Econ 100
Homework 6

1) Suppose that a honey farm is located next to an apple orchard. The farm and the orchard are both competitive firms. Let the amount of apples produced be A, and the amount of honey produced be H. The honey farm's cost function is $C=H^2/100$ and the apple farm's cost function is $C=A^2/100-H$. In other words, as more honey is produced, the cost of producing apples falls. The price of apples is \$3 and the price of honey is \$2.

a) If the firms each operate independently, how much honey and how many apples will be produced?

Orchard $\max \pi = 3A - A^2/100 + H \Rightarrow \frac{\partial \pi}{\partial A} = 3 - \frac{2A}{100} = 0 \Rightarrow$ $\boxed{A = 150}$

Farm $\max \pi = 2H - \frac{H^2}{100} \Rightarrow \partial \pi / \partial H = 2 - \frac{2H}{100} = 0 \Rightarrow$ $\boxed{H = 100}$

Each firm max its own profits by choosing to produce the π max amount of good. It cannot influence how much the other firm produces.

b) Suppose that the honey and apple firms merged. What would be the profit maximizing output for honey from the combined firm?

$$\max \ 2H + 3A - \frac{H^2}{100} - \frac{A^2}{100} + H$$

Now, choose both H & A so find 2 FOC's:

$$\frac{\partial \pi}{\partial A} = 3 - \frac{A}{50} = 0$$
$$\frac{\partial \pi}{\partial H} = 2 - \frac{H}{50} + 1 = 0$$

$\boxed{A = 150, \ H = 150}$

c) What is the socially efficient amount of honey that should be produced?

H = 150 since that amount max total π.

d) If the firms stayed separate, how much would honey production have to be subsized in order to induce an efficient supply?

To produce the socially efficient amount of honey, the honey producer needs to get \$3 per unit of honey ∴ he needs \$1 per unit subsidy

2. Cathy and Healthcliff are a couple. Each has a little habit that annoys the other. Heathcliff's habit is called activity X and Cathy's habit is called activity Y. Cathy and Heathcliff have \$1,000,000 to spend per year. Heathcliff's utility function is $U=C_H+500\ln(x)-10y$, where C_H is the amount of money he spends on goods other than X. Cathy's utility function is $U=C_C+500\ln(y)-10x$, where C_C is the amount of money she spends per year on goods other than Y. Activity X costs \$20 per unit. Activity Y costs \$100 per unit.

a) Suppose that Heathcliff has a right to half of their joint income and Cathy has a right to the other half of their income. If they each pursue their own activity independently, how much X will Heathcliff consume and how much Y will Cathy consume? (Remember that the derivative of lnX is 1/x).

Heathcliff max $C_h + 500\ln(X) - 10y$ given that $C_h = 500{,}000 - 20x$

$$\max\ 500{,}000 - 20x + 500\ln x - 10y$$

$$\frac{\partial U}{\partial x} = -20 + \frac{500}{x} = 0 \qquad \boxed{x = \frac{500}{20} = 25}$$

Cathy max $C_c + 500\ln y - 10x$ where $C_c = 500{,}000 - 100y$

$$\max\ 500{,}000 - 100y + 500\ln y - 10x$$

$$\partial U/\partial y = -100 + 500/y = 0 \qquad \boxed{y = 5}$$

b) Given Cathy and Heathcliff's individual utility functions, write down the family utility function, which is the sum of their individual utility functions.

$$C_h + C_c + 500\ln x + 500\ln y - 10y - 10x$$

c) Now, $C_H+C_C=1{,}000{,}000-20x-100y$. Substitute this into the equation above, and find the values of x and y that would be chosen if Cathy and Heathcliff maximized their family utility function.

$$U = 1{,}000{,}000 - 20x - 100y + 500\ln x + 500\ln y - 10y - 10x$$

$$\partial U/\partial x = -20 + 500/x - 10 = 0 \qquad \boxed{x = 50/3}$$

$$\partial U/\partial y = -100 + 500/y - 10 = 0 \qquad \boxed{y = 50/11}$$

3) A clothing store and a jewelry store are located side by side in a small shopping mall. The number of customers who come to the shopping mall intending to shop at either store depends on the amount of money that the store spends on advertising per day. Each store also attracts some customers who have come to shop at the neighboring store. If the clothing store spends \$C per day on advertising and the jeweler spends \$J per day on advertising, then the total profits per day of the clothing store are $(60+J)C-2C^2$. The total profits per day of the jewelry store are $(105+C)J-2J^2$.

a) If each store acts independently, then we can find the equilibrium amount of advertising for each store by solving two equations in two unknowns. To get started, if each store maximizes its own profit function, what will be the two first order equations?

FOC for jewelry store: $105 + C - 4J = 0$

FOC for clothing store: $60 + J - 4C = 0$

$$J = 4C - 60$$

$$C = 4(4C-60) - 105$$

$$C = 16C - 240 - 105$$

$$15C = 345$$

b) Now, solve these two equations for C and J.

$$\boxed{C = 23 \quad J = 32}$$

c) How much extra profit does the jeweler get if the clothing store spends another dollar on advertising? How much extra profit does the clothing store get if the jeweler spends another dollar on advertising?

π for jeweler $= 105J + CJ - 2J^2$

to find how much $\pi\Delta$ when $C\Delta$'s find $\frac{\partial\pi}{\partial C} = J$

at π max $J = 32$, so π increase by 32 when C changes by 1

π for clothier $= 60C + JC - 2C^2$

$\frac{\partial\pi}{\partial J} = C$, $C = 23$ so $\pi\uparrow$ by 23 when $J\uparrow$ by 1

d) Suppose that the clothing store and jewelry store are owned by the same person, but have the same profit functions as before. The new owner wants to maximize TOTAL profits, which is the sum of the profits from the two stores. Given this objective, how much C and J will the new owner choose?

$$\pi = 60C + JC - 2C^2 + 105J + CJ - 2J^2$$

$$\frac{\partial\pi}{\partial C} = 60 + 2J - 4C = 0 \qquad J = \frac{4C - 60}{2}$$

$$\frac{\partial\pi}{\partial J} = 105 + 2C - 4J = 0$$

$$105 + 2C - 2(4C - 60) = 0$$

$$105 + 2C - 8C + 120 = 0$$

$$105 - 6C + 120 = 0$$

$$\frac{225}{6} = C \qquad \boxed{\begin{array}{c} C = 37 \\ J = 44 \end{array}}$$

e) Will total profits be higher or lower than when each store made decisions independently? By how much? π when combined $= 60(37) + 37 \cdot 44 - 2(37)^2 + 105(44) + 37 \cdot 44 - 2(44$

$= 3486$

total π when seperate $= 60(23) + 23 \cdot 32 - 2(23)^2 + 105(32) + 23 \cdot 32 - 2(32)^2$

$= 3106$

otal π ve higher when combined, by 3486 - 3106 = 380

4) There are 100 families living around the town square. Each family has two neighbors, one on each side. In this neighborhood, nobody mows their lawn. Instead, everyone enjoys long grass. Each family has a grass lawn in their front yard, in full view of their neighbors. Curiously, nobody cares about the height of the grass growing in the yard to their right, but every family is jealous of the nice tall grass that is growing in the yard to their left. Every family has a utility function $U=C-L^2$, where C is the length of grass grown by the family, and L is the length of grass grown by the family on the left. Suppose each family grows grass that is 1" long.

a) Calculate each family's utility level.

$$U = 1 - 1 = 0$$

b) Suppose that each consumer grows only ¾ inch grass. Will all families be better or worse off?

$U = 3/4 - 9/16 = 3/16$ better off

c) If everybody consumes the same amount, what is the optimal amount for society?

If everyone consumes the same amount, then $c = L$.

$\max U = C - C^2$

FOC: $1 - 2C = 0$

$c = 1/2$

d) Suppose that everybody around the square is growing 1 inch grass. Can any two people make themselves better off by either redistributing between them, or choosing not to have a lawn at all?

$\max\ U = L - 1 \longrightarrow$ this person always ↑ U by growing as tall a lawn as possible.

$\max\ U = C - L^2$

So, $\boxed{\text{NO}}$

e) How about a group of three people?

No, same problem as above.