

Entry, Multinational Firms and Exchange Rate Volatility*

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August 2006

Abstract

Recent discussions of exchange rate determination have emphasized the possible role of foreign direct investment in influencing exchange rate behavior. Yet, there are few existing models of multinational enterprises (MNEs) and endogenous exchange rates. This paper demonstrates that the entry decisions of MNEs can influence the volatility of the real exchange rate in countries where there are significant costs involved in maintaining production facilities, even when prices are perfectly flexible. For empirically plausible parameters, MNE activity can make the exchange rate much more volatile than relative consumption.

*The authors thank without implication Joshua Aizenman, Thomas Bishop, Doireann Fitzgerald, Bang Jeon, Andrew Rose, Galina Schwartz, and Juanyi Xu for helpful discussions and participants in the 2005 Western Economics Association International and 2006 Asia-Pacific Economics Association meetings. Any opinions expressed are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

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1 Introduction

Recent discussions of exchange rate determination have increasingly emphasized the possible role of foreign direct investment (FDI) in influencing exchange rate behavior. Yet, there are few existing models of multinational enterprises (MNEs) and endogenous exchange rates. This paper demonstrates that the entry decisions of MNEs influence the volatility of the real exchange rate in countries where there are significant costs involved in maintaining production facilities, even when prices are perfectly flexible. For plausible parameterizations, MNE activity can make the exchange rate more volatile than relative consumption.

In this paper we draw on three different strands of international macroeconomics and trade: (i) the role of returns on foreign direct investment and other assets in determining the exchange rate; (ii) recent work on the behavior of heterogeneous exporters and MNEs; and (iii) studies of the exchange-rate disconnect puzzle. In what follows, we discuss these approaches in turn to highlight how they motivate and inform our approach.

It is well known that the volatility of the exchange rate is much higher than that of other macroeconomic variables, such as the aggregate price level and consumption. This produces a fundamental challenge for optimization-based open economy models that link marginal rates of substitution to international goods prices. For instance, Baxter and Stockman (1989) and Flood and Rose (1995) point out that nominal and real exchange rate volatility is typically ten times higher than the volatility of relative prices and several times greater than the volatility of output or consumption. As demonstrated by Backus, Kehoe, and Kydland (1992), standard open economy business cycle models have difficulty replicating these stylized facts unless implausible substitution elasticities are assumed. This is due to the tight link between marginal rates of substitution and international relative prices that are at the heart of optimization-based frameworks.

The exchange rate volatility puzzle is also related to, in the nomenclature of Rogoff (1996), the exchange rate disconnect puzzle. It stipulates that empirically exchange rates appear to behave virtually independently of underlying economic fundamentals. Consequently, the ability of modern open economy macroeconomics to explain exchange rate movements has not been an unqualified success.¹ We address this issue by approaching exchange rate determination not from the goods side, but rather from a perspective of financial flows generated by the operations of MNEs. This removes the burden of having relative quantities match the volatility of relative prices.

Our argument – that aggregate consumption and prices appear to be much less volatile than the exchange rate because their movement is dampened by the entry of less productive

¹The seminal paper in this literature is Meese and Rogoff (1985). Different perspectives on this issue are given by Clarida and Gali (1994) in a VAR framework, and Lubik and Schorfheide (2005) in an estimated DSGE model.

firms – is akin to a new vein of literature on the exchange rate disconnect puzzle emphasizing the role of transaction costs in trade. Fitzgerald (2005) shows both theoretically and empirically that trade costs based on the geographic distance between countries can explain why relative price levels are much less volatile than the real exchange rate, even when prices are perfectly flexible. Our paper abstracts from trade in goods, all local consumption being produced by either domestic firms or resident branches of MNEs. It nonetheless approaches the disconnect puzzle in a similar spirit, asking not why nominal and real exchange rates are so volatile, but why they appear so volatile relative to consumption and relative price levels. The excess volatility of the nominal exchange rate in this paper, reaching an upper bound of 44 times that of consumption, can not attain the levels produced by recent models of noise trading such as Xu (2005). This study simply brings to light a different factor – entry by heterogeneous firms – which may generate some portion of the observed disconnect without sticky wages or prices.

In order to highlight the entry channel for exchange rate determination and to derive (almost) closed-form solutions, we make two simplifying assumptions. First, we segment markets by allowing no cross-border transfers of wealth via portfolio investment and we shut down any real trade linkages, except for those involving the production and remittance activities of multinational firms. These assumptions leave the nominal exchange rate completely determined by flows of currency involved in paying local costs of production incurred by overseas branches of MNEs and repatriating their profits earned abroad. Foreign direct investment, even in this model without sunk costs or physical capital, is the key driver of real and nominal exchange rate movements. In this sense, the model draws on empirical work by Gourinchas and Rey (2005), who uncover the interaction between returns on net foreign assets, including foreign direct investment, and exchange rate behavior, as well as on Dooley, Folkerts-Landau, and Garber (2004), who stress the potential role of foreign direct investment as a factor influencing exchange rates.

Second, firms' technology is characterized by heterogeneous labor productivity levels, which influences the relative volatility of exchange rates, the aggregate price level, and consumption arising in response to country-specific productivity shocks. A positive country-specific productivity shock allows both native and foreign-owned firms with lower firm-specific levels of productivity to become profitable market participants. Lower idiosyncratic labor productivity in these new entrants dampens the impact of the country-specific shock on total aggregate productivity. Thus, a positive productivity shock can impact the nominal exchange rate at the same time entry by progressively less productive firms dampens the effect of the productivity shock on the aggregate price level and consumption.

This feature places the model in a new category of work uniting advances in trade theory involving firm heterogeneity by Bernard, Eaton, Jensen, and Kortum (2003) and Melitz

(2003) with models of international finance. It does not involve the sunk costs or incomplete asset markets that generate, respectively, endogenous persistence in exchange rate behavior and a role for active monetary policy in the study of heterogeneous exporters and exchange rates as in Ghironi and Melitz (2005). However, it is rich enough to demonstrate that production decisions by multinational firms can explain part of the differential in the variance of exchange rates and other macroeconomic variables without nominal rigidities.

The rest of the paper considers the role that MNEs might play in explaining the disconnect puzzle. It begins by briefly discussing current understanding about the relationship between MNEs and exchange rate variability, as well as the importance of heterogeneity amongst MNEs. Then, a simple, stylized model of multinational production is introduced in Section 3, emphasizing the role of entry in determining the aggregate productivity level and the number of different goods available in the economy. The impacts of a shock to Home technology on both nominal and real exchange rates, as well as on consumption and the ratio of the Home and Foreign price level, are decomposed analytically and mapped numerically in Section 4, followed by conclusions and suggestions for future research.

2 Exchange Rate Volatility and Foreign Direct Investment

The literature analyzing the relationship between the exchange rate and FDI is diverse and at times conflicted, with no clear conclusion as to whether volatility will increase or decrease FDI. Russ (2005) reconciles the conflicting empirical findings by demonstrating that if exchange rates and demand for goods produced by MNEs are linked to common macroeconomic variables, then the relationship between FDI and exchange rate volatility will depend on the source of the volatility. However, she stops short of considering the sort of endogeneity that would arise if flows of FDI directly influenced the supply and demand for currency in foreign exchange markets.

Several papers indicate that investment activity by multinational firms and the returns on this investment, in fact, *are* important in understanding exchange rate behavior and should be integrated into models of the open economy. Aizenman's (1992 and 1994) groundbreaking portrayals of multinational firms with sticky wages depict a fully endogenous exchange rate that is impacted by supply and demand for multinational firms' production, with crucial implications for the optimal choice of exchange rate regime. Kosteletoú and Liargovas (2000) provide empirical evidence that inflows of FDI Granger-cause fluctuations in the real exchange rate for some European countries (Denmark, France, Greece, Portugal, and Spain). Whether FDI generates appreciating or depreciating tendencies varies by country, a disparity that the authors explain as emerging from each country's use of the inflows to finance either consumption or capital accumulation. Shrikhande (2002) builds a

theoretical model that allows for cross-border acquisitions of physical capital. He is able to replicate the observed persistence and time-varying volatility in the real exchange rate using sunk costs in investment.

More recent work by Dooley, Folkerts-Landau, and Garber (2004) suggests that FDI can be an important determinant of exchange rate behavior insofar as it may act as the source country's collateral when borrowing from foreigners. Gourinchas and Rey (2005) find empirical evidence of a recursive relationship between exchange rates and the return on net foreign asset holdings, including foreign direct investment. Cavallari (2005) argues using a representative-firm framework that exchange rate overshooting may be generated by repatriated profits from multinational firms exploiting a positive productivity shock overseas. The model below draws its motivation from this growing body of work stressing the potential role of MNEs as one factor driving exchange rate fluctuations, but adds the additional consideration that entry by heterogeneous firms may dampen fluctuations in prices and consumption, making the exchange rate look more volatile in comparison.

There are important conceptual, empirical, and purely practical reasons for modeling multinational firms characterized by heterogeneous productivity levels. First, it is difficult to explain why some firms – but not all – establish branches abroad, unless there exists some differential in their potential to make a (nonnegative) profit, which occurs when firms have differing labor productivity. Second, there are several stylized facts regarding micro characteristics of MNEs that conflict with the representative firm assumption. Using an extensive dataset that joins figures on firm size and employment with intra- and inter-firm trade data, Bernard, Redding, and Schott (2005) show that multinational firms are larger in size and have greater revenues per worker than firms that do not show evidence of having overseas affiliates. Modeling firm-specific labor productivity as Pareto-distributed generates a pattern of firm sizes that is also Pareto, which reconciles with empirical findings by Helpman, Melitz, and Yeaple (2003). These stylized facts on firm size and distribution are captured by the heterogeneous framework below.

Finally, introducing heterogeneity in the tradition of Melitz (2003) causes the entire solution of the model to rest only on the lowest productivity level among firms producing in a particular period and a set of exogenous parameters. Pinpointing this threshold productivity level using a zero-cutoff profit condition allows the entire model to be solved numerically without linearization and yields analytical results depicting the influence of shocks to a country's general technological state on the nominal and real exchange rate.

3 A Simple Model of Entry and FDI

3.1 The Consumer's Problem

The representative consumer in the Home country maximizes lifetime utility:

$$\max E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \right], \quad (1)$$

subject to the budget constraint:

$$P_t C_t + M_t = W_t L_t + M_{t-1} + \pi_t + T_t, \quad (2)$$

and the cash-in-advance constraint:

$$P_t C_t \leq M_t. \quad (3)$$

C_t is aggregate consumption, L_t is labor input, M_t is the money stock; W_t is the nominal wage, π_t are firm profits accruing to the household, and T_t are transfer payments from the government; P_t is the aggregate price index, which we define below.

We assume that the period utility function is additively separable, $U(C_t, L_t) = \frac{C_t^\rho - 1}{1 - \rho} - \kappa L_t$, where $\rho > 0$, $\kappa > 0$, and that the consumption aggregator is defined as:

$$C_t = \left[\int_0^{n_{h,t}} c_{h,t}(i)^{\frac{\theta-1}{\theta}} di + \int_1^{n_{f,t}} c_{f,t}(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad (4)$$

with $\theta > 1$. The interval $[0, n_{h,t})$ represents the continuum of all goods $c_{h,t}(i)$ that can *possibly* be produced by Home-owned firms for the Home market, and the interval $[1, n_{f,t}]$ represents the continuum of all goods that can be produced by Foreign-owned firms, $c_{f,t}(i)$, for the Home market ($n_{h,t}, n_{f,t} \leq 1$). Furthermore, we assume that the cash-in-advance constraint always binds. This determines aggregate consumption as a function of real money balances, $C_t = \frac{M_t}{P_t}$.

First-order conditions yield a wage relation:

$$W_t = \kappa P_t C_t^\rho, \quad (5)$$

and demand equations for individual goods produced by Home and Foreign firms that are downward sloping in prices. Homothetic preferences imply that the demand for each good

is a constant proportion of aggregate consumption:

$$\begin{aligned} c_{h,t}(i) &= \left(\frac{p_{h,t}(i)}{P_t} \right)^{-\theta} C_t \\ c_{f,t}(i) &= \left(\frac{p_{f,t}(i)}{P_t} \right)^{-\theta} C_t. \end{aligned} \tag{6}$$

3.2 The Firm's Problem

In the Home and Foreign country, there is a continuum of entrepreneurs with plans to put their particular invention into production. Each firm which decides to enter the market during period t produces a unique good and has a firm-specific productivity level, φ . This idiosyncratic component is distinct from the time-varying disturbance A_t , which denotes the country-specific state of technology available to all firms operating in the Home country. Technology is thus characterized by:

$$c_{h,t}(i) = A_t \varphi(i) l_{h,t}(i), \tag{7}$$

where $l_{h,t}(i)$ is the amount of labor used by Home firm i for production in the Home country. The country-specific productivity parameter for the Home country, A_t , is defined by

$$A_t = 1 + \varepsilon_t,$$

where $\varepsilon_t = \phi \varepsilon_{t-1} + v_t$, with $v_t \sim N(0, \sigma_\varepsilon^2)$.

There is also a distribution of idiosyncratic productivity levels, $g(\varphi)$, with support over the interval $(0, \infty)$. Any difference among the pricing rules and production decisions of firms operating in the Home country is due only to differences in φ . It is assumed that $g(\varphi)$ is a continuous distribution, so that the probability of two firms drawing the same productivity level is zero and each firm will have a unique level of labor productivity. Thus, φ is henceforth used to index each good and the firm which produces it, instead of the general subscript i .² Home firms operating in the Home country maximize profits subject to consumer demand. They also bear a fixed overhead cost of production, f_h , denominated in units of output. The Home firm operating in the Home country sets prices to maximize profits:

$$\max_{p_{h,t}(\varphi)} p_{h,t}(\varphi) c_{h,t}(\varphi) - W_t \frac{c_{h,t}(\varphi)}{\varphi A_t} - P_t f_h. \tag{8}$$

²The continuous distribution is used because it allows this notational convenience. The modeling can also be done with a discrete distribution and yield the same results.

The first-order condition for profit maximization is then:

$$\frac{\partial \pi_{h,t}(\varphi)}{\partial p_{h,t}(\varphi)} : c_{h,t}(\varphi) + \frac{\partial c_{h,t}(\varphi)}{\partial p_{h,t}(\varphi)} p_{h,t}(\varphi) - \frac{\partial c_{h,t}(\varphi)}{\partial p_{h,t}(\varphi)} \frac{W_t}{\varphi A_t} = 0. \quad (9)$$

The pricing rule can be derived by substituting the derivative of the demand equation into the firm's first-order condition:

$$p_{h,t}(\varphi) = \left(\frac{\theta}{\theta - 1} \right) \frac{W_t}{\varphi A_t},$$

i.e. firms set prices as a markup over marginal costs. The same process can be used to derive the pricing equation for Foreign-owned firms operating in the Home country:

$$\max_{p_{f,t}(\varphi)} \left(\frac{1}{S_t} \right) (p_{f,t}(\varphi) c_{f,t}(\varphi) - W_t l_{f,t}(\varphi) - P_t f_f).$$

Here, S_t is the nominal exchange rate at time t , measured in units of Home currency per unit of Foreign currency. The term f_f denotes the fixed cost paid by Foreign-owned firms operating in the Home currency, which may or may not be equal to the fixed cost paid by Home-owned firms, f_h . The fixed cost is paid in the local currency of the host country – in this case, the Home currency. It can be thought of as an overhead cost, or more abstractly as the cost of capital with 100 percent depreciation. The pricing rule for Foreign goods produced and sold in the Home currency turns out to be identical, since firms face the same Home-country wage and are influenced by the same country-specific productivity shocks:

$$\frac{\partial \pi_{f,t}(\varphi)}{\partial p_{f,t}(\varphi)} : \left(\frac{1}{S_t} \right) c_{f,t}(\varphi) + \left(\frac{1}{S_t} \right) \frac{\partial c_{f,t}(\varphi)}{\partial p_{f,t}(\varphi)} p_{f,t}(\varphi) - \left(\frac{1}{S_t} \right) \frac{\partial c_{f,t}(\varphi)}{\partial p_{f,t}(\varphi)} \frac{W_t}{\varphi A_t} = 0, \quad (10)$$

from which follows immediately that:

$$p_{f,t}(\varphi) = \left(\frac{\theta}{\theta - 1} \right) \frac{W_t}{\varphi A_t}.$$

More productive firms – those having a high level of labor productivity φ – will charge lower prices, sell more units, and earn higher revenues and profits.

There is a continuum of prospective Home and Foreign entrepreneurs distributed over $[0, 1)$ and $[1, 2]$, respectively, but only firms which can expect to be sufficiently productive to recoup the overhead cost will choose to produce in a particular period. Any firm may enter, depending whether its total productivity, $A_t \varphi$, is high enough to result in revenues sufficient to cover the fixed cost. Let $\eta_{h,t}(\varphi)$ and $\eta_{f,t}(\varphi)$ be the distribution of firm-specific productivity levels observed among these active Home- and Foreign-owned firms. Then the

aggregate price level, P_t , is given by:³

$$P_t = \left[n_{h,t} \int_0^\infty p_{h,t}(\varphi)^{1-\theta} \eta_{h,t}(\varphi) d\varphi + n_{f,t} \int_0^\infty p_{f,t}(\varphi)^{1-\theta} \eta_{f,t}(\varphi) d\varphi \right]^{\frac{1}{1-\theta}}. \quad (11)$$

Substituting the wage relation and pricing rules for individual goods, the expression reduces to:

$$\begin{aligned} P_t &= \left(\frac{\theta}{\theta-1} \right) \frac{W_t}{A_t} \left[n_{h,t} \int_0^\infty \varphi^{\theta-1} \eta_{h,t}(\varphi) d\varphi + n_{f,t} \int_0^\infty \varphi^{\theta-1} \eta_{f,t}(\varphi) d\varphi \right]^{\frac{1}{1-\theta}} \\ &= \left(\frac{\kappa N_t^{\frac{1}{1-\theta}}}{\alpha \bar{\varphi}_t A_t} \right)^{\frac{1}{\rho}} M_t, \end{aligned} \quad (12)$$

where $\alpha = \frac{\theta}{\theta-1}$, the gross markup; $N_t = n_{h,t} + n_{f,t}$, the composite continuum of goods available in the Home economy; and $\bar{\varphi}_t$, the production-weighted average firm-specific level of labor productivity, defined by:⁴

$$\bar{\varphi}_t = \left[\frac{n_{h,t}}{N_t} \bar{\varphi}_{h,t}^{\theta-1} + \frac{n_{f,t}}{N_t} \bar{\varphi}_{f,t}^{\theta-1} \right]^{\frac{1}{\theta-1}}. \quad (13)$$

3.2.1 The Zero-Cutoff Profit Condition

The lowest productivity level, $\hat{\varphi}$, that allows a firm to enter into production expecting nonnegative profits can be described using the Zero-Cutoff Profit (ZCP) condition. The ZCPs for Home- and Foreign-owned firms operating in the Home country are given by:

$$\pi_{h,t}(\hat{\varphi}_{h,t}) = p_{h,t}(\hat{\varphi}_{h,t}) c_{h,t}(\hat{\varphi}_{h,t}) - W_t l_{h,t}(\hat{\varphi}_{h,t}) - P_t f_h = 0 \quad (14)$$

and

$$\pi_{f,t}(\hat{\varphi}_{f,t}) = (p_{f,t}(\hat{\varphi}_{f,t}) c_{f,t}(\hat{\varphi}_{f,t}) - W_t l_{f,t}(\hat{\varphi}_{f,t}) - P_t f_f) = 0, \quad (15)$$

respectively. Analogous expressions apply to entry in the Foreign market.

The ZCP conditions governing entry into the Home market reduce to functions of the

³See Melitz (2003) and Russ (2006), Section 2.4, for a discussion of the computation of the aggregate price level and average firm-specific level of labor productivity.

⁴The average firm-specific productivity level for firms owned by country j is defined by $\bar{\varphi}_{j,t} = \left[\int_0^\infty \varphi^{\theta-1} \eta_{j,t}(\varphi) d\varphi \right]^{\frac{1}{\theta-1}}$.

underlying parameters and the threshold productivity levels, $\hat{\varphi}_{h,t}$ and $\hat{\varphi}_{f,t}$:

$$\begin{aligned}\hat{\varphi}_{h,t} &= \left[f_h(\theta - 1) \left(\frac{\theta}{\theta - 1} \right)^\theta \left(\frac{\kappa}{A_t} \right)^{\frac{1}{\rho}} \left(\frac{\bar{\varphi}_t}{N_t} \right)^{\frac{\rho(\theta-1)-1}{\rho}} \right]^{\frac{1}{\theta-1}}, \\ \hat{\varphi}_{f,t} &= \left[f_f(\theta - 1) \left(\frac{\theta}{\theta - 1} \right)^\theta \left(\frac{\kappa}{A_t} \right)^{\frac{1}{\rho}} \left(\frac{\bar{\varphi}_t}{N_t} \right)^{\frac{\rho(\theta-1)-1}{\rho}} \right]^{\frac{1}{\theta-1}}.\end{aligned}\tag{16}$$

Once $g(\varphi)$ is specified, Eq. (16) is sufficient to pinpoint the minimum level of labor productivity for Home firms entering the Home market. Note that the level of the Home (and Foreign) money supply, M , does not appear in these equations. The money supply is completely neutral, as is common in flexible-price frameworks. It does not affect the threshold level of productivity and therefore bears no influence on the entry behavior of firms. It is used solely to specify the size of the market – necessary to discern the number of firms that can enter – by limiting the maximum level of aggregate nominal expenditure. Furthermore, the difference between the threshold productivity levels for Home- and Foreign-owned firms depends only on the ratio of the fixed costs they pay to produce in the Home market:

$$\hat{\varphi}_{f,t} = \left(\frac{f_f}{f_h} \right)^{\frac{1}{\theta-1}} \hat{\varphi}_{h,t}.$$

As described in Helpman, Melitz, and Yeaple (2003) and Russ (2006), the equilibrium distribution of firm-specific productivity levels for firms owned by country $j \in [h, f]$ can now be characterized as truncated, so that firms with productivity levels too low to earn at least zero profits do not produce in period t .⁵ These low-productivity firms are plucked from the formulation of the aggregate price and output levels, leaving a truncated equilibrium distribution:

$$\eta_{j,t}(\varphi) = \left\{ \begin{array}{l} 0 \text{ for } \varphi < \hat{\varphi}_{j,t} \\ \frac{g(\varphi)}{1-G(\hat{\varphi}_{j,t})} \end{array} \right\}.$$

The average productivity levels in the Home- and Foreign-owned sector of both economies—and therefore the aggregate productivity and price levels, as well—are all functions of the cutoff productivity levels. Average productivity for j -owned firms operating in the Home market can now be written as

$$\bar{\varphi}_{j,t} = \left[\frac{1}{1-G(\hat{\varphi}_{j,t})} \int_{\hat{\varphi}_{j,t}}^{\infty} \varphi^{\theta-1} g(\varphi) d\varphi \right]^{\frac{1}{\theta-1}}$$

⁵See Appendix for proof of existence of $\hat{\varphi}_{j,t}$.

Similar expressions emerge from the ZCP conditions governing entry in the Foreign market:

$$\begin{aligned}\hat{\varphi}_{h,t}^* &= \left[f_h^*(\theta - 1) \left(\frac{\theta}{\theta - 1} \right)^\theta \left(\frac{\kappa}{A_t^*} \right)^{\frac{1}{\rho}} \left(\frac{\bar{\varphi}_t^*}{N_t^*} \right)^{\frac{\rho(\theta-1)-1}{\rho}} \right]^{\frac{1}{(\theta-1)}} \\ \hat{\varphi}_{f,t}^* &= \left[f_f^*(\theta - 1) \left(\frac{\theta}{\theta - 1} \right)^\theta \left(\frac{\kappa}{A_t^*} \right)^{\frac{1}{\rho}} \left(\frac{\bar{\varphi}_t^*}{N_t^*} \right)^{\frac{\rho(\theta-1)-1}{\rho}} \right]^{\frac{1}{(\theta-1)}}\end{aligned}\tag{17}$$

It is assumed that the fixed cost involved in production abroad is sufficiently large that a firm producing abroad will always produce in its native country, as well ($\hat{\varphi}_{f,t}^* \leq \hat{\varphi}_{f,t}$). Thus, this benchmark model does not capture issues of geographic preference in firm location.

3.2.2 The Number and Size of Firms

It is important to stress that the threshold productivity levels directly determine the proportion of prospective Home and Foreign entrants who actually undertake production in the Home and Foreign markets. This proportion, denoted $n_{j,t}$ for firms owned by residents of country j who enter the Home market ($j \in [f, h]$),⁶ is simply the probability that a firm holds an idiosyncratic productivity parameter greater than $\hat{\varphi}_{j,t}$. Specifically, $n_{j,t} = 1 - G(\hat{\varphi}_{j,t})$. As $\hat{\varphi}_{f,t}$ increases, for instance, the proportion of Foreign-owned firms entering the Home market falls. Such an increase means that a Foreign firm must have a greater idiosyncratic level of labor productivity to expect to enter without incurring a loss. Implicit differentiation of Eq. (16) (see the Appendix for explicit derivation) demonstrates that $\frac{\partial \hat{\varphi}_{j,t}}{\partial A_t} < 0$: an increase in Home's aggregate technology level makes it easier for both Home- and Foreign-owned firms to produce profitably for the Home market. More firms will enter when A_t rises – both n_h and n_f will increase, as will the variety of goods available in the Home market. If the size of the underlying pool of prospective entrants remains constant, then the proportion of entrants can be interpreted loosely as the number of firms and varieties in the market.⁷

⁶The proportion is denoted n_j^* for those that enter the Foreign market.

⁷The size of this underlying pool is not specified in this model, nor do we consider the possibility that its size may change in response to changes in technology, population, tax policy etc.

4 Productivity Shocks, Entry, and the Exchange Rate

4.1 Some Analytical Results

The foreign exchange market equilibrium requires that the number of units of Home currency being offered for exchange by overseas branches of Foreign multinationals repatriating their profits equal the number of units of Home currency demanded by overseas branches of Home multinationals repatriating their own profits. This condition is the multinational analog to the condition for a world with exporters described in Bacchetta and Van Wincoop (2000):⁸

$$S_t n_{h,t}^* \pi_{h,t}^* (\bar{\varphi}_{h,t}^*) = n_{f,t} \pi_{f,t} (\bar{\varphi}_{f,t}).$$

Rearranging, one obtains an expression for the nominal exchange rate.

$$S_t = \frac{n_{f,t} \pi_{f,t} (\bar{\varphi}_{f,t})}{n_{h,t}^* \pi_{h,t}^* (\bar{\varphi}_{h,t}^*)}. \quad (18)$$

Equation (18) shows that the nominal exchange rate is entirely driven by the local costs paid and revenues repatriated by the overseas branches of MNEs. Given the importance of FDI in influencing the exchange rate here, it may seem objectionable not to model capital expenditures more carefully than the simplistic repeated fixed cost enfolded in the profit function. However, income earned by foreign branches of U.S. firms exceeded capital outflows from the United States in the form of foreign direct investment in 2002 and 2003 (Lowe 2004, p.103),⁹ so it is not far-fetched to suppose that if the exchange rate is influenced at all by foreign direct investment activity, it may be as heavily impacted by the repatriation of net income as by capital expenditures.

The response of the nominal exchange rate to a positive country-wide shock to Home's productivity can be decomposed analytically into competing effects. First, it is useful to note that due to the segmented markets, i.e. goods can not be traded internationally, the flexible exchange rate, and the cash-in-advance constraint, there is no transmission of productivity shocks across borders. Thus, the denominator in equation (18) will not move when A_t changes. Second, using the implicit function rule, it is shown in the Appendix that $\frac{\partial \hat{\varphi}_{j,t}}{\partial A_t} < 0$, implying that the number of Foreign-owned firms producing in the Home market, $n_{f,t}$, increases in response to a positive innovation in Home technology. Then, it is left only

⁸See Russ (2006) for a derivation of the aggregation of profits, also described in Melitz (2003).

⁹This has been the case even for US multinational activity in China (Mataloni 2004, p.24 and 25).

to compute the impact on average profits earned by Foreign-owned firms, which is:¹⁰

$$\begin{aligned} \frac{\partial \pi_{f,t}(\bar{\varphi}_{f,t})}{\partial A_t} &= \left(\frac{f_f}{\rho}\right) \left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\rho}} M_t \left[A_t^{\frac{\rho-1}{\rho}} \bar{\varphi}_t^{\frac{-1}{\rho(\theta-1)}} + \left(\frac{1}{\theta-1}\right) \bar{\varphi}_t^{\frac{-[1+\rho(\theta-1)]}{\rho(\theta-1)}} \frac{\partial \bar{\varphi}_t}{\partial \bar{\varphi}_{f,t}} \frac{\partial \hat{\varphi}_{f,t}}{\partial A_t} \right] \\ &\quad + \left(\frac{M_t \bar{\varphi}_{f,t}^{\theta-1}}{\theta \bar{\varphi}_t}\right) \left[\left(\frac{1}{\bar{\varphi}_{f,t}}\right) \left(\frac{\partial \bar{\varphi}_{f,t}}{\partial \bar{\varphi}_{f,t}}\right) \left(\frac{\partial \hat{\varphi}_{f,t}}{\partial A_t}\right) - \left(\frac{1}{\bar{\varphi}_t}\right) \left(\frac{\partial \bar{\varphi}_t}{\partial \bar{\varphi}_{f,t}}\right) \left(\frac{\partial \hat{\varphi}_{f,t}}{\partial A_t}\right) \right] \end{aligned} \quad (19)$$

where $\tilde{\varphi}_t = \int_{\hat{\varphi}_{h,t}}^{\infty} \varphi^{\theta-1} g(\varphi) d\varphi + \int_{\hat{\varphi}_{f,t}}^{\infty} \varphi^{\theta-1} g(\varphi) d\varphi$.

The top line in expression (19) manifests two ways that a positive shock to Home technology increases the average profits of Foreign-owned firms operating there. Both mechanisms operate by making entry easier. They both reduce the fixed cost firms must pay to produce by pushing down the aggregate price level.¹¹ The first term in the top brackets represents the direct downward effect that technology growth exerts on aggregate prices. This effect is obvious from the definition of the aggregate price level, Eq. (12), where all else equal, an increase in A_t reduces P_t . The second term reveals an additional competitive effect through which entry pushes the aggregate price level down further than the change in A_t by itself.

The bottom line in Eq. (19) represents two downward effects that entry exerts on the revenues of the average Foreign-owned firm. First, it reduces the average idiosyncratic productivity level of active Foreign firms in the Home market ($\left(\frac{\partial \bar{\varphi}_{f,t}}{\partial \bar{\varphi}_{f,t}}\right) \left(\frac{\partial \hat{\varphi}_{f,t}}{\partial A_t}\right) < 0$), which means lower variable profits for the average Foreign firm. Second, because new entering firms charge a higher price than existing firms due to their lower idiosyncratic productivity (generating higher marginal costs), they sell fewer units. Thus, the average firm now operates on a smaller scale regardless of whether it is Home- or Foreign-owned. Unless the burden of the fixed cost is sufficiently lowered by the drop in the aggregate price level depicted in the top line of equation (19), a smaller scale for the average firm means lower profits for the average firm. These competing effects on repatriated firm profits and the exchange rate are illustrated in more detail below.

4.2 Model Calibration and Baseline Numerical Results

We now proceed to analyse the model by assigning values to the parameters and computing the responses of selected variables to productivity shocks. The model's calibration is summarized in Table 1.

¹⁰See the Appendix for the derivation.

¹¹This is because the fixed cost is denominated in units of aggregate output, making the burden of paying it depend on the price of a unit of the aggregate consumption bundle, P_t .

Table 1: Parameter Values

Parameter		Value
θ	Substitution Elasticity	11
ρ	Relative Risk Aversion	2
κ	Labor Disutility	1
f_h	Home Fixed Cost	0.0182
f_f	Foreign Fixed Cost	0.0182
k	Pareto Location	11
M	Money Supply	100
ϕ	AR(1) coefficient	0.9

The fixed costs are calibrated in this and the following sections such that 25% of prospective entrants from the Home and Foreign country decide to produce in the Home market. Based on the empirical findings in Helpman, Melitz and Yeaple (2003) we model the distribution of idiosyncratic productivity shocks as Pareto. The Pareto shape parameter for the distribution of labor productivity levels, k , is chosen such that the index of firm dispersion is as close to 1 as possible to fit the range of estimates presented in Helpman, Melitz and Yeaple (2003) from a regression of firm size and rank.¹²

Our results are robust to alternative specifications the size of the coefficient of relative risk aversion, ρ ; the fixed costs; the relative disutility of labor, κ ; and—since money is completely neutral in this flexible-price framework—the money supply, M . The elasticity of substitution, θ , is chosen from a range of estimates in Feenstra (1994) and implies a 10% markup over marginal cost. Implications of higher markups (lower θ) are explored in Section 4.3 below.

The benchmark results are reported in Figure 1. The variables shown are responding to an exogenous 10% shock to Home’s country-specific productivity parameter, A_t . The jump in Home’s aggregate productivity level (defined as $A_t\bar{\varphi}_t$), however, is only 5.3%. This is because the average firm-specific level of labor productivity, $\bar{\varphi}_t$, actually *falls* more than 4%, as the increase in A_t boosts the profitability of firms with low idiosyncratic productivity, enticing some of them to enter the Home market. Entry of firms that have lower φ than existing producers lessens the effect of the country-wide productivity shock on aggregate

¹²Firm size dispersion can be seen graphically by plotting the logarithm of the rank of each firm’s size in comparison to all active firms against the logarithm of its size, with size measured in total sales. The slope of this line represents the index of dispersion for firm size (Helpman, Melitz, and Yeaple, 2003). A flatter line indicates higher dispersion. Firm size dispersion can also be defined as the variance of firm size within the population of active firms (Kremp and Mairesse, 1992).

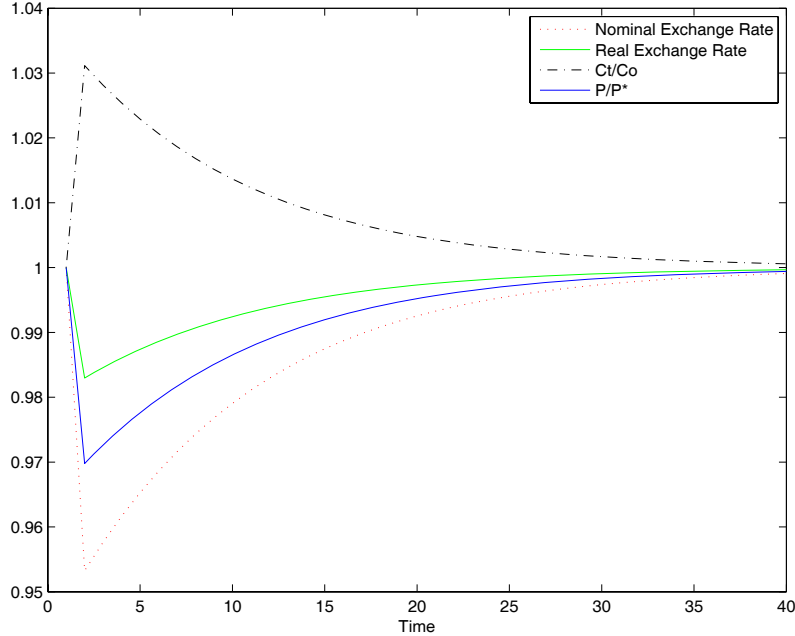


Figure 1: Impulse responses to a 10% increase in A_t - Baseline

productivity in the Home country. This is important because it is total productivity that impacts the aggregate price level, P_t , and aggregate consumption, C_t . Since aggregate productivity reflects only half of the country-wide productivity shock, the responses of P_t and C_t are also muted.

Movement in the nominal exchange rate is one-and-a-half times as large as the response in consumption, although it is still a bit smaller than the change in aggregate productivity. Home's nominal exchange rate appreciates (S_t falls) because Foreign firms as a whole are repatriating less net profit. The number of Foreign firms producing in the Home market after the shock increases by 4%, so entry exerts some positive effect on the amount of profit repatriated in terms of the sheer number of MNEs remitting income back to their Foreign owners. However, the reduction in the average idiosyncratic productivity and scale of the average Foreign-owned firm operating in the Home market causes the level of profit repatriated by the average firm to drop by over 9%. The net impact is a 4.7% appreciation of Home's nominal exchange rate.¹³ The Home price level also falls, acting as a counterweight to this fluctuation in the nominal exchange rate and subduing the response of the real exchange rate to the Home technology shock; the real exchange rate falls only 1.4%.

¹³Since S_t is measured in units of Home currency per unit of Foreign currency, a decrease reflects an appreciation of the Home currency.

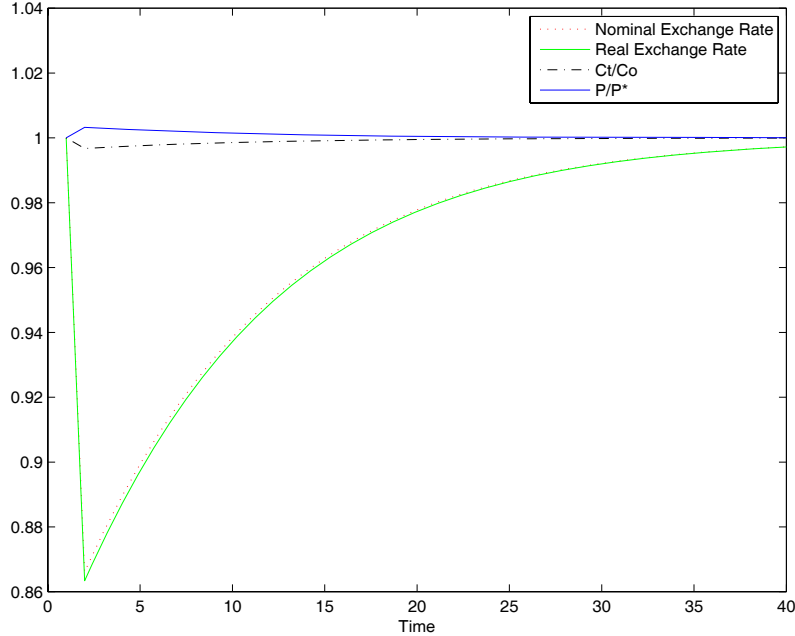


Figure 2: Impulse Responses to a 10% increase in A_t - Fixed Cost Shock

4.3 Technology Shocks and the Fixed Cost of Production

The innovation in productivity considered so far is a very specific type of shock, affecting only labor productivity in the Home country. Such a shock generates entry effects only indirectly, through its influence on the nominal value of the fixed cost, $P_t f_f$. Corsetti, Pesenti, and Martin (2005) point out that a shock to the size of the fixed cost of production, as occurs in Ghironi and Melitz (2005), yields a more direct effect, since it alters the size of this barrier to entry which acts as the primary limiting factor in determining the number of firms. Redefining the fixed cost such that profits for firms owned by country $j \in [f, h]$ operating in the Home market are given by:

$$\pi_{j,t}(\varphi_{j,t}) = p_{j,t}(\varphi_{j,t})c_{j,t}(\varphi_{j,t}) - W_t l_{j,t}(\varphi_{j,t}) - \frac{P_t f_j}{A_t}$$

boosts the sensitivity of the real and nominal exchange rates to innovations in Home technology A_t , but diminishes the response of consumption and the price level.

Figure 2 and Table 2 show that the nominal and real exchange rates fall more than 13%, while consumption rather surprisingly responds with a miniscule drop to the positive shock to Home technology. How can this happen? The response of aggregate consumption/output is sluggish because a dramatic increase in firm entry (15%) eats away the entire impact of the technological innovation on aggregate productivity, which actually drops 3% after a

10% increase in A_t . Moreover, the large number of new firms exerts a competitive entry effect that erodes the market share of the average Foreign firm in the Home country by 27% which, in combination with the lower-productivity of new entrants, diminishes the profits repatriated by the average Foreign firm by almost a quarter and total repatriated profits by 13%. Thus, total profits repatriated to the Foreign country, the principal driver of the nominal exchange rate, fall dramatically relative to consumption, despite the increase in the number of Foreign firms operating overseas. Finally, the subdued response of aggregate productivity generates only a one percent drop in the Home aggregate price level, allowing the nominal appreciation to spur a large real appreciation, as well. In this case, exchange rate volatility is about 44 times greater than the volatility of Home consumption.

4.4 Exchange Rate Volatility and the Markup

The degree to which entry mitigates movements in prices and consumption relative to the nominal exchange rate in this model of monopolistically competitive firms depends on the markup. As discussed above, progressively less productive new entrants lower the average idiosyncratic labor productivity among producers, which smothers the increase in aggregate productivity and the reduction in the price level that follows a country-wide productivity shock. However, entry by new firms also introduces a downward competitive effect on prices, even if they are a bit less productive than existing firms. When markups are high, many more new firms have the opportunity to produce profitably following a positive country-wide productivity shock than when markups are low. A high markup creates a competitive effect on prices large enough to outweigh the drop in the average idiosyncratic productivity level, $\bar{\varphi}$. Thus, Figure 3 shows that when the elasticity of substitution between goods is quite low (about 2), the Home-specific technology shock induces much less dramatic fluctuations in the real and nominal exchange rate relative to consumption, even when the technological innovation is applied to the fixed cost, as in Section 5.2.

The proportion of entrants increases by almost 30 percent, which drives the price level down by about 26%. Even though the plummeting price level boosts consumption by 35%, the competitive effects from entry reduce the Home market share of the average Foreign-owned firm by a whopping 65%. This large competitive effect squeezes profits such that the nominal exchange rate falls as well, by 40%. The competitive effect is much larger here than the previous example, where the shock structure is identical but the elasticity of substitution is 11. The high elasticity of substitution lowers profit margins and slows the increase in entry, so that changes in the number of entrants and profit margins are only half as large as when the elasticity is 2. It is worth noting here that although $g(\varphi)$, the underlying distribution of labor productivity levels, remains the same as in the previous two sections for this third experiment, changing the elasticity of substitution, θ , alters the

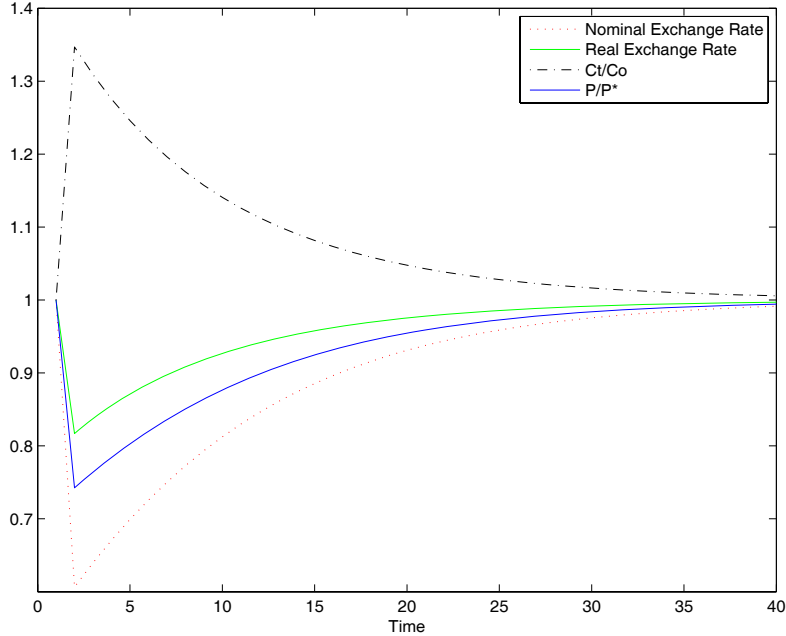


Figure 3: Impulse Responses to a 10% increase in A_t - Low Substitutability

dispersion of firm size. Nonetheless, the results in this section remain qualitatively the same, even if k is also changed to preserve the degree of firm dispersion.

Table 2 presents a comparison of the results for all three experiments. Exchange rate volatility is highest relative to that of consumption, output, and the price level when country-wide productivity shocks impact not only labor productivity, but also the fixed cost of production, f_j ($j = [f, h]$). This is the case because reductions in the fixed cost of production have a stronger impact on the number of producers than increases in labor productivity. Since each additional firm is a bit less productive than the last, the entry counteracts the impact of the country-wide shock on total aggregate productivity in the economy. When consumers are very price-responsive and less willing to substitute away from cheaper goods for the sake of enjoying more variety, as is the case when $\theta = 11$, the entry is just enough to dampen the response of the aggregate price level and consumption to the country-wide productivity shock. Because entry has a more dramatic effect on market share, it has a bigger impact on profits repatriated by foreign firms, which drive the real and nominal exchange rate. The net effect is nominal and real exchange rates that are much more volatile than consumption.

Entry responds most dramatically, and both market share and profit margins are squeezed

Table 2: Numerical Results

	10 %. increase in A_t		
	Labor prod. only $\theta = 11$	Labor prod. and f_j $\theta = 11$	Labor prod. and f_j $\theta = 2$
Percentage Change:			
N_t	4.9	15.4	30.0
$\hat{\varphi}_{j,t}$	-0.4	-1.3	-2.4
$A_t \bar{\varphi}_t$	5.3	-3.5	-25.8
P_t	-3.0	0.3	-25.8
C_t	3.1	-0.3	34.7
$\frac{c_{f,t}(\bar{\varphi}_{f,t})}{C_t}$	-10.0	-27.0	-65.0
$\bar{\pi}_{f,t}(\bar{\varphi}_{f,t})$	-9.1	-25.0	-53.3
S_t	-4.7	-13.4	-39.4
$\frac{S_t P_t^*}{P_t}$	-1.7	-13.7	-18.3
Rel. Volatility:			
$\frac{\sigma_{rer}}{\sigma_c}$	0.6	45	0.7
$\frac{\sigma_s}{\sigma_c}$	1.6	44	1.7

the most when the elasticity of substitution is low. A low elasticity allows a high markup, inviting opportunistic firms with much lower idiosyncratic productivity (and higher prices) to enter into production and yielding an enormous downward effect on total aggregate productivity. However, the flood of new producers in combination with consumers eager for variety produces a fierce competitive effect, despite the high markups, demonstrated by plunging market shares for Foreign-owned firms ($\frac{c_{f,t}(\bar{\varphi}_{f,t})}{C_t}$). Repatriated profits, as well as the level of the nominal exchange rate, plummet. But amidst the intense competition, the aggregate price level falls as well, so that consumption responds almost as strongly as the nominal exchange rate and considerably more than the real exchange rate, since the real exchange rate is tempered by the change in the Home and Foreign relative price levels. Thus, the net effect of conflicting impacts from falling average idiosyncratic productivity and increasing competitive pressures depends not only on the specification of the technology shocks, but also on consumer preferences.

5 Conclusion

The results from this stylized model of multinational firms suggest that entry decisions by heterogeneous firms may partly explain why cross-country consumption ratios are observed to be much less volatile than real and nominal exchange rates, even when prices are perfectly flexible. It does not strive to replace, but complement explanations in other studies, such as sticky goods prices, varying trade costs, and noise trading in foreign exchange markets. The basic intuition underlying the results is that under reasonable parameter choices, entry by less productive firms may dampen the effect of country-wide technological innovation on aggregate prices and consumption, while generating larger movements in nominal and, under some circumstances, real exchange rates as the amount of net revenues repatriated by multinational firms fluctuates. It is noteworthy that the larger the entry effect, the greater the response of the nominal and real exchange rate will be relative to the response of prices and consumption when country-specific technology shocks occur, though the relationship is weaker when demand is inelastic.

Allowing only multinational firms and no trade in goods, the model here may be considered to embody infinitely high trade costs. Its results would likely be mitigated by allowing traded goods, as in work by Bergin and Glick (2003) and Naknoi (2006) on endogenous tradability. If either Home- or Foreign-owned firms operating in the Home country were allowed to supply the Foreign market, depending on the size of trade costs, the Foreign price level may also fall after a positive innovation in Home-specific technology. In this context, an extension of the model allowing for goods trade may correspond with the findings of Fitzgerald (2005), which indicate that high real exchange rate volatility is most likely to

emerge between countries with high bilateral trade costs.

Another drawback of the model is that persistence in exchange rate movements arises only from persistence in the exogenous productivity shock. Shrikhande (2002) shows that explicitly adding capital with partial depreciation could produce endogenous persistence, as the number of firms entering would become a state variable rather than a jump variable. Ghironi and Melitz (2005) generate endogenous persistence in the real exchange rate using a one-time sunk cost of entry. Nonetheless, using a simple model, this study carries the important message that if the production and investment decisions of multinational firms are considered as a factor driving exchange rate behavior, then heterogeneous labor productivity among firms may explain part of the excess volatility observed in both the nominal and real exchange rate, particularly when technological innovation directly reduces barriers to entry.

References

- [1] Aizenman, J., 1992. Exchange Rate Flexibility, Volatility, and Domestic and Foreign Direct Investment. *IMF Staff Papers* 39, 890–922.
- [2] Aizenman, J., 1994. Monetary and Real Shocks, Productive Capacity and Exchange Rate Regimes. *Economica* 61, 407-434.
- [3] Backus, David K., Patrick J. Kehoe, and Finn E. Kydland (1992): “International Real Business Cycles”. *Journal of Political Economy*, 100(4), pp. 745-775.
- [4] Bacchetta, Philippe, and Eric van Wincoop (2000): “Does Exchange-Rate Stability Increase Trade and Welfare?” *American Economic Review*, 90(5), pp. 1093-1109.
- [5] Baxter, Marianne, and Alan Stockman (1989): “Business Cycles and the Exchange-Rate Regime: Some International Evidence”. *Journal of Monetary Economics* 23, pp. 377-400.
- [6] Bernard, Andrew B., Stephen Redding, and Peter K. Schott (2005): “Importers, Exporters, and Multinationals: A Portrait of Firms in the U.S. that Trade Goods”. Manuscript, Dartmouth College Tuck School of Business.
- [7] Bernard, Andrew B., Jonathan Eaton, J. Bradford Jensen, and Samuel Kortum (2003): “Plants and Productivity in International Trade”. *American Economic Review*, 93(4), pp. 1268-1290.
- [8] Bergin, Paul and Reuven Glick (2003): “Endogenous Tradability and Macroeconomic Implications”. NBER Working Paper No. 9739.

- [9] Cavallari, Lilia (2005): “Globalisation and the International Monetary Transmission”. Manuscript, University of Rome III.
- [10] Clarida, Richard, and Jordi Galí (1994): “Sources of Real Exchange Rate Fluctuations: How Important are Nominal Shocks?” Carnegie-Rochester Conference Series on Public Policy, 41, pp. 1-56.
- [11] Corsetti, Giancarlo, Philippe Martin, and Paolo Pesenti (2005): “Productivity Spillovers, Terms of Trade and the ‘Home Market Effect’”. NBER Working Paper No. 11165.
- [12] Dooley, Michael P., David Folkerts-Landau, and Peter M. Garber (2004): “The US Current Account Deficit and Economic Development: Collateral for a Total Return Swap”. NBER Working Paper No. 10727.
- [13] Feenstra, Robert C. (1994): “New Product Varieties and Price Indexes”. American Economic Review, 84(1), pp.157-177.
- [14] Fitzgerald, Doireann (2004): “A Gravity View of the Exchange Rate Disconnect”. SCCIE Working Paper 04-19.
- [15] Flood, Robert, and Andrew Rose (1995): “Fixing Exchange Rates: A Virtual Quest for Fundamentals”. Journal of Monetary Economics.
- [16] Gourinchas, Pierre-Olivier and Helene Rey (2005): “International Financial Adjustment”. NBER Working Paper No. 11155.
- [17] Ghironi, Fabio and Marc J. Melitz (2005): “International Trade and Macroeconomic Dynamics with Heterogeneous Firms”. Quarterly Journal of Economics, 120(3), pp. 865-915.
- [18] Helpman, Elhanan, Marc J. Melitz and Stephen R. Yeaple (2004): “Export versus FDI with Heterogeneous Firms”. American Economic Review 94(1): pp. 300-316.
- [19] Kosteletou, Nikolina and Panagiotis Liargovas (2000): “Foreign Direct Investment and Real Exchange Rate Interlinkages”. Open Economies Review, 11, pp. 135-148.
- [20] Lowe, Jeffrey H (2004): “U.S. Direct Investment Abroad: Detail for Historical-Cost Position and Related Capital and Income Flows, 2003”. Survey of Current Business (September): 101-144.
- [21] Lubik, Thomas A. and Frank Schorfhedie (2005): “A Bayesian Look at New Open Economy Macroeconomics”. NBER Macroeconomics Annual, pp. 313-366.

- [22] Mataloni, Raymond J. Jr. (2004): “U.S. Multinational Companies”. *Survey of Current Business* (July): 10-29.
- [23] Melitz, Marc J. (2003): “The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity”. *Econometrica*, 71(6), pp. 1695-1725.
- [24] Naknoi, Kanda (2006): “Real Exchange Rate Fluctuations, Endogenous Tradability and Exchange Rate Regime”. Forthcoming, *Journal of Monetary Economics*.
- [25] Rogoff, Kenneth S. (1996): “The Purchasing Power Parity Puzzle”. *Journal of Economic Literature*, 34, pp.647-668.
- [26] Russ, Katheryn N. (2006): “The Endogeneity of the Exchange Rate as a Determinant of FDI: A Model of Money, Entry, and Multinational Firms”. Forthcoming, *Journal of International Economics*.
- [27] Shrikhande, Milind M. (2002): “A General Equilibrium Analysis of Foreign Direct Investment and the Real Exchange Rate”. *International Journal of Finance and Economics*, 7, pp. 309-325.
- [28] Xu, Juanyi (2005): “Noise Traders and the Exchange Rate Disconnect Puzzle”. Manuscript, Simon Fraser University.

Appendix

A The Existence of $\hat{\varphi}_{j,t}$

The cutoff level of productivity for a firm owned by country j ($j \in [f, h]$) can be found using the zero-cutoff profit condition:

$$\pi_{j,t}(\hat{\varphi}_{h,t}) = p_{j,t}(\hat{\varphi}_{j,t})c_{j,t}(\hat{\varphi}_{j,t}) - W_t l_{j,t}(\hat{\varphi}_{j,t}) - P_t f_j = 0$$

Substituting the (rearranged) pricing rule and the specification for production technology yields

$$\begin{aligned} p_{j,t}(\hat{\varphi}_{j,t})c_{j,t}(\hat{\varphi}_{j,t}) - \left(\frac{\theta-1}{\theta}\right) A_t \hat{\varphi}_{j,t} p_{j,t}(\hat{\varphi}_{j,t}) \left(\frac{c_{j,t}(\hat{\varphi}_{j,t})}{A_t \hat{\varphi}_{j,t}}\right) &= P_t f_j \\ p_{j,t}(\hat{\varphi}_{j,t})c_{j,t}(\hat{\varphi}_{j,t}) &= \theta P_t f_j. \end{aligned} \quad (20)$$

Substituting the pricing rule, demand equation, and wage relation and rearranging, equation (20) can be written as

$$\begin{aligned} \hat{\varphi}_{j,t} &= (\theta f_j)^{\frac{1}{\theta-1}} \left(\frac{\kappa}{\alpha A_t}\right) M_t^{\frac{\rho(\theta-1)-1}{\theta-1}} P_t^{\frac{1-\rho(\theta-1)}{\theta-1}} \\ &= (\theta f_j)^{\frac{1}{\theta-1}} \left(\frac{\kappa}{\alpha A_t}\right)^{\frac{1}{\rho(\theta-1)}} \tilde{\varphi}_t^{\frac{\rho(\theta-1)-1}{\rho(\theta-1)^2}}, \end{aligned}$$

where $\tilde{\varphi}_t = \int_{\hat{\varphi}_{h,t}}^{\infty} \varphi^{\theta-1} g(\varphi) d\varphi + \int_{\hat{\varphi}_{f,t}}^{\infty} \varphi^{\theta-1} g(\varphi) d\varphi$ and $\alpha = \frac{\theta-1}{\theta}$.

Define $\Gamma_j = (\theta f_j)^{\frac{1}{\theta-1}} \left(\frac{\kappa}{\alpha A_t}\right)^{\frac{1}{\rho(\theta-1)}}$ and

$$Z_{j,t} = \hat{\varphi}_{j,t} - \Gamma_j \left(\frac{1}{A_t}\right)^{\frac{1}{\rho(\theta-1)}} \tilde{\varphi}_t^{\frac{\rho(\theta-1)-1}{\rho(\theta-1)^2}}.$$

Then, to prove the existence of $\hat{\varphi}_{j,t}$, it is sufficient to show that $Z_{j,t}$ is monotonic in $\hat{\varphi}_{j,t}$. $Z_{j,t}$ is, in fact, monotonically increasing in $\hat{\varphi}_{j,t}$. The case of $\hat{\varphi}_{h,t}$ is shown below and the existence of $\hat{\varphi}_{f,t}$ can be proven in the same way:

$$\begin{aligned} \frac{dZ_{h,t}}{d\hat{\varphi}_{h,t}} &= 1 + \Gamma_0 \left(\frac{\rho(\theta-1)-1}{\rho(\theta-1)^2}\right) \left(\frac{1}{A_t}\right)^{\frac{1}{\rho(\theta-1)}} \tilde{\varphi}_t^{\frac{\rho(\theta-1)-\rho(\theta-1)^2-1}{\rho(\theta-1)^2}} \left(\hat{\varphi}_{h,t} g(\hat{\varphi}_{h,t}) + \left(\frac{f_f}{f_h}\right)^{\frac{1}{\theta-1}} g(\hat{\varphi}_{f,t})\right) > 0 \\ &\text{for all } \hat{\varphi}_{h,t}, \hat{\varphi}_{f,t} > 0. \end{aligned}$$

B The Impact of an Increase in A_t on Entry

The implicit function rule tells us that $\frac{d\hat{\varphi}_{h,t}}{dA_t} = -\frac{Z'_{A_t}}{Z'_{\hat{\varphi}_{h,t}}}$. Both Z'_{A_t} and $Z'_{\hat{\varphi}_{h,t}}$ are positive. That $Z'_{\hat{\varphi}_{h,t}} > 0$ is shown in Appendix A and

$$Z'_{A_t} = \left(\frac{1}{\rho(\theta-1)} \right) \Gamma_j A_t^{\frac{-\theta}{\theta-1}} \tilde{\varphi}_t^{\frac{\rho(\theta-1)-1}{\rho(\theta-1)^2}} > 0.$$

Thus, $\frac{d\hat{\varphi}_{h,t}}{dA_t} = -\frac{Z'_{A_t}}{Z'_{\hat{\varphi}_{h,t}}} < 0$. It can be shown in the same manner that $\frac{d\hat{\varphi}_{f,t}}{dA_t} = -\frac{Z'_{A_t}}{Z'_{\hat{\varphi}_{f,t}}} < 0$, as well.

C The Impact of an Increase in A_t on Repatriated Profits for the Average Foreign Firm

$$\begin{aligned} \pi_{f,t}(\bar{\varphi}_{f,t}) &= \left(\frac{1}{\theta} \right) p_{f,t}(\varphi_{j,t}) c_{f,t}(\varphi_{f,t}) - P_t f_f \\ &= \frac{\alpha^{\theta-1}}{\theta} \left(\frac{\kappa P_t^{1-\rho} M_t^\rho}{\bar{\varphi}_{f,t} A_t} \right)^{1-\theta} P_t^{\theta-1} M_t - P_t f_f \\ &= \frac{1}{\theta} \left(\frac{\alpha}{\kappa} \right)^{\theta-1} (A_t \bar{\varphi}_{f,t})^{\theta-1} M_t^{1-\rho(\theta-1)} P_t^{\rho(\theta-1)} - P_t f_f \\ &= \frac{1}{\theta} \left(\frac{\alpha A_t}{\kappa} \right)^{\theta-1} \bar{\varphi}_{f,t}^{\theta-1} M_t^{1-\rho(\theta-1)} \left[\left(\frac{\kappa}{\alpha A_t} \right)^{\frac{1}{\rho}} M_t \tilde{\varphi}_t^{\frac{-1}{\rho(\theta-1)}} \right]^{\rho(\theta-1)} - \left(\frac{\kappa}{\alpha A_t} \right)^{\frac{1}{\rho}} M_t \tilde{\varphi}_t^{\frac{-1}{\rho(\theta-1)}} f_f \\ &= \left(\frac{M_t}{\theta} \right) \bar{\varphi}_{f,t}^{\theta-1} \tilde{\varphi}_t^{-1} - \left(\frac{\kappa}{\alpha A_t} \right)^{\frac{1}{\rho}} M_t \tilde{\varphi}_t^{\frac{-1}{\rho(\theta-1)}} f_f \end{aligned}$$

$$\begin{aligned} \frac{\partial \pi_{f,t}(\bar{\varphi}_{f,t})}{\partial A_t} &= \left(\frac{M_t}{\alpha} \right) \bar{\varphi}_{f,t}^{\theta-2} \tilde{\varphi}_t^{-1} \left(\frac{\partial \bar{\varphi}_{f,t}}{\partial \hat{\varphi}_{h,t}} \right) \left(\frac{\partial \hat{\varphi}_{f,t}}{\partial A_t} \right) - \left(\frac{M_t}{\theta} \right) \bar{\varphi}_{f,t}^{\theta-1} \tilde{\varphi}_t^{-2} \left(\frac{\partial \tilde{\varphi}_t}{\partial \hat{\varphi}_{f,t}} \right) \left(\frac{\partial \hat{\varphi}_{f,t}}{\partial A_t} \right) \\ &\quad + \left(\frac{1}{\rho} \right) \left(\frac{\kappa}{\alpha} \right)^{\frac{1}{\rho}} M_t A_t^{\frac{\rho-1}{\rho}} \tilde{\varphi}_t^{\frac{-1}{\rho(\theta-1)}} f_f + \left(\frac{\kappa}{\alpha A_t} \right)^{\frac{1}{\rho}} M_t \tilde{\varphi}_t^{\frac{-[1+\rho(\theta-1)]}{\rho(\theta-1)}} f_f \left(\frac{\partial \tilde{\varphi}_t}{\partial \hat{\varphi}_{f,t}} \right) \left(\frac{\partial \hat{\varphi}_{f,t}}{\partial A_t} \right) \\ &= \left(\frac{M_t \bar{\varphi}_{f,t}^{\theta-1}}{\theta \tilde{\varphi}_t} \right) \left[\left(\frac{1}{\bar{\varphi}_{f,t}} \right) \left(\frac{\partial \bar{\varphi}_{f,t}}{\partial \hat{\varphi}_{h,t}} \right) \left(\frac{\partial \hat{\varphi}_{f,t}}{\partial A_t} \right) - \left(\frac{1}{\tilde{\varphi}_t} \right) \left(\frac{\partial \tilde{\varphi}_t}{\partial \hat{\varphi}_{f,t}} \right) \left(\frac{\partial \hat{\varphi}_{f,t}}{\partial A_t} \right) \right] \\ &\quad + \left(\frac{f_f}{\rho} \right) \left(\frac{\kappa}{\alpha} \right)^{\frac{1}{\rho}} M_t \left[A_t^{\frac{\rho-1}{\rho}} \tilde{\varphi}_t^{\frac{-1}{\rho(\theta-1)}} + \left(\frac{1}{\theta-1} \right) \tilde{\varphi}_t^{\frac{-[1+\rho(\theta-1)]}{\rho(\theta-1)}} \frac{\partial \tilde{\varphi}_t}{\partial \hat{\varphi}_{f,t}} \frac{\partial \hat{\varphi}_{f,t}}{\partial A_t} \right] \end{aligned}$$