-extremal acceptable prior is a supporting prior $\mu(Y, \Pi)$ for some choice-set Y; in this way, the decision maker's suspension of judgement among acceptable priors is faithfully reflected in his SIMEU choices.

3. SIMEU AND LEXIMIN: DEFINITION AND BASIC PROPERTIES

3.1. *Framework and notation*

Let Ω denote an infinite universe of states, and let $\mathcal{F}$ be the set of finite partitions $F = \{S\}_{S \in F}$ of Ω into infinite subsets S. Note that, by definition, any $F \in \mathcal{F}$ is infinitely divisible in the sense that any event of any partition in $\mathcal{F}$ can be broken up into arbitrarily many subevents;[12] the role of this assumption is explained in Remark 1 following Theorem 2. An *act* x maps states to consequences $c \in K : x : \Omega \to K$. For expositional simplicity, we will assume that $K = [0,1]$, interpreting c as cardinal utility (normalized von Neumann Morgenstern utility, "payoff"); such an interpretation can be justified by standard arguments along the lines of Anscombe-Aumann's (1963) two-stage "horse-lottery" approach.[13] In a world with only two final consequences ("winning" and "losing", with winning preferred), x_ω can be identified with the objective probability of winning conditional on ω. A well-defined *choice set* is assumed to be closed with respect to the inclusion of mixed acts, and is therefore formally represented as a convex set of acts $X \subseteq [0,1]^\Omega$. To canonically include mixed acts is technically necessary and seems to be the more conservative way to proceed outside SEU-theory.[14]

For $F \in \mathcal{F}$, let $[0,1]^F$ denote the class of F-measurable[15] acts, and denote $[0,1]^{\mathcal{F}} = \cup_{F \in \mathcal{F}}[0,1]^F$, the class of *simple* acts. A choice-set X is *simple* if it is a closed (hence compact)[16] and convex set of simple acts; let $\mathcal{X}$ denote their class. It is not very difficult to show that a closed convex set $X \subseteq [0,1]^{\mathcal{F}}$ is simple if and only if all acts in X are measurable with respect to a common finite partition, i.e. if $X \subseteq [0,1]^F$ for some $F \in \mathcal{F}$. This fact is technically important and will be used throughout.[17] Some additional notation: "$cl\ X$" is the closure of X, "$co\ X$" is the convex hull of X, and $[x, y] = co\ \{x, y\}$. "$x<y$" holds if $x \leqslant y$ and $x_\omega<y_\omega$ for some $\omega \in \Omega$, "$x \ll y$" if $x_\omega<y_\omega$ for all $\omega \in \Omega$; e^S denotes the indicator-function of S, i.e., $e^S_\omega = 1$ if $\omega \in S$, and $e^S_\omega = 0$ otherwise.

A *decision problem under Complete Ignorance* ("CI problem") is a pair $(X, R_\emptyset)$, where X is a choice set and $R_\emptyset$ denotes the Complete Ignorance preference relation defined by

$$x\ R_\emptyset\ y \iff \left[x_\omega \geqslant y_\omega \quad \text{for all } \omega \in \Omega\right].$$

Since $R_\emptyset$ is assumed fixed in almost all of the following, we will normally identify a CI problem $(X, R_\emptyset)$ with its choice set X, and define a *choice function* as a non-empty-valued mapping C on $\mathcal{X}$ such that $C(X) \subseteq X$ for all $X \in \mathcal{X}$. We will write "$x\ P_\emptyset\ y$" for "$x\ R_\emptyset\ y$ and not $y\ R_\emptyset\ x$", as well as "$x\ N_\emptyset\ y$" for "neither $x\ R_\emptyset\ y$ nor $y\ R_\emptyset\ x$".

3.2. *SIMEU and leximin*

The following sections are devoted to an axiomatization of SIMEU for Complete-Ignorance problems, σ^{CI}. Along the way, we also obtain a choice-functional characterization of the lexicographic maximin rule LM defined as follows, with min $\emptyset = -\infty$.

$$LM(X) = \{x \in X \mid \text{For all } y \in X : \min_{\omega : x_\omega \neq y_\omega} x_\omega \geqslant \min_{\omega : x_\omega \neq y_\omega} y_\omega\}.$$

As it reads, we have defined $LM(X)$ as Barbera-Jackson's (1988) "protective criterion". Since the following proposition shows it to coincide (on *convex* sets) with the lexicographic maximin, we denote it by LM and refer to it by the latter, more informative name.

The SIMEU rule σ^{CI} modifies LM by normalizing ex-post utilities; the normalization yields "degrees of implementation" $\lambda_\omega(x)$ of x within X in state ω (respectively: "for each extremal prior e^ω"),

$$\lambda_\omega(x) = \frac{x_\omega - \inf_{y \in \mathcal{A}(X)} y_\omega}{\sup_{y \in \mathcal{A}(X)} y_\omega - \inf_{y \in \mathcal{A}(X)} y_\omega},$$

with $0/0 = 1$ by convention.

Also, define

$$\sigma^{CI}(X) = \{x \in X \mid \text{For all } y \in X : \min_{\omega : \lambda_\omega(x) \neq \lambda_\omega(y)} \lambda_\omega(x) \geqslant \min_{\omega : \lambda_\omega(x) \neq \lambda_\omega(y)} \lambda_\omega(y)\}.^{18}$$

EXAMPLE 1. The following matrix describes the payoffs of two acts in terms of the event partition $\{S_1, S_2\}$.

	S_1	S_2
x	0.90	0
y	0	0.10

Consider choices from the set $X = [x, y]$. The leximin-rule equalizes payoffs across states, selecting $LM(X) = (0.09, 0.09) = 0.1x + 0.9y$, which can be interpreted as randomized choice of y

with a probability of 90%. Measured in terms of degrees of implementation, $LM(X)$ favors the event S_2, and is thus non-robust with respect to the possibility of S_1, with $\lambda_\omega((0.09, 0.09)) = 0.90$ for any $\omega \in S_2$, whereas $\lambda_\omega((0.09, 0.09)) = 0.10$ for any $\omega \in S_1$. By comparison, $SIMEU(X) = (0.45, 0.045) = 0.5x + 0.5y$, equalizing degrees of implementation across states.

It is instructive to compare the selection of SIMEU to that of Savage's (1951) "minimax loss" rule ("MML"), its closest kin in the literature, with $MML(X)$ denoting the set of acts x that minimize $\max_{\omega\in\Omega}(\max_{y\in X} y_\omega - x_\omega)$. MML equalizes losses across states: $MML(X) = (0.81, 0.01) = 0.9x + 0.1y$. This is also non-robust, this time with respect to the possibility of S_2, with $\lambda_\omega((0.81, 0.01)) = 0.10$ for any $\omega \in S_2$, and $\lambda_\omega((0.81, 0.01)) = 0.90$ for any $\omega \in S_1$. MML relies heavily on the comparison of utility-differences across states, arguably more so than is warranted in view of the absence of any bound on the relative weight of S_1 and S_2; this is further discussed in Example 3 below. □

PROPOSITION 1. *(i) If $X \in \mathcal{X}$, $LM(X)$ and $\sigma^{CI}(X)$ are non-empty and single-valued.*

(ii) Moreover, if $x = LM(X)$ and $y \in X\backslash\{x\}$,

$$\min_{\omega: x_\omega \neq y_\omega} x_\omega > \min_{\omega: x_\omega \neq y_\omega} y_\omega.$$

Similarly, if $x = \sigma^{CI}(X)$ and $y \in X\backslash\{x\}$,

$$\min_{\omega:\lambda_\omega(x_\omega)\neq\lambda_\omega(y_\omega)} \lambda_\omega(x_\omega) > \min_{\omega:\lambda_\omega(x_\omega)\neq\lambda_\omega(y_\omega)} \lambda_\omega(y_\omega).$$

Remark. The convexity assumption on X is indispensable, as the counter-example of $X = \{(1, 0), (0, 1)\}$ shows, for which $LM(X) = SIMEU(X) = X$.[19] Convexity is also necessary to ensure (via part ii) of the Proposition) satisfaction of the consistency conditions defined below, of WARP for LM and of WAREP for SIMEU, respectively.

4. AXIOMATIZATION OF SIMEU AND LEXIMIN

This section characterizes SIMEU and LM in complete ignorance problems; while the relevant axioms on choice functions are given a first-round motivation, a more extensive discussion is reserved for the next section. The most basic rationality-requirement is compatibility with asserted preferences.

AXIOM 1 (*Admissibility*). For all $X \in \mathcal{X}$ and $x, y \in X$: $x \ P_{\emptyset} \ y$ implies $y \notin C(X)$.

If one rewrites the condition "$x \ P_{\emptyset} \ y$" in utility-terms as "for all $\omega \in \Omega$, $x_\omega \geqslant y_\omega$, and for some $\omega \in \Omega,\ x_\omega > y_\omega$", it is evident that this axiom amounts to the standard concept of *strict* admissibility.

The two key axioms of the theory are axioms of structural equivalence. The first is based on the symmetry of $R_{\emptyset}$ in events. For any one-to-one map $\phi : F \to F'$ on event partitions $F, F' \in \mathcal{F}$, define an associated one-to-one map on acts $\Phi : [0, 1]^F \to [0, 1]^{F'}$ by $\Phi(x)_{\phi(S)} = x_S$, for $S \in F$. $\Phi(x)$ is the act that results if the consequence x_S occurs in the event $\phi(S)$ instead of in the event S.

AXIOM 2 (*Symmetry, SY*). For all $X \in \mathcal{X}$, any $F \in \mathcal{F}$ such that X is F-measurable, and any $\phi : F \to F$ that is one-to-one: $\Phi(X) = X$ implies $C(X) = \Phi(C(X))$.

SY requires that symmetry of the choice set in events implies a corresponding symmetry of the chosen set. It is a weak version of the hallmark axiom of the CI literature (see Remark 1 following Theorem 1); it rules out representability of the choice function by some (as-if) subjective probability, as shown by the following example.

EXAMPLE 2. The following matrix describes the payoffs of four acts in terms of the event-partition $F^* = \{S_1, S_2, S_3\}$.

	S_1	S_2	S_3
w	1	0	1
x	1	1	0
y	0	1	1
z	1	0	0

Suppose C to be representable by the as-if subjective probability vector (π_1, π_2, π_3). SY applied to the choice set $[w, x]$, with $F = F^*$ and ϕ given by $\phi(S_1) = S_1$, $\phi(S_2) = S_3$ and $\phi(S_3) = S_2$, implies $x \in C([w, x]) \Leftrightarrow w \in C([w, x])$, and thus $\pi_2 = \pi_3$. An analogous application of SY to the choice set $[w, y]$ yields $\pi_1 = \pi_2$, and thus $\pi_1 = \pi_2 = \pi_3 = \frac{1}{3}$. However, applying SY to $[y, z]$ with $F = \{S_1, S_2 \cup S_3\}$ and ϕ given by $\phi(S_1) = S_2 \cup S_3$, and $\phi(S_2 \cup S_3) = S_1$ implies $y \in C([y, z]) \Leftrightarrow z \in C([y, z])$, and thus $\pi_1 = \pi_2 + \pi_3$, a contradiction. □

Symmetry can be viewed as expressing a decision-theoretic "principle of insufficient reason". It is desirably weaker than its classical Laplacian counterpart by merely asserting context-dependent equivalences of choice, rather than equal probabilities. As illustrated by Example 2, this makes it possible to apply this principle to arbitrary event partitions simultaneously and to thereby capture *complete* ignorance.[20]

A second invariance axiom called CISO (for "Consequence Isomorphism"), "dual" to Symmetry, considers transformations of payoffs within states. It hinges critically on an understanding of optimal choice as compromise, and is a natural consequence of the bargaining metaphor: the optimal choice should be invariant to positive affine transformations of state (fictitious players') utilities. In 5.3, a more detailed justification of the axiom is given. To define CISO formally, let an *affine consequence-isomorphism*[21] be a mapping θ from $[0, 1]^{\mathcal{F}}$ to $[0, 1]^{\mathcal{F}}$ (not necessarily onto) of the form $\theta(x) = (\alpha_\omega x_\omega + \beta_\omega)_{\omega \in \Omega}$, for appropriate $\alpha_\omega > 0$ and β_ω.

AXIOM 3 (*CISO*). For all $X \in \mathcal{X}$ and any affine consequence-isomorphism θ such that $\theta(X) \in \mathcal{X}$: $C(\theta(X)) = \theta(C(X))$.

EXAMPLE 3. Consider a typical instance of CISO.

	S_1	S_2
x	1	0
y	0	1
y^ϵ	0	ϵ

Let $X = [x, y]$, $X^\epsilon = [x, y^\epsilon]$, and assume $0 < \epsilon < 1$. Since $x\ N_\emptyset\ y$ as well as $x\ N_\emptyset\ y^\epsilon$, and since X^ϵ can be obtained from X by positive affine transformation of payoffs, CISO implies $y \in C(X) \Leftrightarrow y^{\epsilon} \in C(X^{\epsilon})$. Holding for arbitrarily small positive ε, this implication seems wild at first blush: while it seems perfectly reasonable to choose y in X, who would not choose x over y^{ϵ} in X^{ϵ}? After all, x might be much better than y^{ϵ} (in S_1) which at best might only be slightly better (in S_2). Such a reaction forgets, however, that the decision-maker could have asserted this preference himself, but explicitly declined to do so by asserting $x\ N_{\emptyset}\ y^{\epsilon}$. CISO ensures that the asserted non-comparabilities are fully respected by the choice-function. □

As discussed in more detail below, the preceding three axioms are incompatible with traditional context-independent choice-consistency conditions such as WARP.

CONDITION 1 (*WARP*). For all $x, y \in X \cap Y$:
$x \in C(X) \Rightarrow [y \in C(Y) \Rightarrow x \in C(Y)]$.

In words: if x is chosen in X, x is "revealed" to be at least as choiceworthy as any other alternative y in X, hence must be chosen in Y whenever y is. It seems natural to contain the extent of context-dependence by restricting WARP to "range-equivalent" pairs of decision problems for which it is unproblematic. X and X' are *range-equivalent* if $\mathrm{proj}_{\omega}\ cl\ \mathcal{A}(X) = \mathrm{proj}_{\omega}\ cl\ \mathcal{A}(X')$ for all $\omega \in \Omega$, that is, if they agree on the set of "admissible consequences" in each state.[22]

AXIOM 4 (*WAREP*). For any range-equivalent $X, X' \in \mathcal{X}$ and $x, x' \in X \cap X'$: $x \in C(X) \Rightarrow (x' \in C(X') \Rightarrow x \in C(X'))$.

While WAREP does not rest on quite as compelling a foundation as the other axioms, it has the definite merit of leading to a tractable and nicely interpretable solution. Moreover, it is weak in being satisfied by all major CI-solutions proposed in the literature, and in not determining the qualitative character of the choice rule, for which SY and CISO are responsible.

THEOREM 1. *σ^{CI} is uniquely characterized by Admissibility, Symmetry, Consequence-Isomorphism and WAREP.*

If one insists on preserving context-independence, at least one of the other axioms has to go. If one drops CISO, a characterization of leximin is obtained by a much simplified proof.

THEOREM 2. *LM is uniquely characterized by Symmetry, Admissibility and WARP.*

REMARK 1. Theorems 1 and 2 appear to be unique in the literature in using only symmetry besides the shared assumptions of admissibility and choice-consistency (as well as CISO in the case of Theorem 1). From Milnor (1954) on (see also Luce-Raiffa 1957), most use in addition an axiom that express some idea of description-invariance. This conceptually not unproblematic requirement can be dispensed with due to the infinite-divisibility assumption on the partitions $F \in \mathcal{F}$. It has been the main reason for making that assumption in the first place.[23]

REMARK 2. Theorems 1 and 2 are also unique among axiomatizations of "maximin-type" solutions in that they do not make any (explicit) assumption of "uncertainty-aversion," be it in the form of a quasi-concavity condition on preferences, as Milnor (1954) and Barbera-Jackson (1988) do, or as convex-valuedness of the choice function. We are enabled to drop such a condition by Lemma 2 in the proof of Theorem 1, for which *strict* Admissibility is crucial.

In the literature, Complete Ignorance is defined in terms of finite universes of events; part 1 of the appendix shows how the two theorems apply to finite universes via an embedding argument.

5. DISCUSSION

5.1. *Incompleteness as non-comparability*

Does it really make sense in situations of truly *complete* ignorance to determine a single-valued choice function, as we have done?[24] To legitimately obtain *any* determinate restriction on choice beyond ex-post undominatedness, it would seem that *some* knowledge on part of the decision-maker must be assumed, at least implicitly. Indeed, it *has been* assumed that, when asserting the preference relation $R_{\emptyset}$, the decision-maker acknowledges and, in this sense, "knows of" his complete ignorance about events. In other words, for SIMEU theory to be applicable, incompleteness of preferences must be given an *exhaustive* interpretation on which absence of weak preference (of both x over y and y over x) is equivalent to a judgment of *non-comparability* ("I decline to prefer one alternative over another"). In terms of beliefs, this active suspension of judgment involves accepting each "acceptable prior" as fully compatible with one's total view of the evidence; non-comparability thus corresponds to *self-aware* ignorance, as in "I know that I don't know".[25] An exhaustive interpretation of incompleteness as non-comparability contrasts with a *partial elicitation* interpretation of incompleteness as *non-comparedness*, that is: as mere absence of comparing judgment ("I have not figured out / made up my mind").[26]

Thus the possibility to meaningfully select among admissible acts is based on an active suspension of judgment. Conversely, it can be argued that the notion of a rationally motivated suspension of judgment makes pragmatic sense only if it is supported by a choice rule that selects among admissible acts. For if admissibility were the *only* criterion of rational choice with incomplete preference ordering R_Π, it would be legitimate – in terms of the exclusively relevant admissibility criterion itself! – to arrive at a decision by replacing (more or less arbitrarily) the given partial order R_Π with any complete order $R_{\{\pi\}}$ that extends it, with $\pi \in \Pi$. Since $\mathcal{A}(\cdot, R_{\{\pi\}}) \subseteq \mathcal{A}(\cdot, R_\Pi)$, any choice optimal under $R_{\{\pi\}}$ would then also be admissible, hence optimal, under the original partial order R_Π. A decision-maker could thus never go wrong by adopting complete preferences: some decision must be made – some act will be chosen, after all – so what use would it be to suspend judgment since you cannot suspend choice? At worst, some preference judgments entailed in the completion might be arbitrary. The concept of non-comparability would be useless for the purpose of decision-making.

5.2. *On the rationale for context-dependence*

It follows easily from examples 2 and 3 that for single-valued choice-functions the conjunction of Symmetry and CISO implies

$$x \; N_{\emptyset} \; y \Rightarrow C\left([x, y]\right) = \left\{\frac{1}{2}x + \frac{1}{2}y\right\}.$$

This "coin-flip property" endows judgments of non-comparability with well-defined *operational meaning*. It also entails that one cannot reconcile these axioms with traditional context-independent choice-consistency conditions such as WARP.

In the present non-comparability-based approach, the necessity of violating WARP should come as no surprise. Indeed, since CISO and Symmetry reflect the requirement that the choice-function take proper account of the (*non-transitive*!) non-comparability inherent in the structure of the underlying partial order $R_{\emptyset}$, WARP's incompatibility with these axioms simply reflects its inappropriateness.

Rather than being an embarrassment or impasse, the inherent context-dependence of SIMEU plays a crucial conceptual role by resolving an apparent tension between the assumed exhaustive interpretation of the underlying partial order and the single-valuedness of the derived choice-rule: *how can an act x be legitimately chosen over another act (y) when the decision maker has suspended judgment between them?* The answer is that suspension of judgment involves abstention only from expressing a *definite* preference of x over y, i.e. abstention from context-*independent* choice of x over y. On the other hand, suspension of preference judgment is entirely compatible with choice of x over y and of y over x on a "case-by-case" basis. This happens under SIMEU: it is not difficult to show that for any x, y such that $x N_{\emptyset} y$, *any* choice of x over y is context-*dependent*, i.e. that there exist $X', X'' \supseteq \{x, y\}$ such that $\{x\} = \sigma^{CI}(X')$ and $\{y\} = \sigma^{CI}(X'')$. Intuitively, non-comparability rules out the choice of one act over another as *intrinsically better*, but is compatible with the choice of one act over another *as a superior compromise* in the context of a particular choice-set.

A particularly clear-cut instance of this distinction occurs in the choice among just two non-comparable alternatives, where SIMEU recommends the flipping of a fair coin. The only apparent advantage of such randomization is the symmetric treatment of both alternatives; this may not seem much. On the other hand, given the assumed suspension of judgment one cannot really hope to do better. Psychologically though, some dissatisfaction may still remain, as it does for the author. But perhaps such dissatisfaction reveals just how hard it is to honestly face genuine ignorance and to suspend judgment accordingly. In this vein, Elster (1989, pp. 54–59) argues that as a rule there is a psychological bias against its acknowledgment. He makes a strong case for the existence of a human tendency to exaggerate the support of many decisions by "reasons," summarizing (on p. 58): "The toleration of ignorance, like the toleration of ambiguity more generally, does not come easily."[27]

An understanding of optimal choice as best compromise is also helpful in getting an intuitive grip on how to endow context-dependence with more structure, especially on how to contain its extent in terms of axioms such as WAREP. The following example is intended to flesh out the motivation for that axiom.

EXAMPLE 4. Consider three choice-sets X_1, X_2, X_3, with $X_i = co\{x, y, z_i\}$ defined in terms of the following five acts:

	S_1	S_2
x	$\frac{1}{2}$	1
y	$\frac{3}{4}$	$\frac{3}{4}$
z_1	1	$\frac{1}{2}$
z_2	$\frac{5}{6}$	$\frac{1}{2}$
z_3	1	0

For the SIMEU, LM and MML choice functions, $C(X_1) = y$. Noting that $X_1 = \mathcal{A}(co\{x, y, z_1, z_2, z_3\})$, the conjunction of Admissibility and WARP entails $y = C(X_2)$ as well as $y = C(X_3)$. Neither implication is appealing from a compromise perspective. Intuitively, in X_2, a choice of y favors the possibility of S_1, and comparatively neglects that of S_2, since it almost achieves maximal utility in the former, but not in the latter. An optimal compromise should yield less utility than y in S_1 and more in S_2; SIMEU in fact selects the act $(\frac{7}{10}, \frac{4}{5})$. By contrast, in X_3 y intuitively favors the possibility of S_2, since now some admissible choices, in particular that of z_2, might entail much lower utility in S_2 than any admissible act did before (in X_1 where y was an optimal compromise). Correspondingly, an optimal compromise should yield more utility than y

in S_1 and less in S_2; SIMEU selects the act $(\frac{7}{9}, \frac{7}{12})$. Note that MML moves in the desired direction in X_2 but not in X_3. □

The replacements of z_1 by z_2 and z_3 in X_2 respectively X_3 both illustrate both the context-dependence of an optimal compromise. At least as an approximation, it seems reasonable to attribute this context-dependence in the case of X_2 to the decrease of *maximal* S_1-utility, and in the case of X_3 to the decrease of the *minimal admissible* S_2-utility.[28] WAREP entails that the context-dependence of SIMEU choices is entirely driven by such changes in the state-wise ranges of admissible utilities. If there is no such change, i.e. if the two choice sets are "range-equivalent", WAREP requires context-independence; this is formalized by the condition that the choice in range-equivalent sets cannot reveal contradictory compromise rankings. WAREP thus assumes as much context-independence as is possible.[29]

5.3. *A deeper justification: the principle of preference-basedness*

Conceptually, we have attributed the context-dependence of SIMEU to an interpretation of optimal choice as compromise. Mathematically, the context-dependence of the solution arises from the two invariance conditions underlying the solution, especially CISO. These two perspectives will now be linked, with the purpose of achieving a deeper justification of the two key invariance axioms based on the compromise interpretation. The intuitive point of departure is the idea that a good compromise-choice *fully* exploits all available preference information, and that this information consists not only in the asserted preference comparisons, but also in the preference comparisons abstained from, that is: in the asserted suspensions of judgment. This leads to the informal requirement that the structure of the choice function should reflect the structure of the entire preference relation.

It is a non-trivial issue how to formalize this requirement, which we shall refer to as the "Principle of Preference Basedness" (PPB). While a comprehensive treatment of this issue goes beyond the scope of this paper, we will argue that SY and CISO are natural consequences of the PPB. The discussion will initially be phrased in terms of conditions on choice-functions $C(X, R)$ that involve appropriate general partial orders R;[30] we will later specialize to the complete-ignorance ordering $R_\emptyset$, for which the PPB turns out to be especially powerful.

Note first that it would be mistaken to formalize the PPB in context-*independent* manner by way of a condition of the following type.[31]

CONDITION 2. For any x, y, X such that $x, y \in X$: if either yRz and zRy or not yRz and not zRy, then $z \in C(X, R)$ if and only if $y \in C(X, R)$.

This condition asserts choice-equivalence between two acts in any choice set X whenever they are treated symmetrically by the preference relation R, i.e. whenever the acts are either indifferent or non-comparable. Conceptually, the condition is inappropriate as it effectively equates non-transitive non-comparability with transitive indifference. The mismatch is reflected in the mathematics, as Condition 2 is not even consistent with Admissibility! Setting $X = co\{x, y, z\}$ in Example 2 of Section 4, for instance, Condition 2 implies both $x \in C(X, R_\emptyset) \Leftrightarrow y \in C(X, R_\emptyset)$ and $z \in C(X, R_\emptyset) \Leftrightarrow y \in C(X, R_\emptyset)$, hence also $x \in C(X, R_\emptyset) \Leftrightarrow z \in C(X, R_\emptyset)$, in conflict with Admissibility.

This example also points to the source of error in Condition 2, which stems from the fact that, *in the context of the choice set* X, the preference ordering $R_\emptyset$ does *not* treat z and y fully symmetrically, since z is $R_\emptyset-$inferior to some feasible act in X, namely x, whereas y is inferior to none. Condition 2 needs to be reformulated in a manner that *allows the context to matter;* a prototype is the following "*invariance under preference-isomorphism*" condition.

CONDITION 3. Let θ be any mapping from $[0,1]^{\mathcal{F}}$ to $[0,1]^{\mathcal{F}}$ (not necessarily onto) that preserves $R-$order, i.e. such that $\theta(x)\ R\ \theta(y) \Leftrightarrow x\ R\ y \quad \forall x, y \in [0,1]^{\mathcal{F}}$.

Then, for all $X \in \mathcal{X}$ such that $\theta(X) \in \mathcal{X}$, $C(\theta(X), R) = \theta(C(X, R))$.

The normative force of a condition of this kind resides in taking an isomorphism of choice problems in terms of the ordering R to be *sufficient* for choice equivalence; this means that *no other* *information is allowed to matter.* In particular, the PPB formalized in this way rules out approaches such as Levi's (1980, ch. 7), who proposes to select among admissible acts on the bases of additional (non-expectational) "security considerations".

While Condition 3 conveys the general idea of the PPB correctly, it needs further refinement; for one thing, its presupposition is still too weak by neglecting cardinal information (see clause ii) in the reformulation of CISO below and its discussion). From now on, we will specialize Condition 3 to $R = R_\emptyset$, and show that by requiring the mappings θ to have additional structure, one obtains both a version of Symmetry as well as CISO, thus showing these two key axioms to emanate from the PPB properly formulated.[32] In the notation of Section 4, consider first mappings $\theta = \Phi$ ("event-isomorphisms") based on some permutation of events $\phi : F \to F'$. Note that *any* such Φ is $R_\emptyset$-order-preserving[33]; Condition 3 thus specializes to the following condition of "Event-Isomorphism" which is slightly stronger than Symmetry.

CONDITION 4 (*EISO*). For all $X \in \mathcal{X}$ and $\phi : F \to F'$ one-to-one such that X is F-measurable: $C(\Phi(X), R_\emptyset) = \Phi(C(X, R_\emptyset))$.

In complementary fashion, CISO can be viewed as a condition of invariance with respect to preference-isomorphisms that assign different payoffs to given events. More formally and precisely, let a *consequence-isomorphism* be a mapping θ from $[0, 1]^{\mathcal{F}}$ to $[0, 1]^{\mathcal{F}}$ (not necessarily onto) that preserves order as well as mixture-information about acts and is separable in states, i.e., that satisfies

(i) $\theta(x)\ R_\emptyset\ \theta(y) \Leftrightarrow x\ R_\emptyset\ y \quad \forall x, y \in [0, 1]^{\mathcal{F}}$,
(ii) $\theta(\lambda x + (1 - \lambda)y) = \lambda\theta(x) + (1 - \lambda)\theta(y) \quad \forall x, y \in [0, 1]^{\mathcal{F}}$, $0 \leqslant \lambda \leqslant 1$, and
(iii) There exist $(\theta_\omega)_{\omega\in\Omega}, \theta_\omega : [0, 1] \to [0, 1]$ such that $\theta(x) = (\theta_\omega(x_\omega))_{\omega\in\Omega}$.

It is easily verified that θ is a consequence-isomorphism with respect to $R_\emptyset$ if and only if each θ_ω is of the form $\theta_\omega(c) = \alpha_\omega c + \beta_\omega$, with $\alpha_\omega > 0$. CISO amounts therefore to restricting Condition 3 to consequence isomorphisms. The mixture-condition ii) reflects the need to preserve *cardinal* utility information; as is well-known from bargaining theory, without it, no interesting theory could be developed. Note also that it is automatically satisfied by the event-isomorphisms considered in EISO. Just as EISO, CISO in extremely powerful in the context of CI-problems due to their extreme richness in asserted non-comparabilities. In particular, if the decision-maker had asserted any preference other than $R_{\emptyset}$, invariance with respect to arbitrary positive affine state-by-state transformations would no longer be entailed.

While CISO has been motivated heuristically by the bargaining metaphor, it is fully justified only by (something like) the PPB. The PPB explains why the bargaining metaphor is appropriate.[34] Without a justification of this kind, CISO would be open to the critique that it forces the decision-maker to ignore prima-facie relevant information, namely utility differences across states. The PPB counters this critique (recall the discussion of Example 3 in Section 4) by insisting that the choice rule should make *full* use of the preference relation $R_{\emptyset}$, and in particular, that it should respect the entailed non-comparabilities $N_{\emptyset}$.

5.4. *Extension to partial ignorance*

In view of their extreme nature, Complete Ignorance problems are relevant for applications not so much in themselves, but primarily because they can be viewed as "reduced forms" of general d.p.u.s. The reduction of general d.p.u.s is achieved by a condition of "Complete Ignorance Reduction" (CIR).[35] CIR associates to each d.p.u. an equivalent CI problem "in expected utility profiles"; these are obtained from taking the expected utility of an act with respect to each extremal prior.[36] In the two-event case, it reads as follows (in the notation of Section 2).

CONDITION 5 (*CIR*). $C(X, \Pi) = \Psi^{-1}(C(\Psi(X), \Delta^2))$.

As far as we know, the first contribution extending choices in CI-problems to a reasonably general class of decision problems under partial ignorance is Jaffray's (1989) using a mixture-space approach; see also Hendon et al. (1994) for further work along this line. Two points of comparison seem particularly noteworthy. Mathematically, the mixture-space approach applies to "belief-functions" which correspond to a rather restrictive class of belief sets.[37] Conceptually, the mixture-space approach takes the underlying belief-function (respectively lower probability) as representing *given* evidence, whereas an agent's incomplete preference relation is viewed in our approach as the outcome of the agent's judgment, and, in this sense, as something *chosen*. The appeal to the agent's active suspension of judgment has been central to our justification of the key axioms Symmetry and CISO via the PPB.

In justifying the key axioms via the PPB, we have frequently appealed to the decision maker's "prior", "hypothetically given" preference relation R. This "priority" is to be understood logically, not temporally. In particular, there is no presumption that the decision maker comes already fully equipped with an incomplete preference relation. On the contrary: to know what preferences to adopt (in particular: when to suspend judgment), the decision maker needs to know the *choice content* of preference judgments. Indeed, in view of the extreme nature of SIMEU choices under Complete Ignorance and their apparent contrariness to common sense, it will rarely if ever be reasonable to assert Complete Ignorance preferences $R_{\emptyset}$, even in situations in which there seems to be no tangible evidence at all.[38] Contemplating what rationally would have to be chosen if one *were* completely ignorant brings to light that one generally has beliefs over many events, that is: that one is prepared to bet if betting one must.

6. APPENDIX

6.1. *Extension of Theorems 1 and 2 to finite universes*

To derive versions of Theorems 1 and 2 for finite universes, one has to interpret $\mathcal{F}$ as a class of conceivable "universes" F described by finite sets of "states" (atomic events); each F may be thought of as a "framework of description" related by the common "language" Ω.

A CI-problem is now defined as a pair $(X, R_{\emptyset}^{F})$ such that $F \in \mathcal{F}$ and X is a compact convex subset of $[0, 1]^F$. Let $D^F = \{(X, R_{\emptyset}^{F}) \mid X \subseteq [0, 1]^F\}$; a solution is defined on the class of such problems $D = \cup_{F \in \mathcal{F}} D^F$. The axioms are now applied to each subdomain separately.

The subdomains can be linked by an embedding condition.

AXIOM 5 (*EMB*). If $X \subseteq [0,1]^F$ and G is a refinement of F, $C(X, R_\emptyset^F) = C(X, R_\emptyset^G)$.

EMB can be read as saying that if a given frame F with complete ignorance $R_\emptyset^F$ is refined to G, that refinement should not affect the chosen set per se, i.e., as long as no preference is asserted beyond those affirmed by $R_\emptyset^F$ and implied by the consistency axioms on preferences. Following the terminology of Walley (1991, ch. 3.1), this may be described as "Natural Extension" property. Noting that for any $F, G \in \mathcal{F}$ there exists $H \in \mathcal{F}$ that is a refinement of both F and G, EMB implies that

$$C(X, R_\emptyset^F) = C(X, R_\emptyset^G), \text{ whenever } X \subseteq [0,1]^F \cap [0,1]^G.$$

C may thus be viewed as defined on X only, and, with EMB in place, the axioms defined on $\bigcup_{F \in \mathcal{F}} D^F$ turn out to be equivalent to those defined on $\mathcal{X} \times \{R_\emptyset\}$. It follows that Theorems 1 and 2 carry over.

Remark: Although one now needs to refer to CI-problems that reside in different hypothetical universes of events, just as the traditional CI-literature does, the present approach still has the significant conceptual advantage that it does not make the assumption that the frame of reference is irrelevant. Such an assumption is implicit in the traditional treatment of events as "generic events without names" which can be formalized in the current setting by the following condition:

"For all F,$G \in \mathcal{F}$ and any one-to-one map $\phi : F \to G$: $\Phi(C(X, R_\emptyset^F)) = C(\Phi(X), R_\emptyset^G)$".

6.2. *Proofs*

For future reference, a set $X \subseteq [0,1]^\Omega$ is called *normalized* if, for all $\omega \subseteq \Omega$, $\text{proj}_\omega\, cl\, \mathcal{A}(X) = [0,1]$ or $\text{proj}_\omega\, cl\, \mathcal{A}(X) = \{1\}$.

Proof of Proposition 1:

Since LM and σ^{CI} agree on normalized choice-sets, it evidently suffices to prove the Proposition for LM. Let $F \in \mathcal{F}$ be any partition such that X is F-measurable.

For $G \subseteq F$, define $\mu(X, G) = \max_{x\in X} \min_{S\in G} x_S$ and $MM(X, G) = \arg\max_{x\in X} \min_{S\in G} x_S$. The key to the proof is the following lemma.

LEMMA 1. If X is convex, then there exists $T \in G$ such that, for all $x \in X : x \in MM(X, G) \Rightarrow x_T = \mu(X, G)$.

Proof of Lemma:

The following simple fact will be used repeatedly:

$$\text{For any } x \in MM(X, G) \text{ and } S \in G : x_S \geqslant \mu(X, G). \tag{1}$$

Suppose the claim of the lemma to be false, i.e. that for every $T \in G$ there exists $z^T \in MM(X, G)$ such that $z^T_T > \mu(X, G)$. Then, setting $z' = \sum_{T\in G} \frac{1}{\#G} z^T$ ($\in X$ by convexity), in view of (1), $\min_{S\in G} z'_S > \mu(X, G)$, a contradiction. □

Let $F(0) = F$, $X^{(0)} = X$, and $n = \#F$.

For $k = 0, \ldots, n-1$, define inductively $X^{(k+1)} = MM(X^{(k)}, F^{(k)})$, and $F^{(k+1)} = F^{(k)} \backslash \{S^{(k)}\}$, where $S^{(k)}$ is any $T \in F^{(k)}$ satisfying the property asserted in the lemma for $(X^{(k)}, F^{(k)})$.

It is easily verified by induction that for all $k \leqslant n-1$ $X^{(k)}$ is non-empty, compact and convex. Fix some $\xi \in X^{(n-1)}$, and consider any $y \in X \backslash \{\xi\}$.

We will show that

$$\min_{\omega:\xi_\omega \neq y_\omega} \xi_\omega > \min_{\omega:\xi_\omega \neq y_\omega} y_\omega. \tag{2}$$

This implies $y \notin LM(X)$, and, since y is arbitrary and $LM(X)$ is non-empty, indeed $LM(X) = \{\xi\}$, from which the asserted properties of LM follow in view of (2).

To show (2), assume that $y_S > \xi_S$ for some $S \in F$; otherwise (2) is satisfied trivially. Let $\nu = \min_{S\in F}\{\xi_S \mid \xi_S < y_S\}$, and let k^* be the largest integer k such that $\xi_{S^{(k)}} \leqslant \nu$.

We will show that for some $k \leqslant k^*$, $y_{S^{(k)}} < \xi_{S^{(k)}}$. From this (2) follows, since $k \leqslant k'$ implies, for any k, k', $\mu(X^{(k)}, F^{(k)}) \leqslant$

$\mu(X^{(k')}, F^{(k')})$ (by definition) which in turn implies $\xi_{S^{(k)}} \leqslant \xi_{S^{(k')}}$ by Lemma 1.

Suppose that the last claim is false, i.e. that

$$\text{for all } k \leqslant k^*, \; y_{S^{(k)}} \geqslant \xi_{S^{(k)}}. \tag{3}$$

Let $z^\varepsilon = \varepsilon \cdot y + (1 - \varepsilon) \cdot \xi$. For sufficiently small but strictly positive ε, the following three properties are satisfied:

(i) $z^\varepsilon_{S^{(k)}} \geqslant \xi_{S^{(k)}}$, for all $k \leqslant k^*$.
(ii) $z^\varepsilon_{S^{(k)}} > \xi_{S^{(k)}}$, for some $k \leqslant k^*$.
(iii) $z^\varepsilon_{S^{(k)}} > \nu$, for all $k > k^*$.

(i) is straightforward from (3); (ii) follows from the definition of k^* and (3); (iii) finally follows from the fact that $\xi_{S^{(k)}} > \nu$, for all $k > k^*$ if ε is chosen sufficiently small.

(i) and (iii) imply $z^\varepsilon \in X^{(k)}$, for all $k \leqslant k^*$. But then (ii) contradicts Lemma 1, the desired contradiction. □

Proof of Theorem 1:

Necessity of the first three properties is straightforward, and that of WAREP is implied by part (ii) of Proposition 1.

To show sufficiency, note first that WAREP implies the following property IDA (“Independence of Dominated Alternatives”):

(IDA) $\mathcal{A}(X) = \mathcal{A}(X') \Rightarrow C(X) = C(X') \quad \forall\, X, X' \in \mathcal{X}$.

It thus involves no loss of generality to restrict attention to normalized choice-sets. A choice set $Y \subseteq [0,1]^F$ will be called *F-comprehensive* if $x' \leqslant x$, $x \in Y$, and $x' \in [0,1]^F$ imply $x' \in Y$.

Essential to the proof are the following two lemmas:

LEMMA 2. *If Y is F-measurable and Y is symmetric with respect to all Φ: $[0,1]^F \to [0,1]^F$ that leave events outside $G \subseteq F$ invariant (i.e. such that $\Phi(x)_T = x_T \; \forall T \in F \setminus g$), then any $x \in C(X)$ is constant on $\cup G$.*

Proof. By CISO and IDA, Y can assumed to be normalized and F-comprehensive. The proof is by contradiction: suppose that $C(Y)$ contains an act ξ that is not constant on $\cup G$. Let $\nu = \min_{S \in G} \xi_S$, and let S_0 be any $S \in G$ such that $\xi_S = \nu$. Also, let $F' \in \mathcal{F}$ be any partition obtained from F by splitting S_0 into $\{S_1, S_2\}$: $F' = \{S \in F \mid S \neq S_o\} \cup \{S_1, S_2\}$.

Define $\eta : [0, 1]^F \to [0, 1]^{F'}$ by

$$\eta(x)_S = \begin{cases} \min_{T \in G} x_T & \text{if } S = S_1, \\ x_{S_0} & \text{if } S = S_2, \\ x_S & \text{otherwise,} \end{cases}$$

define: $Z \subseteq [0, 1]^{F'}$ as

$$Z = co\ \left(\{\eta(x) \,|x \in Y\} \cup e^{S_1}\right),$$

and let $Y' = \{x \in [0, 1]^{F'} | x \leqslant y$ for some $y \in Y\}$, the "F'-comprehensive hull" of Y.

Z has the following properties:

(i) $\xi \in Z \subseteq Y'$.
(ii) $\forall S \in F'$: $\text{proj}_S\ cl\ \mathcal{A}(Z) = \text{proj}_S\ cl\ \mathcal{A}(Y) = \text{proj}_S\ cl\ \mathcal{A}(Y') = [0, 1]$.
(iii) Z is symmetric w.r.t. all event-isomorphisms $\Phi : [0, 1]^{F'} \to [0, 1]^{F'}$ that leave all events in $(F \setminus g) \cup \{S_1\}$ invariant.

Note that (i) follows from the definition of S_0, (ii) hinges on the inclusion of e^{S_1} in Z, and (iii) follows from the symmetry assumption on Y.

Since $\mathcal{A}(Y') = \mathcal{A}(Y)$, from IDA,

$$\xi \in C(Y'). \tag{4}$$

Hence, using properties (i) and (ii) of Z, by WAREP also

$$\xi \in C(Z). \tag{5}$$

Since ξ is non-constant, for some $S_3 \in G : \xi_{S_0} < \xi_{S_3}$. Let $\phi : F \to F$ permute S_0 and S_3, leaving other events invariant, and let $\phi' : F' \to F'$ permute S_2 and S_3, leaving other events invariant, with associated Φ respectively Φ'. By property (iii) of Z, $\Phi'(Z) = Z$; using SY, it thus follows from (5) that

$$\Phi'(\xi) \in C(Z). \tag{6}$$

By WAREP, from (4), (6) and properties (i) and (ii) of Z also

$$\Phi'(\xi) \in C(Y'). \tag{7}$$

However, by the symmetry assumption on Y, Y and hence Y' contain also $\Phi(\xi)$. Noting $\Phi(\xi)_{S_1} = \xi_{S_3} > \xi_{S_1} = \Phi'(\xi)_{S_1}$ and $\Phi(\xi)_{-S_1} = \Phi'(\xi)_{-S_1}$, one has $\Phi(\xi) > \Phi'(\xi)$. By admissibility, $\Phi'(\xi) \notin C(Y')$, in contradiction to (7). □

LEMMA 3. *Consider any normalized X, $y \in X$, and F such that X is F-measurable. If there exists $z \in X$ such that:*

(i) $z_S > 0 \quad \forall S \in F$,
(ii) *z is constant on* $\{S \in F \mid z_S \neq y_S\}$, *and*
(iii) *for some* $S \in F : z_S > y_S$,

then $\quad y \notin C(X)$.

Proof. Take any X, F and $y, z \in X$ with the properties assumed in the statement of the lemma. Partition F into the following three collections of events, fixing some S' such that $z_{S'} > y_{S'}$.

$$F' = \{S'\},$$
$$F'' = \{S \in F \setminus \{S'\} \mid z_S \neq y_S\}, \text{ and}$$
$$F''' = \{S \in F \mid z_S = y_S\}.$$

It is clear that events S such that $\#\mathrm{proj}_S\, X = 1$ make no difference; hence, assume w.l.o.g. that there are no such events. Take any sufficiently large integers l and m such that

$$m > \frac{2}{\min_S\, z_S} \quad \text{and} \quad l \geqslant \frac{\#F \cdot m}{(z_{S'} - y_{S'})}. \tag{8}$$

Let $G \in \mathcal{F}$ be a refinement of F such that S' is "replicated" l times (i.e. such that $\#\{T \in G \mid T \subseteq S'\} = l$) and any $S \neq S'$ is replicated m times. Also, let G' (resp. G'', G''') denote the corresponding refinement of F' (resp. F'', F''').

Let ϕ^* be the class of permutations ϕ of G that leave events outside $G' \cup G''$ invariant (i.e. events such that $T \notin G' \cup G'' \Rightarrow \phi(T) = T$). Likewise, let ϕ^{**} be the class of those permutations ϕ of G such that, for all $T \in g$, $\phi(T)$ is a "replica" of the same event in F as T (i.e. such that $\forall T \in g, \forall S \in F : S \supseteq T \Rightarrow S \supseteq \phi(T)$), and let Φ^*, Φ^{**} denote the associated classes of event-isomorphisms $\Phi : [0, 1]^G \to [0, 1]^G$.

Define a choice-set Z as follows:

$$Z = co\ (\{z\} \cup \{\Phi\,(y)\}_{\Phi\in\Phi^*} \cup E)\,,$$
$$\text{with } E = \left\{e^H\,|H = T_1 \cup T_2 \text{ for some } T_1, T_2 \in G, T_1 \neq T_2\right\}.$$

If $F'' = \emptyset$, the claim follows directly from admissibility; assume thus $F'' \neq \emptyset$ which implies $z_{s'} < 1$ in view of assumption (ii). Hence any $e^H \in E$ such that $H \cap S' \neq \emptyset$ is admissible, which implies $\mathrm{proj}_T\ \mathcal{A}(Z) = [0,1] \quad \forall T \in G$. Z is thus range-equivalent to X.

Take $w \in C(Z)$ and express w as convex combination:

$$w = \lambda_z z + \sum_{\Phi\in\Phi^*} \lambda_\Phi \Phi\,(y) + \sum_{e^H\in E} \lambda_H e^H.$$

For any $S \in F,\ Z$ is symmetric under all permutations $\phi : G \to G$ leaving events outside S invariant. By Lemma 2, w must thus be constant on each $S \in F$, i.e. F-measurable.

It is also not difficult to verify that, for any F-measurable act $x,\ x = \sum\limits_{\Phi\in\Phi^{**}} \frac{1}{\#\Phi^{**}}\Phi(x)$, and, in view of (8), that $z{>}\frac{2}{m}e^{\Omega} \geqslant \sum\limits_{\Phi\in\Phi^{**}} \frac{1}{\#\Phi^{**}}\Phi(e^H)$ for all $e^H \in E$.

Thus, by the admissibility of w, $\lambda_H = 0$ for all H such that $e^H \in E$ (for otherwise $w{<}(\lambda_z + \sum_{e^H\in E}\lambda_H)z + \sum_{\Phi\in\Phi^*}\lambda_\Phi\Phi\,(y)$, contradicting the admissibility of w).

This shows $w \in Z' = co\ (\{z\}\cup\{\Phi(Y)\}_{\Phi\in\Phi^*})$.

By the admissibility of w in Z', the fact that for any $x \in Z'$: $x_{-\cup(G'\cup G'')} = z_{-\cup(G'\cup G'')}$, and the convexity of Z', it follows from a standard supporting-hyperplane argument that w must maximize $\sum\limits_{T\in G'\cup G''} \pi_T x_T$ in Z' for appropriate non-negative coefficients π_T. Since Z' is symmetric under all permutations $\phi \in \phi^*$ by construction, w must be constant on $\cup(G'\cup G'')$ by Lemma 2; moreover, the π_T can assumed to be constant $(= 1)$ as well; it follows that w must in fact maximize $\sum\limits_{T\in G'\cup G''} x_T$ in Z'. Since this is uniquely done by z in view of the assumption on l in (8), it follows $C(Z) = \{z\}$, and in particular $y \notin C(Z)$. Since $z \in X$, the claim then follows from WAREP. $\square$

Proof of Theorem 1, ctd.: Fix any F such that X is F-measurable. By IDA, X can be assumed F-comprehensive. Let $\sigma^{CI}(X) = \{\xi\}$.

Take any $y \neq \xi$, and define z by

$$z_\omega = \begin{cases} y_\omega & \text{if } y_\omega = \xi_\omega \\ \min_{\omega' :\, y_{\omega'} \neq \xi_{\omega'}} \xi_{\omega'} & \text{if} y_\omega \neq \xi_\omega \end{cases} .$$

By Proposition 1, for some $S \in F$, $z_S > y_S$. Since $z \leqslant \xi$ and by the $F-$comprehensiveness of X, it follows that $z \in X$. Thus X, y, z, F satisfy the properties assumed by Lemma 3 which yields $y \notin C(X)$. It follows that $C(X) = \sigma^{CI}(X)$ by the non-emptiness of C. $\square$

Proof of Theorem 2:

Theorem 2 can be demonstrated using a significantly simplified version of the proof of the Theorem 1. $\square$

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NOTES

1. Noncomparability is distinguished from genuine indifference by its *lack of transitivity*. Indeed, non-comparability is typically robust with respect to small

(unambiguous) changes in the value of the alternatives. This is a typical feature of "hard" choices. For example, if you find it difficult to decide whether to accept a job-offer at a salary of x dollars per year or to stay put, you will find it just as difficult to decide at $x + 1$ dollars, probably also at $x + 100$, maybe even at $x + 10,000$ dollars. (While you will probably be able to tell the difference between x and $x + 10,000$ dollars, this may not settle the matter for you, as money may simply not be the real issue.)

2. This follows from standard representation theorems, e.g. Smith (1961) and Bewley (1986). Partial orders with the assumed structure have received a mathematically comprehensive and conceptually profound treatment in Walley's monograph "Statistical Reasoning with Imprecise Probabilities" (1991). Belief-functions and upper-and lower probabilities, other frequently endorsed generalizations of the probability calculus, can be viewed as special (and restrictive) instances of assessing such partial orders (see Walley 1991, ch. 4, especially pp. 182–184 and 197–199).
3. The classical reference is de Finetti (1937); for a discussion of exchangeability in the context of partial orders, see Walley (ch. 9.5).
4. MML can be viewed as applying the maximin solution after normalizing consequence utilities by subtracting, for each state, the maximal achievable utility in that state.
5. Under Complete Ignorance, i.e. requiring Symmetry, these exhaust the set of all preference maximizing choice rules, as shown in Arrow and Hurwicz (1972).
6. In the literature on statistical decision theory, this is often phrased as violation of the "likelihood principle" (Barnard 1949, Birnbaum 1962).
7. And of its extension to convex sets of priors often referred to as Γ-minimax loss rule.
8. These can be derived from a standard representation theorem (cf. Section 3).
9. Note that in the limiting case of Complete Ignorance, the parallelogram of Figure 1 becomes a rectangle whose sides are parallel to the axes.
10. $\overline{\xi}$ provides an easy way to thematize the role of extremal priors. A plausible alternative to the definition of SIMEU as σ would be as $\sigma^{\infty}(X, \Pi) = \overline{\xi}(X, co\ \Pi)$; this is discussed in detail in Nehring (1991, ch. 2.5), with arguments suggesting the superiority of the adopted specification of SIMEU as σ. For the moment, just note that while in higher dimensions the two specifications may differ, in two dimensions they are always identical; this has been shown in Nehring (1991, ch. 2), Proposition 6.
11. WARP is formally defined in Section 4.
12. I.e., for each $F \in \mathcal{F}$ and each $\#F$-tuple of natural numbers $(n_s)_{S\in F}$, there exists a refinement G of F in $\mathcal{F}$ such that $\#\{T \in G | T \subseteq S\} = n_s$.
13. For an exposition of the theory that does not assume (but effectively reduces to) [0,1]-valued consequences, see Nehring (1995).
14. Note that otherwise uncertainty-averse choice rules such as maximin and SIMEU may recommend giving up significant amounts of utility for access

to a random device. If the pure acts are (1,0) and (0,1) (in natural "partition notation"), for example, randomization would be worth up to 0.5 utiles for a decision-maker using either of these rules.

15. An act x is F*-measurable* iff it is constant on each cell $S \in F$.
16. With $[0,1]^{\mathcal{F}}$ being endowed with the product topology; since $[0,1]^{\mathcal{F}}$ is compact in this topology (by Tychonoff's Theorem), so is any simple choice-set $X \in \mathcal{X}$.
17. I owe this fact to the intervention of a referee; note that it would clearly be false for non-convex X.
18. Note that $\lambda_{\cdot}(x)$ is F-measurable whenever $X \subseteq [0,1]^F$, hence simple; note also that $\{\omega : \lambda_\omega(x) \neq \lambda_\omega(y)\} = \{\omega : x_\omega \neq y_\omega\}$.
19. Taking any F such that $X \subseteq [0,1]^F$, and viewing $[0,1]^F$ as a *finite-dimensional* unit-cube, the proposition also implies that, for *convex* X, the unique $x \in LM(X)$ coincides with the lexicographic maximin act as defined ordinarily for finite-dimensional Euclidean spaces.
20. Dating back to the nineteenth century, there has been a long tradition of criticisms of the principle *in its Laplacian form* which has been revived in recent years under the name of "non-informative Bayesian priors"; see Berger (1985, ch. 3) for a review and Walley (1991, ch. 5) for an extended critique of non-informative priors.
21. For the terminology, see Section 5.3.
22. Two remarks on the technical definition of WAREP:
    1. One might consider defining range-equivalence alternatively by: "$\forall \omega \in \Omega : \mathrm{proj}_\omega X = \mathrm{proj}_\omega X'$ ". However, this would make the choice rule highly dependent on the addition or deletion of strictly dominated acts. The present formulation avoids this, implying the condition " $\mathcal{A}(X) = \mathcal{A}(X') \Rightarrow C(X) = C(X') \quad \forall X, X'$ ".
    2. It would be preferable to specify range-equivalence without using the topological concept of closure, i.e., as "$\forall \omega \in \Omega$: $\mathrm{proj}_\omega\ \mathcal{A}(X) = \mathrm{proj}_\omega\ \mathcal{A}(X')$". This is not possible in general, since compactness of X fails to imply that of $\mathcal{A}(X)$ in more than two dimensions (see Arrow et al. 1953). Compactness of $\mathcal{A}(X)$ is guaranteed, on the other hand, if X is a polyhedron.
23. The two theorems are also the first in the literature that make Symmetry and *strict* Admissibility compatible without an *ad-hoc* qualification of the axioms. The problem of their apparent incompatibility has in fact been (at least implicitly) a major issue of the CI-literature in the 1980s. Maskin (1979) imposes an *ad-hoc* restriction on the applicability of "Column Duplication", Barbera-Jackson (1988) in effect restrict the requirement of preference completeness, and Cohen-Jaffray (1980, 1983) demand only "approximate satisfaction" of certain conditions.
24. I thank Louis Makowski for articulating a skepticism along this line.
25. This is the property of "negative introspection" in the language of epistemic logic. Complete ignorance in the sense of this paper has therefore nothing to do with "unawareness" in the sense of the recent literature on that topic, for

which violation of negative introspection is deemed essential (cf. Modica–Rustichini 1994, Dekel–Lipman–Rustichini 1998.

26. For a good exposition of the distinction between an exhaustive and a partial eliciation interpretation, see Walley (1991, Section 2.10).
27. Elster also supports the "Solomonic" use of randomization in situations of ignorance.
28. From the point of view of a bargaing theoretic interpretation, the example shows the relevance of both the imputed ideal and disagreement points.
29. Indeed, it assumes perhaps too much context-independence. A/(the?) "perfectly rational" theory will probably need to replace WAREP by some subtler set of conditions, presumably at the price of substantially increased complexity.
30. The technical details are omitted; generalizing the two-state case presented in Section 2, it should suffice to think of the partial orders R as intersections (unanimity relations) of sets of expected-utility orders $R_{\{\pi\}}$, with π denoting a probability measure on Ω and
$$x \; R_{\{\pi\}} \; y \iff \int x_\omega d\pi \geqslant \int y_\omega d\pi .$$
Such classes can be axiomatized along the lines of standard representation theorems in the literature; see Smith (1961), Bewley (1986) and in great generality Walley (1991), as well as Nehring (1995) for a statement directly appropriate to SIMEU theory.
31. We say "of the following *type*", since the domain of the choice-function $C(X, R)$ has not been formally defined.
32. The motivation for imposing this additional structure is expositional and technical: Symmetry and CISO are independently interpretable, and they are what matters mathematically. Conceptually, the formulation of a general, integrated "invariance under preference-isomorphism" seems desirable and non-trivial and is left to future research.
33. This property is unique to $R_\emptyset$ and reflects the extreme richness in symmetries of $R_\emptyset$ which makes EISO/Symmetry so powerful.
34. Since justified acceptance of CISO relies on the PPB, it must be accompanied by acceptance of EISO. Thus, the class of bargaining solutions that make sense in the present context is severely restricted; in particular, EISO implies that the solution cannot depend on the number of players with identical preferences, as for instance adaptations of the Nash solution would imply. When WAREP is assumed in addition, the lexicographic Kalai–Smorodinsky solution is already uniquely singled out.
35. See Nehring (1992), for a brief published statement, and Nehring (1991), ch. 2 for a more extensive discussion; it is also effectively shown there (in a slightly different setting) that a choice rule defined on the class of CI problems has a CIR extension if and only if it satisfies EISO.
36. We note that CIR strengthens the case for CISO. Specifically, it is shown in Nehring (1991, ch. 1), that in the presence of CIR, CISO is equivalent to condition STP ("sure-thing principle") which determines for a simple class of

decision problems how choices respond to the "conditioning" of preferences that results from a partial resolution of the uncertainty.

37. For example, it has been shown in Nehring (1999) that Choquet Expected Utility maximization with convex capacities (which includes the class of maxmin-preference orderings based on belief-functions) imposes severe restrictions on the familiy of "unambiguous events" (those for which the capacity of an event and its complement add up to one).
38. In line with this conclusion, Complete Ignorance has been defined here in terms of the preference relation, without reference to an informal epistemic notion of "total absence of information". This contrasts both with the classical literature on Complete Ignorance and with more recent viewpoints such as Walley's (1996, p. 4).

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