Appendix A: Deriving the Intratemporal Demand Equations

The representative consumer in the Home country maximizes lifetime utility subject to an intertemporal budget constraint:

$$\max E_t \left[\sum_{t=0}^{\infty} \beta^t U_t \right]$$

$$s.t.\ P_t C_t + M_t + \sum_{z^{t+1}|z^t} q(z^{t+1}|z^t) B(z^{t+1}) = W_t L_t + \pi_t + M_{t-1} + B_t + T_t$$

where U is a function of aggregate consumption, C, and labor, L,

$$U_t = \frac{1}{1-\rho} C_t^{1-\rho} + \chi \ln\left(\frac{M_t}{P_t}\right) - \kappa L_t,$$

and $q(z^t|z^{t-1})$ is the price at time $t-1$ of the bond $B(z^t)$, which is denominated in Home currency and has a payoff of one unit of home currency given that one of a set z of possible states of the macroeconomy is realized at the end of time t.[1] Aggregate consumption is an index reflecting preferences with constant elasticity of substitution (CES) across the set of all available goods,

$$C_t = \left[\int_0^{n_H(t)} c_H(i,t)^{\frac{\mu-1}{\mu}} di + \int_1^{1+n_F(t)} c_F(i,t)^{\frac{\mu-1}{\mu}} di \right]^{\frac{\mu}{\mu-1}}.$$

A.1. The Aggregate Price Index

The aggregate price index is defined as the minimum expenditure required to purchase one unit of the consumption index, C_t. To find P_t, one can solve the following minimization problem:[2]

$$\begin{aligned} \pounds &= \min \int_0^{n_H(t)} p_H(i,t) c_H(i,t) di + \int_1^{1+n_F(t)} p_F(i,t) c_F(i,t) di \\ &\quad + \psi_t \left(1 - \left[\int_0^{n_H(t)} c_H(i,t)^{\frac{\mu-1}{\mu}} di + \int_1^{1+n_F(t)} c_F(i,t)^{\frac{\mu-1}{\mu}} di \right]^{\frac{\mu}{\mu-1}} \right), \end{aligned}$$

where ψ_t is the Lagrange multiplier. First-order conditions:

$$\frac{\partial \pounds}{\partial c_H(i,t)} : p_H(i,t) - \psi_t \left[\int_0^{n_H(t)} c_H(i,t)^{\frac{\mu-1}{\mu}} di + \int_1^{1+n_F(t)} c_F(i,t)^{\frac{\mu-1}{\mu}} di \right]^{\frac{1}{\mu-1}} c_H(i,t)^{\frac{-1}{\mu}} = 0 \quad \text{(a.1)}$$

$$\frac{\partial \pounds}{\partial c_F(i,t)} : p_F(i,t) - \psi_t \left[\int_0^{n_H(t)} c_H(i,t)^{\frac{\mu-1}{\mu}} di + \int_1^{1+n_F(t)} c_F(i,t)^{\frac{\mu-1}{\mu}} di \right]^{\frac{1}{\mu-1}} c_F(i,t)^{\frac{-1}{\mu}} = 0 \quad \text{(a.2)}$$

$$\frac{\partial \pounds}{\partial \psi_t} : 1 - \left[\int_0^{n_H(t)} c_H(i,t)^{\frac{\mu-1}{\mu}} di + \int_1^{1+n_F(t)} c_F(i,t)^{\frac{\mu-1}{\mu}} di \right]^{\frac{\mu}{\mu-1}} = 0 \quad \text{(a.3)}$$

[1]That is, $\sum_{z^{t+1}|z^t} B(z^{t+1})$ represents a complete set of state-contingent bonds denominated in the Home currency.
[2]See Obstfeld and Rogoff (1996, Chapter 4).

Rearranging (a.1),

$$\begin{aligned}
p_H(i,t) - \psi_t C_t^{\frac{1}{\mu}} c_H(i,t)^{\frac{-1}{\mu}} &= 0 \\
\frac{1}{\psi_t} p_H(i,t) C_t^{\frac{-1}{\mu}} &= c_H(i,t)^{\frac{-1}{\mu}} \\
c_H(i,t) &= \left(\frac{1}{\psi_t}\right)^{-\mu} p_H(i,t)^{-\mu} C_t.
\end{aligned}$$

By definition, to find P_t, $C_t = 1$, so that

$$c_H(i,t) = \psi_t^{\mu} p_H(i,t)^{-\mu}. \tag{a.4}$$

Similarly, a counterpart equation for $c_F(i,t)$ emerges,

$$c_F(i,t) = \psi_t^{\mu} p_F(i,t)^{-\mu}. \tag{a.5}$$

Substituting (a.4) and (a.5) into (a.3) it is possible to solve for ψ_t^{μ}:

$$\begin{aligned}
1 - \left(\int_0^{n_H(t)} \left(\psi_t^{\mu} p_H(i,t)^{-\mu}\right)^{\frac{\mu-1}{\mu}} di + \int_1^{1+n_F(t)} \left(\psi_t^{\mu} p_F(i,t)^{-\mu}\right)^{\frac{\mu-1}{\mu}} di \right)^{\frac{\mu}{\mu-1}} &= 0 \\
\psi_t^{\mu} \left(\int_0^{n_H(t)} \left(p_H(i,t)^{-\mu}\right)^{\frac{\mu-1}{\mu}} di + \int_1^{1+n_F(t)} \left(p_F(i,t)^{-\mu}\right)^{\frac{\mu-1}{\mu}} di \right)^{\frac{\mu}{\mu-1}} &= 1 \\
\left(\begin{array}{c} \int_0^{n_H(t)} \left(p_H(i,t)^{-\mu}\right)^{1-\mu} di \\ + \int_1^{1+n_F(t)} \left(p_F(i,t)^{-\mu}\right)^{1-\mu} di \end{array} \right)^{\frac{1}{1-\mu}} &= \psi_t
\end{aligned}$$

Since ψ_t, the Lagrangian multiplier, represents the value of one extra unit of the consumption index, C_t, it is by definition an expression of the aggregate price index. Therefore, the two are equal:

$$P_t = \psi_t = \left(\int_0^{n_H(t)} \left(p_H(i,t)\right)^{1-\mu} di + \int_1^{1+n_F(t)} \left(p_F(i,t)\right)^{1-\mu} di \right)^{\frac{1}{1-\mu}}. \tag{a.6}$$

A.2 The Demand for an Individual Good

To determine the demand for $c_H(i,t)$ when $C_t = 1$, one can isolate ψ_t from () to obtain the relation

$$P_t = \psi_t = p_H(i,t) c_H(i,t)^{\frac{1}{\mu}},$$

yielding the demand equation

$$c_H(i,t) = \left(\frac{p_H(i,t)}{P_t}\right)^{-\mu}.$$

Since CES preferences are homothetic, the demand for any particular good is a constant proportion of aggregate consumption. Therefore, if it is true that

$$\frac{c_H(i,t)}{C_t} = \left(\frac{p_H(i,t)}{P_t}\right)^{-\mu}$$

for the case where $C_t = 1$, the ratio must be constant for all levels of aggregate consumption. Thus, the demand for an individual good is given by

$$c_H(i,t) = \left(\frac{p_H(i,t)}{P_t}\right)^{-\mu} C_t. \tag{a.7}$$

An analogous equation holds for individual foreign goods produced in the home country:

$$c_F(i,t) = \left(\frac{p_F(i,t)}{P_t}\right)^{-\mu} C_t$$

The demand relation also implies that expenditure on an individual home or foreign good will be proportional to total consumer expenditure. Multiplying by $p_H(i,t)$,

$$\begin{aligned} p_H(i,t)c_H(i,t) &= p_H(i,t)^{1-\mu} P_t^{\mu-1} P_t C_t \\ &= \left(\frac{p_H(i,t)}{P_t}\right)^{1-\mu} P_t C_t. \end{aligned}$$

Appendix B: First-Order Conditions and the Exchange Rate

Given the consumer's maximization problem above, let λ be the Lagrange multiplier for the intertemporal budget constraint.

$$\frac{\partial \pounds}{\partial C_t} : C_t^{-\rho} - \lambda_t P_t = 0 \Rightarrow \lambda_t = \frac{1}{P_t C_t^{\rho}} \tag{b.1}$$

$$\frac{\partial \pounds}{\partial M_t} : \frac{\chi}{M_t} - \lambda_t + \beta E_t\left[\lambda_{t+1}\right] = 0 \tag{b.2}$$

$$\frac{\partial L}{\partial L_t} : -\kappa + \lambda_t W_t = 0 \tag{b.3}$$

$$\frac{\partial \pounds}{\partial B_{t+1}} : \beta E_t\left[\lambda_{t+1}\right] - q(z^{t+1}|z^t)\lambda_t = 0 \tag{b.4}$$

Substituting (b.1) into (b.3) provides the wage relation (also equation (4))

$$W_t = \kappa P_t C_t^{\rho}. \tag{b.5}$$

Substituting (b.1) into (b.2) and rearranging yields the demand equation (equation (5) in the main text) for real money balances:

$$\begin{aligned} \frac{\chi}{M_t} &= \frac{1}{P_t C_t^{\rho}} - \beta E_t\left[\frac{1}{P_{t+1} C_{t+1}^{\rho}}\right] \\ \frac{\chi P_t C_t^{\rho}}{M_t} &= 1 - E_t\left[\frac{\beta P_t C_t^{\rho}}{P_{t+1} C_{t+1}^{\rho}}\right] \\ \frac{M_t}{P_t} &= \frac{\chi C_t^{\rho}}{1 - E_t\left[d_{t+1}\right]}, \end{aligned} \tag{b.6}$$

where $d_{t+1} = \frac{\beta P_t C_t^{\rho}}{P_{t+1} C_{t+1}^{\rho}}$. Again substituting (b.1) into (b.4) gives a bond-pricing equation,[3]

$$q(z^{t+1}|z^t) = \frac{\beta P_t C_t^{\rho}}{P_{t+1} C_{t+1}^{\rho}}, \tag{b.7}$$

B.1 The Foreign Consumer

The Foreign consumer's intertemporal budget constraint will be similar to that of the Home consumer, namely

$$P_t^* C_t^* + M_t^* + \frac{1}{S_t} \sum_{z^{t+1}|z^t} q(z^{t+1}|z^t) B^*(z^{t+1}) = W_t^* L_t^* + \pi_t^* + M_{t-1}^* + S_t B_t^*.$$

As mentioned in the text, S_t is the nominal exchange rate at time t. Thus, the Foreign consumer's problem yields first-order conditions

$$W_t^* = \kappa P_t^* C_t^{*\rho}, \tag{b.8}$$

$$\frac{M_t^*}{P_t^*} = \frac{\chi C_t^{*\rho}}{1 - E_t\left[d_{t+1}^*\right]}, \tag{b.9}$$

$$q(z^{t+1}|z^t) = \beta \left(\frac{S_t}{S_{t+1}}\right)\left(\frac{P_t^* C_t^{*\rho}}{P_{t+1}^* C_{t+1}^{*\rho}}\right). \tag{b.10}$$

B.2 Solving for the Exchange Rate

Arbitrage forces the price of a bond, $q(z^{t+1}|z^t)$ to be equal in both countries. Setting (b.7) equal to (b.10), both iterated backward, we have

$$\frac{P_{t-1} C_{t-1}^{\rho}}{P_t C_t^{\rho}} \equiv \left(\frac{S_{t-1}}{S_t}\right)\left(\frac{P_{t-1}^* C_{t-1}^{*\rho}}{P_t^* C_t^{*\rho}}\right).$$

One can iterate each side backward one period and multiply,

$$\begin{aligned}
\left(\frac{P_{t-1} C_{t-1}^{\rho}}{P_t C_t^{\rho}}\right)\left(\frac{P_{t-2} C_{t-2}^{\rho}}{P_{t-1} C_{t-1}^{\rho}}\right) &= \left(\frac{S_{t-1}}{S_t}\right)\left(\frac{P_{t-1}^* C_{t-1}^{*\rho}}{P_t^* C_t^{*\rho}}\right)\left(\frac{S_{t-2}}{S_{t-1}}\right)\left(\frac{P_{t-2}^* C_{t-2}^{*\rho}}{P_{t-1}^* C_{t-1}^{*\rho}}\right) \\
\frac{P_{t-1} C_{t-1}^{\rho}}{P_t C_t^{\rho}} &= \left(\frac{S_{t-2}}{S_t}\right)\left(\frac{P_{t-2}^* C_{t-2}^{*\rho}}{P_t^* C_t^{*\rho}}\right).
\end{aligned}$$

Repeating this step continually yields the expression[4]

$$\frac{P_0 C_0^{\rho}}{P_t C_t^{\rho}} = \left(\frac{S_0}{S_t}\right)\left(\frac{P_0^* C_0^{*\rho}}{P_t^* C_t^{*\rho}}\right),$$

or

$$S_t = S_0 \left(\frac{P_0^* C_0^{*\rho}}{P_t^* C_t^{*\rho}}\right)\left(\frac{P_t C_t^{\rho}}{P_0 C_0^{\rho}}\right).$$

Assuming that both the Home and Foreign country have identical initial conditions, so that $S_0 P_0^* C_0^{*\rho} = P_0 C_0^{\rho}$, one obtains an equation for the real exchange rate, equation (19) in the text,

[3]State-specific probabilities associated with bond prices are omitted here for notational convenience, but are implicit and can be seen in Chari, Kehoe, and McGrattan (2002).

[4]This solution process is described by Chari, Kehoe, and McGrattan (2002).

$$\frac{S_t P_t^*}{P_t} = \frac{C_t^\rho}{C_t^{*\rho}}. \tag{b.11}$$

Rearranging (b.6) and (b.9), one obtains expressions for consumption in each country,

$$C_t^\rho = \frac{M_t}{P_t}\left(\frac{1 - E_t\left[d_{t+1}\right]}{\chi}\right) \qquad C_t^{*\rho} = \frac{M_t^*}{P_t^*}\left(\frac{1 - E_t\left[d_{t+1}\right]}{\chi}\right). \tag{b.12}$$

Using the specification for the money supply growth rate described in Section 2.2,

$$E_t\left[\frac{M_t}{M_{t+1}}\right] = \theta \qquad E_t\left[\frac{M_t^*}{M_{t+1}^*}\right] = \theta^*,$$

(so that $\theta = \frac{e^{\sigma_m^2}}{(1+\psi)}$) Obstfeld and Rogoff (1998, p.39) show that consumption is a function of real money balances and the underlying parameters,

$$C_t^\rho = \frac{M_t}{P_t}\left(\frac{1 - \beta\theta}{\chi}\right) \qquad C_t^{*\rho} = \frac{M_t^*}{P_t^*}\left(\frac{1 - \beta\theta^*}{\chi}\right). \tag{b.13}$$

Subsituting (b.13) into (b.11), an expression for the nominal exchange rate emerges,

$$S_t = \frac{M_t(1-\beta\theta)}{M_t^*(1-\beta\theta^*)}$$

Appendix C: Aggregation and Equilibrium Conditions

C.1.1 Revenues, Profits, and Aggregate Employment

Like the price level, total revenues and total profits for firms operating on Home-country soil can be written as a function of aggregate sectoral productivity. The actual revenue, on average, for a firm picked at random from firms owned by country j producing inside the Home economy at the end of period t can be computed as

$$\bar{r}_j(t) = \int_0^\infty r_j(\varphi, t)\eta_j(\varphi, t)\, d\varphi$$

Using expression (18) and the definition of $\bar{\varphi}_{jt}$ in (21), this simplifies as shown in Melitz (2003):

$$\begin{aligned}
\bar{r}_j(t) &= \int_0^\infty r_j(\bar{\varphi}_j, t)\frac{r_j(\varphi, t)}{r_j(\bar{\varphi}_j(t), t))}\eta_j(\varphi, t)\, d\varphi. \\
&= r_j(\bar{\varphi}_j(t), t)\int_0^\infty \left(\frac{\varphi}{\bar{\varphi}_j(t)}\right)^{\mu-1}\eta_j(\varphi, t)\, d\varphi \\
&= r_j(\bar{\varphi}_j(t), t)\left(\frac{1}{\bar{\varphi}_j(t)}\right)^{\mu-1}\int_0^\infty \varphi(t)^{\mu-1}\eta_j(\varphi, t)\, d\varphi \\
&= r_j(\bar{\varphi}_j(t)).
\end{aligned}$$

To compute total revenues as a function of aggregate sectoral productivity, it is possible to treat all firms in the Home economy owned by residents of country j as though they earned the average level of revenue in the sector of all j-owned firms,

$$\begin{aligned}
R_t &= \int_0^{n_H} r_H(\bar{\varphi}_H(t), t)di + \int_1^{1+n_F} r_F(\bar{\varphi}_F(t), t)di \\
&= n_H r_H(\bar{\varphi}_H(t), t) + n_F r_F(\bar{\varphi}_F(t), t).
\end{aligned}$$

Labor enters linearly in the representative consumer's utility function, making the supply of labor perfectly inelastic with respect to the wage, so that equilibrium in the labor market is fully characterized by the cutoff productivity levels and the (exogenous) parameters underlying the model. If countries are not identical, the labor market equilibrium can be calculated numerically once the cutoff productivity levels are determined. Symmetry is not a necessary assumption for the model and it can be solved under asymmetric conditions as described in the main text, as all endogenous variables, including the level of labor supplied, follows from the solution for the threshold levels. However, to show analytically how the labor market clears, it is useful to take the special case in which countries are identical. In this case, an equation depicting employment as a function of aggregate consumption can be derived analytically as follows:

Let the total level of profit earned by all firms (Home- and Foreign-owned) operating in the Home economy be denoted by Π. Appendix C.1.2 shows that using a similar process, Π_t is given by

$$\Pi_t = n_H \pi_H(\bar{\varphi}_H(t), t) + n_F \pi_F(\bar{\varphi}_F(t), t).$$

Suppose that the labor force is composed of manufacturing workers and entrepreneurial workers, who are engaged in investment and managerial activities. Then aggregate expenditure on manufacturing labor in the Home economy equals total revenues, less the profits distributed to entrepreneurial workers,

$$W_t L_t^P = R_t - \Pi_t,$$

where L_t^P is the amount of labor hired in the production of goods in the Home economy. Profits are distributed to entrepreneurial workers at the same wage rate,[5] so that if L_t^E is the level of entrepreneurial labor hired in period t, and countries have identical monetary processes and fixed costs[6]

$$W_t L_t^E = \Pi_t.$$

Thus, the total income received by all workers is equal to total revenues. Defining aggregate employment as $L_t = L_t^P + L_t^E$ this relation can be written

$$W_t L_t = R_t = P_t C_t.$$

Using the labor supply relation in equation (5), one can solve for the aggregate level of employment as a function of aggregate consumption:

$$L_t = \frac{P_t C_t}{\kappa P_t C_t^{\rho}} = \frac{1}{\kappa} C_t^{1-\rho}.$$

C.1.2 Aggregation of Profits Earned by All Firms Operating in the Home Economy

The actual period-t profits of a firm owned by country j and earned in the Home country can be expressed as a function of revenues:

$$\begin{aligned}
\pi_j(\varphi, t) &= p_j(\varphi, t) c_j(\varphi, t) - W_t \left(\frac{c_j(\varphi, t)}{\varphi} \right) - P_t f \\
&= p_j(\varphi, t) c_j(\varphi, t) - p_j(\varphi, t) c_j(\varphi, t) \left(\frac{W_t}{\varphi p_j(\varphi, t)} \right) - P_t f \\
&= \Gamma_0 r_j(\varphi, t) - P_t f,
\end{aligned}$$

[5] Alternatively, one could specify that entrepreneurial workers receive a wage that is larger than the manufacturing wage by some constant factor without substantively affecting the results.

[6] The assumption that countries are identical implies that $n_F \pi_F(\bar{\varphi}_F(t)) = n_H^* S_t \pi_H^*(\bar{\varphi}_H(t))$.

where, substituting the firm's pricing rule from Section 2.3, $\Gamma_0 = 1 - \frac{\alpha\chi E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}\right]W_t}{\kappa(1-\beta\theta)E_{t-1}\left[M_t^{\frac{1}{\rho}}\right]}$. The profit of a firm picked at random from the country j-owned sector of the Home economy is then computed using the equilibrium distribution of firm productivity levels:

$$
\begin{aligned}
\bar{\pi}_j &= \int_0^\infty \left[\Gamma_0 r_j(i,t) - P_t f\right] \eta_j(\varphi,t)\, d\varphi \\
&= \Gamma_0 \int_0^\infty r_j(i,t)\eta_j(\varphi,t)\, d\varphi - P_t f \\
&= \Gamma_0 \int_0^\infty r_j(\bar{\varphi}_j(t),t)\frac{r_j(i,t)}{r_j(\bar{\varphi}_{jt})}\eta_j(\varphi,t)\, d\varphi - P_t f \\
&= \Gamma_0 r_j(\bar{\varphi}_j(t),t)\int_0^\infty \left(\frac{\varphi}{\bar{\varphi}_{jt}}\right)^{\mu-1} \eta_j(\varphi,t)\, d\varphi - P_t f \\
&= \Gamma_0 r_j(\bar{\varphi}_j(t),t)\left(\frac{1}{\bar{\varphi}_j(t)}\right)^{\mu-1}\int_0^\infty \varphi^{\mu-1}\eta_j(\varphi,t)d\varphi - P_t f \\
&= \Gamma_0 r_j(\bar{\varphi}_j(t),t) - P_t f.
\end{aligned}
$$

Substituting back in the definition of Γ_0 and rearranging, it is clear that the average level of profit is also the level of profit earned by a firm with the average productivity level, or $\bar{\pi}_j = \pi_j(\bar{\varphi}_j(t),t)$. All firms in each sector can now be treated as though they earned this average level of profit. Total profits earned by all Home- and Foreign-owned firms in the Home economy are then computed as

$$
\begin{aligned}
\Pi_t &= \int_0^{n_H(t)} \pi_H(\bar{\varphi}_H(t),t)di + \int_1^{1+n_F(t)} \pi_F(\bar{\varphi}_F(t),t)di \\
&= n_H(t)\pi_H(\bar{\varphi}_H(t),t) + n_F(t)\pi_F(\bar{\varphi}_F(t),t).
\end{aligned}
$$

C.2 Expressing Expected Discounted Profits in Terms of Revenues

As explained in the text, since each Home firm has a different productivity draw, $\varphi_H(i)$, firm subscripts are omitted in the following derivations. Expected discounted profits for the Home firm earned in the Home market in period t can be expressed in terms of revenues:

$$\begin{aligned}
E_{t-1}[d_t\pi_H(\varphi,t)] &= E_{t-1}[d_t(p_H(\varphi,t)c_H(\varphi,t) - W_t l_H(\varphi,t) - P_t f)] \\
&= E_{t-1}[d_t(p_H(\varphi,t)c_H(\varphi,t) - \frac{W_t c_H(\varphi,t)}{\varphi} - f)] \\
&= E_{t-1}[d_t(p_H(\varphi,t)c_H(\varphi,t) \\
&\quad -\frac{\kappa P_t C_t^{\rho} c_H(\varphi,t)}{\varphi}\left(\frac{p_H(\varphi,t)}{p_H(\varphi,t)}\right) - f)] \\
&= E_{t-1}[d_t(p_H(\varphi,t)c_H(\varphi,t) - \frac{\kappa P_t C_t^{\rho} r_H(\varphi,t)}{\varphi p_H(\varphi,t)} - f)] \\
&= E_{t-1}\left[d_t r_H(\varphi,t)\left(\begin{array}{c} 1 \\ -\frac{\kappa P_t\left(\frac{M_t}{P_t}\left(\frac{1-\beta m}{\chi}\right)\right)}{\varphi\left(\frac{\kappa(1-\beta m)E_{t-1}\left[M_t^{\frac{1}{\rho}}\right]}{\alpha\chi\varphi E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}\right]}\right)} - f\end{array}\right)\right] \\
&= E_{t-1}\left[d_t\left(1 - \frac{\alpha M_t E_{t-1}\left[M_t^{\frac{1}{\rho}}\right]}{E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}\right]}\right) r_H(\varphi,t)\right] - fE_{t-1}[d_t].
\end{aligned} \tag{1}$$

C.3 Reducing the Zero-Cutoff Profit Conditions

The Zero-Cutoff Profit Condition (ZCP) implies that

$$E_{t-1}[d_t\pi_H(\hat{\varphi}_H(t),t)] = E_{t-1}[d_t\left(1 - \frac{\alpha M_t E_{t-1}\left[M_t^{\frac{1}{\rho}}\right]}{E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}\right]}\right) r_H(\hat{\varphi}_H(t),t)] - P_t f E_{t-1}[d_t] \equiv 0.$$

Thus, one can write

$$E_{t-1}[d_t\left(1 - \frac{\alpha M_t E_{t-1}\left[M_t^{\frac{1}{\rho}}\right]}{E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}\right]}\right) r_H(\hat{\varphi}(t))] = P_t f E_{t-1}[d_t].$$

Noting from Appendix A that $p_H(\varphi,t)c_H(\varphi,t) = p_H^{1-\mu}(\varphi,t)P_t^{\mu}C_t = r_H(\varphi,t)$ and $d_t = E_{t-1}\left[\frac{\beta P_{t-1}C_{t-1}^{\rho}}{P_tC_t^{\rho}}\right]$,

$$\begin{aligned}
E_{t-1}[d_t\left(1-\frac{\alpha M_t E_{t-1}\left[M_t^{\frac{1}{\rho}}\right]}{E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}\right]}\right)p_H(\hat{\varphi}_H(t),t)^{1-\mu}P_t^{\mu}C_t] &= P_t f E_{t-1}[d_t] \\
E_{t-1}\left[\frac{\beta P_{t-1}C_{t-1}^{\rho}}{P_tC_t^{\rho}}\left(1-\frac{\alpha M_t E_{t-1}\left[M_t^{\frac{1}{\rho}}\right]}{E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}\right]}\right)p_H^{1-\mu}(\hat{\varphi}_H(t),t)P_t^{\mu-1}C_t\right] &= fE_{t-1}\left[\frac{\beta P_{t-1}C_{t-1}^{\rho}}{P_tC_t^{\rho}}\right] \\
\beta P_{t-1}C_{t-1}^{\rho}E_{t-1}\left[\frac{1}{P_tC_t^{\rho}}\left(1-\frac{\alpha M_t E_{t-1}\left[M_t^{\frac{1}{\rho}}\right]}{E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}\right]}\right)p_H^{1-\mu}(\hat{\varphi}_H(t),t)P_t^{\mu-1}C_t\right] &= \beta P_{t-1}C_{t-1}^{\rho} f E_{t-1}\left[\frac{1}{P_tC_t^{\rho}}\right] \\
E_{t-1}\left[\frac{1}{P_tC_t^{\rho}}\left(1-\frac{\alpha M_t E_{t-1}\left[M_t^{\frac{1}{\rho}}\right]}{E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}\right]}\right)p_H^{1-\mu}(\hat{\varphi}_H(t),t)P_t^{\mu-1}C_t\right] &= fE_{t-1}\left[\frac{1}{P_tC_t^{\rho}}\right].
\end{aligned}$$

Using equation (b.13) to substitute for consumption, one finds

$$E_{t-1}\left[\frac{1}{P_t\left(\frac{M_t}{P_t}\left(\frac{1-\beta\theta}{\chi}\right)\right)}\left(1-\frac{\alpha M_t E_{t-1}\left[M_t^{\frac{1}{\rho}}\right]}{E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}\right]}\right)p_H(\hat{\varphi}(t))^{1-\mu}P_t^{\mu-1}\left(\frac{M_t}{P_t}\left(\frac{1-\beta\theta}{\chi}\right)\right)^{\frac{1}{\rho}}\right] = fE_{t-1}\left[\frac{1}{P_t\left(\frac{M_t}{P_t}\left(\frac{1-\beta\theta}{\chi}\right)\right)}\right]$$

$$\left(\frac{1-\beta\theta}{\chi}\right)^{\frac{1}{\rho}}p_H(\hat{\varphi}(t))^{1-\mu}P_t^{\left(\mu-1-\frac{1}{\rho}\right)}E_{t-1}\left[\left(\frac{1}{M_t}\right)\left(1-\frac{\alpha M_t E_{t-1}\left[M_t^{\frac{1}{\rho}}\right]}{E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}\right]}\right)M_t^{\frac{1}{\rho}}\right] = fE_{t-1}\left[\frac{1}{M_t}\right]$$

$$\left(\frac{1-\beta\theta}{\chi}\right)^{\frac{1}{\rho}}p_H(\hat{\varphi}(t))^{1-\mu}P_t^{\left(\mu-1-\frac{1}{\rho}\right)}E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}\left(1-\frac{\alpha M_t E_{t-1}\left[M_t^{\frac{1}{\rho}}\right]}{E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}\right]}\right)\right] = fE_{t-1}\left[\frac{1}{M_t}\right]$$

Substituting the definitions for $p_H(\hat{\varphi}(t),t)$ and P_t given in equations (15) and (22) from the main text, the expression becomes

$$N^{\frac{1-\rho(\mu-1)}{\rho(\mu-1)}}\left(\frac{\alpha}{\kappa}\right)^{\frac{1}{\rho}}\left(\frac{E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}\right]}{E_{t-1}\left[M_t^{\frac{1}{\rho}}\right]}\right)^{\frac{1}{\rho}}\hat{\varphi}_H^{\mu-1}(t)\bar{\varphi}_t^{\frac{1-\rho(\mu-1)}{\rho}}E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}\left(1-\frac{\alpha M_t E_{t-1}\left[M_t^{\frac{1}{\rho}}\right]}{E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}\right]}\right)\right] = fE_{t-1}\left[\frac{1}{M_t}\right].$$

It is now possible to isolate $\hat{\varphi}_H(t)$ and obtain equation (27) in the main text:

$$\hat{\varphi}_H(t) = \left(\frac{fE_{t-1}[\frac{M_{t-1}}{M_t}]}{a_1 M_{t-1}(1-\alpha) E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}} \right]} \right)^{\frac{1}{\mu-1}} \bar{\varphi}_t^{\frac{\rho(\mu-1)-1}{\rho(\mu-1)}}, \tag{c.1}$$

where $a_1 = N^{\frac{1-\rho(\mu-1)}{\rho(\mu-1)}} \left(\frac{\alpha E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}} \right]}{\kappa E_{t-1}\left[M_t^{\frac{1}{\rho}} \right]} \right)^{\frac{1}{\rho}}$.

The same process using the ZCP for Foreign firms operating in the Home country yields equation (28),

$$\hat{\varphi}_F(t) = \left(\frac{f_{MNE}E_{t-1}[\frac{M^*_{t-1}}{M^*_t}]}{a_1 M_{t-1}(1-\alpha) E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}} \right]} \right)^{\frac{1}{\mu-1}} \bar{\varphi}_t^{\frac{\rho(\mu-1)-1}{\rho(\mu-1)}}. \tag{c.2}$$

C.4 The Existence of $\hat{\varphi}_H(t)$

Let Z be defined by

$$Z = \hat{\varphi}_H(t) - \left(\frac{fE_{t-1}[\frac{M_{t-1}}{M_t}]}{a_1 M_{t-1}(1-\alpha) E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}} \right]} \right)^{\frac{1}{\mu-1}} \bar{\varphi}_t^{\frac{\rho(\mu-1)-1}{\rho(\mu-1)}}.$$

Using the definition of $\bar{\varphi}_j(t)$ given in Section 2.5 and recalling that $n_j = 1 - G(\hat{\varphi}_j(t))$, Z can be rewritten:

$$= \hat{\varphi}_H(t) - \Gamma_1 \left(\int_{\hat{\varphi}_H(t)}^{\infty} \varphi_H^{\mu-1}(t) g(\varphi) d\varphi + \int_{\frac{1}{\gamma}\hat{\varphi}_H(t)}^{\infty} \varphi_F^{\mu-1}(t) g(\varphi) d\varphi \right)^{\frac{\rho(\mu-1)-1}{\rho(\mu-1)^2}},$$

where $\Gamma_1 = \left(\frac{fE_{t-1}[\frac{M_{t-1}}{M_t}]}{a_1 M_{t-1}(1-\alpha)E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}} \right]} \right)^{\frac{1}{\mu-1}}$, which is always positive as long as $\mu > 1$. The partial derivative of Z with respect to $\hat{\varphi}_H(t)$ is

$$\frac{\partial Z}{\partial \hat{\varphi}_H(t)} = 1 - \Gamma_1 \left(\frac{\rho(\mu-1)-1}{\rho(\mu-1)^2} \right) *$$
$$\left(\int_{\hat{\varphi}_H(t)}^{\infty} \varphi_H^{\mu-1}(t) g(\varphi) d\varphi + \int_{\frac{1}{\gamma}\hat{\varphi}_H(t)}^{\infty} \varphi_F^{\mu-1}(t) g(\varphi) d\varphi \right)^{\frac{\rho(\mu-1)-1-\rho(\mu-1)^2}{\rho(\mu-1)^2}} *$$
$$(-\hat{\varphi}_H(t) g(\hat{\varphi}_H(t)) - \frac{1}{\gamma}\hat{\varphi}_H(t) g(\hat{\varphi}_H(t))),$$

or, rearranging,

$$\frac{\partial Z}{\partial \hat{\varphi}_H(t)} = 1 + \Gamma_1 \left(\frac{\rho(\mu-1)-1}{\rho(\mu-1)} \right) *$$

$$\left(\int_{\hat{\varphi}_H(t)}^{\infty} \varphi_H^{\mu-1}(t) g(\varphi) d\varphi + \int_{\frac{1}{\gamma}\hat{\varphi}_H(t)}^{\infty} \varphi_F^{\mu-1}(t) g(\varphi) d\varphi \right)^{\frac{\rho(\mu-1)-1-\rho(\mu-1)^2}{\rho(\mu-1)^2}} *$$

$$\left(\frac{\gamma+1}{\gamma} \right) \hat{\varphi}_H(t) g(\hat{\varphi}_H(t)).$$

The derivative implies that Z is monotonically increasing as long as Γ_1 is a real number and has the same sign as $\frac{\rho(\mu-1)-1}{\rho(\mu-1)}$. This will be the case if both Γ_1 and $\frac{\rho(\mu-1)-1}{\rho(\mu-1)}$ are greater than zero, which will be the case if (1) $\mu > 1$ and (2) $\rho(\mu-1)-1 > 0$. The first is true by assumption and fits empirical estimates of μ estimated in Feenstra (1994) and Bergin (2003). The second condition implies that $\mu > \frac{1}{\rho} + 1$ which, based on previous empirical estimates of μ, is a mild or nonbinding restriction for estimated values of ρ ($1 < \rho < 6$, as reported in Deaton (1992, p.73).

Appendix D: The Case of Incomplete Markets– A Static Example

D.1 Consumer Behavior

In this one-period version of the model without bonds and with logarithmic utility in consumption, the representative consumer in the Home country maximizes lifetime utility subject to an income and cash-in-advance (CIA) constraint:

$$\max U(C, L)$$

$$s.t.\ PC = wL + \Pi = M \tag{d.1}$$

where U is a function of aggregate consumption, C, and labor, L,

$$U = \ln(C) - \kappa L.$$

Aggregate consumption is still an index reflecting preferences with a constant elasticity of substitution (CES) across the set of all available goods,

$$C = \left[\int_0^{n_H(t)} c_H(i)^{\frac{\mu-1}{\mu}} di + \int_1^{1+n_F(t)} c_F(i)^{\frac{\mu-1}{\mu}} di \right]^{\frac{\mu}{\mu-1}}.$$

First order conditions yield the same wage relation,

$$w = \kappa PC. \tag{d.2}$$

The aggregate price index and demand equations are unchanged:

$$P = \left(\int_0^{n_H(t)} p_H(i)^{1-\mu} di + \int_1^{1+n_F(t)} p_F(i)^{1-\mu} di \right)^{\frac{1}{1-\mu}} \tag{d.3}$$

$$c_H(i) = \left(\frac{p_H(i)}{P} \right)^{-\mu} C \qquad c_F(i) = \left(\frac{p_F(i)}{P} \right)^{-\mu} C \tag{d.4}$$

Multiplying by $p_H(i)$, the expression for total expenditure on a particular Home good in the Home country, $r_H(i)$, is derived

$$r_H(i) = \left(\frac{p_H(i)}{P}\right)^{1-\mu} R,$$

where R is total consumer expenditure in the Home country. Note that in the presence of a cash-in-advance constraint, total expenditure equals the money supply, ($M = PC = R$) so that

$$r_H(i) = \left(\frac{p_H(i)}{P}\right)^{1-\mu} M, \quad \text{(d.5)}$$

or $r_H(i) = Mp_H(i)^{1-\mu}P^{\mu-1}$.

D.2 Firm Behavior

Firms each produce a unique good and a have a different productivity index, φ, drawn each period as an independent, identically distributed random variable at the point in time the firm decides to enter the industry. Production is linear in labor and is characterized by the technology

$$c_H(i) = \varphi_H(i)l_H(i), \quad \text{(d.6)}$$

where $\varphi_H(i)$ is the productivity draw of a Home firm i and $l_H(i)$ is the quantity of labor used by this firm in its domestic plant. Variables for consumption and production activity in the Foreign country are denoted by an asterisk, so that the identical technology for production by a Home firm abroad is represented by

$$c^*_H(i) = \varphi_H(i)l^*_H(i).$$

The firm seeks to maximize the expected market value of total nominal profits from domestic and overseas plants. Producers anticipate potential fluctuations in demand and wages in the host country as a result of volatility in the host-country money supply. In addition, they consider potential fluctuations in the exchange rate, which could occur due to shocks to either the host-country or its own money supply (that is, to M or M^*), when deciding whether to enter the overseas market. Firms put a subjective value on each potential state of the economy using a stochastic discount factor, U_c, for Home firms and U_{c^*} for Foreign firms. Marginal utility is used as a discount factor because it measures how much a shock will impact the well-being of the consumers who own the firm (Cochrane 2001, Chapter 1). Thus, the firm's problem is to decide whether and how much to produce in its own country or in both countries:

$$\max E[U_c\pi^T_H(i)],$$

where

$$\pi^T_H(i) = E[U_c\pi_H(i)] + \max\left\{0, E[U_c\pi^*_H(i)]\right\},$$

$$\pi_H(i) = p_H(i)c_H(i) - wl_H(i) - f, \quad \text{(d.7)}$$

and

$$\pi^*_H(i) = Sp^*_H(i)c^*_H(i) - Sw^*l^*_H(i) - f^*_{MNE}. \quad \text{(d.8)}$$

Each Home company pays a fixed overhead cost, f, in the Home country and f^*_{MNE} in the Foreign country (if it invests overseas to become a multinational enterprise) that must be subtracted to determine profit. Labor is hired and fixed *overhead* costs are paid after the firm finds out its unique productivity draw, $\varphi_H(i)$. Therefore, to calculate profits from operations abroad, a Home firm takes into account the exchange rate, S, at which it will have to pay wages for Foreign workers and repatriate revenues earned overseas, as well as the fixed overhead costs. It is useful to emphasize here that this is a model of *horizontal* direct investment, where a firm produces a unique good in multiple countries, but for local consumption in each country. The model abstracts from cross-border flows of physical capital.[7] FDI in the case here is the payment of some fixed cost (f^*_{MNE}, denominated in the currency of the payee– in this case, the Home firm) to gain entry into the local market for a particular period.

Substituting $\frac{c_H(i)}{\varphi_H(i)}$ for $l_H(i)$ and taking the derivative of $E[U_c\pi^T_H(i)]$ with respect to $c_H(i)$, one can determine the firm's pricing behavior:

$$\frac{\partial E[U_c\pi^T_H(i)]}{\partial c(i)} : E\left[U_c\left(c_H(i) + \left(\frac{\partial p_H(i)}{\partial c_H(i)}\right)c_H(i) - \frac{w}{\varphi(i)}\right)\right] = 0$$

$$E\left[C^{-\rho}\left(c_H(i) + \left(-\frac{1}{\mu}\right)c_H(i) - \frac{wc_H(i)}{\varphi(i)p_H(i)}\right)\right] = 0$$

$$p_H(i) = \left(\frac{\mu}{\mu-1}\right)\frac{E\left[C^{-\rho}\left(\frac{wc_H(i)}{\varphi(i)}\right)\right]}{E\left[C^{-\rho}c_H(i)\right]}.$$

A firm will set a price for its unique good equal to a fixed markup over the expected discounted marginal cost. Defining $\alpha = \frac{\mu-1}{\mu}$– the inverse of the markup– and noting that the firm knows its own productivity level at the time it makes its production decisions, the Home firm's pricing rule for its good in the domestic market can be written as

$$p_H(\varphi(i)) = \frac{E\left[U_c w c_H(i)\right]}{\alpha\varphi(i)E\left[U_c c_H(i)\right]}.$$

An analogous equation applies for the firm's pricing in the Foreign market. Expressed in terms of Foreign currency, the Home firm's price abroad is

$$p^*_H(\varphi(i)) = \frac{E\left[U_c S w^* c^*_H(i)\right]}{\alpha\varphi(i)E\left[U_c S c^*_H(i)\right]}$$

For foreign firms, the pricing rules are

$$\begin{aligned} p_F(\varphi(i)) &= \frac{E\left[U_{c*}\left(\frac{w}{S}\right)\right]}{\alpha\varphi(i)E\left[U_{c*}\left(\frac{1}{S}\right)\right]} \\ p^*_F(\varphi(i)) &= \frac{E\left[U_{c*}w^*c^*_F(i)\right]}{\alpha\varphi(i)E\left[U_{c*}c^*_F(i)\right]} \end{aligned}$$

Substituting the CIA constraint and the wage relation, equation (d.2), the pricing rules in (d.9)

[7] Cross-border capital flows, where overseas capital is used in domestic production using domestic technology, are distinct from FDI, where firms use a common technology to produce identical goods in multiple countries. See Russ (2002) for a more detailed discussion and a literature review of studies modeling cross-border capital flows.

become

$$p_H(\varphi(i)) = \frac{\kappa E[M]}{\alpha\varphi(i)} \tag{d.9a}$$

$$p_H^*(\varphi(i)) = \frac{\kappa E\left[\left(\frac{M^*}{M}\right) SM^*\right]}{\alpha\varphi(i) E\left[\left(\frac{M^*}{M}\right) S\right]} \tag{d.9b}$$

$$p_F(\varphi(i)) = \frac{\kappa E\left[\left(\frac{M}{M^*}\right)\frac{M}{S}\right]}{\alpha\varphi(i) E\left[\left(\frac{M}{M^*}\right)\left(\frac{1}{S}\right)\right]} \tag{d.9c}$$

$$p_F^*(\varphi(i)) = \frac{\kappa E[M^*]}{\alpha\varphi(i)} \tag{d.9d}$$

D.3 Aggregation

In equilibrium, there will be a continuum of goods produced by Home firms over $[0, n_H(t))$ and by Foreign firms over $[1, n_F(t)]$ $(n_H(t), n_F(t) \leq 1)$in the Home market. There is also an equilibrium distribution of productivity levels– $\eta_H(\varphi, t)$ for Home firms and $\eta_F(\varphi, t)$ for Foreign firms, each with positive support over a subset of $(0, \infty)$. As in Section 2.4 of the main text, the aggregate price index is computed as

$$P = \left[\int_0^{n_H(t)}\int_0^{\infty} p_H(\varphi(i), t)^{1-\mu}\eta_H(\varphi)\, d\varphi di + \int_1^{1+n_F(t)}\int_0^{\infty} p_F(\varphi(i), t)^{1-\mu}\eta_F(\varphi)\, d\varphi di\right]^{\frac{1}{1-\mu}}$$

The pricing rules, (d.9a) and (d.9b); the wage relation (d.2); and the CIA constraint can be used to reduce this expression to a function of the number of firms and the average per-unit labor cost. That is,

$$P = \left[\begin{array}{c} \int_0^{n_H}\int_0^{\infty}\left(\frac{\kappa E[M]}{\alpha\varphi(i)}\right)^{1-\mu}\eta_H(\varphi)\, d\varphi di + \\ \int_1^{1+n_F}\int_0^{\infty}\left(\frac{\kappa E\left[\left(\frac{M}{M^*}\right)\frac{M}{S}\right]}{\alpha\varphi(i) E\left[\left(\frac{M}{M^*}\right)\left(\frac{1}{S}\right)\right]}\right)^{1-\mu}\eta_F(\varphi)\, d\varphi di \end{array}\right]^{\frac{1}{1-\mu}}.$$

The distribution of productivity levels from which firms draw is the same for each firm (for all i), so that the expression can be simplified:

$$\begin{aligned} P &= \frac{\kappa}{\alpha}\left[n_H b_1 \int_0^{\infty}\varphi^{\mu-1}\eta_H(\varphi)\, d\varphi + n_F b_2 \int_0^{\infty}\varphi^{\mu-1}\eta_F(\varphi)\, d\varphi\right]^{\frac{1}{1-\mu}} \\ &= \frac{\kappa}{\alpha}\left[n_H b_1 \int_0^{\infty}\varphi^{\mu-1}\eta_H(\varphi)\, d\varphi + n_F b_2 \int_0^{\infty}\varphi^{\mu-1}\eta_F(\varphi)\, d\varphi\right]^{\frac{1}{1-\mu}}, \end{aligned}$$

where $b_1 = (\kappa E[M])^{1-\mu}$ and $b_2 = \left(\frac{\kappa E[(\frac{M}{M^*})\frac{M}{S}]}{E[(\frac{M}{M^*})(\frac{1}{S})]}\right)^{1-\mu}$. Let $\bar{\varphi}_H$ and $\bar{\varphi}$, defined by

$$\begin{aligned}\bar{\varphi}_H &= \left(\int_0^{\infty} \varphi_H{}^{\mu-1}\eta_H(\varphi)\,d\varphi\right)^{\frac{1}{\mu-1}} \\ \bar{\varphi}_F &= \left(\int_0^{\infty} \varphi_F{}^{\mu-1}\eta_H(\varphi)\,d\varphi\right)^{\frac{1}{\mu-1}},\end{aligned}$$

be the average level of productivity of Home and Foreign firms, respectively, operating in the Home economy.[8] Then, the average per-unit labor cost for the entire Home economy is

$$\bar{\varphi} = \left[\frac{n_H b_1}{N}\bar{\varphi}_H + \frac{n_F b_2}{N}\bar{\varphi}_H\right]^{\frac{1}{\mu-1}}, \tag{d.10}$$

This average per-unit labor cost, $\bar{\varphi}$, is the aggregate level of productivity in the Home economy weighted by Home and Foreign-owned firms' expectations of the subjectively discounted nominal wage. It is useful to note that the weights b_1 and b_2 would be equal if $S = \frac{M}{M^*}$.

Using (d.10), the aggregate price level can now be expressed as

$$P = \frac{N^{\frac{1}{1-\mu}}\kappa}{\alpha\bar{\varphi}}. \tag{d.11}$$

D.4 Equilibrium Conditions

D.4.1 The Cutoff Productivity Level

Given the distribution of productivity levels, there will be some Home firm with productivity level $\hat{\varphi}_H$ such that profits equal zero. Any producer with a productivity level below $\hat{\varphi}_H$ will immediately exit the market, knowing that it will realize negative profits should it remain in the industry. Hence, a zero-cutoff profit condition (ZCP) can be defined for the Home firm operating in the Home and Foreign markets,

$$E[U_c\pi_H(\hat{\varphi}_H)] = 0 \tag{d.12a}$$
$$E[U_c\pi_H^*(\hat{\varphi}_H^*)] = 0, \tag{d.12b}$$

and the Foreign firm in the Foreign in the Home and Foreign markets,

$$E[U_{c^*}\pi_F(\hat{\varphi}_F)] = 0 \tag{d.12c}$$
$$E[U_{c^*}\pi_F^*(\hat{\varphi}_F^*)] = 0 \tag{d.12d}$$

where $\hat{\varphi}_j^*$ $(j \in \{H, F\})$ represents the cutoff level for the Home- and Foreign-owned firm in the Foreign market. Using the ZCP condition, one can derive the profit level of the average Home firm in the domestic market as a function of $\hat{\varphi}_H$ and f.

Beginning with the definition of domestic profits from equation (d.7) and making the appropriate substitutions using the CIA constraint and the wage relation (expressions (d.1) and (d.2)), we have

$$E[U_c\pi_H(\varphi)] = E[U_c(p_H(\varphi)c_H(\varphi) - wl_H(\varphi) - f)]$$

[8] In actuality, $\bar{\varphi}_H$ and $\bar{\varphi}_F$, are expressions of the production-weighted harmonic mean of productivity levels for Home and Foreign firms operating in the Home economy.

Substituting the expressions for revenue (d.5), the aggregate price level (d.11), and the pricing rule (d.9a), a firm's expected discounted profits can be written in terms of the domestic money supply, the markup, and its relative productivity level,

$$E[U_c \pi_H(\varphi(i))] = \left(\frac{\varphi(i)}{\bar{\varphi}}\right)^{\mu-1} E[(1-\alpha\left(\frac{1}{E[M]}\right)M)] - fE[\frac{1}{M}]. \tag{d.13}$$

Similarly, for a Foreign firm investing in the home country as a multinational enterprise, expected discounted profits from operations in the Home country (expressed in terms of the Foreign-country currency) are represented by

$$E[U_{c^*}\pi_F(\varphi(i))] = \left(\frac{\varphi(i)}{\bar{\varphi}}\right)^{\mu-1} E[\left(\frac{M}{M^*}\right)\left(\frac{1}{S}\right)M(1-\alpha\left(\frac{\kappa E\left[\left(\frac{M}{M^*}\right)\frac{1}{S}\right]}{\alpha\varphi_F E\left[\left(\frac{M}{M^*}\right)\left(\frac{M}{S}\right)\right]}\right)M)] - f_{MNE}E[\frac{1}{M^*}]. \tag{d.14}$$

The equations for expected discounted profits are very similar for Home- and Foreign-owned firms. The principle differences lie in two important places: (1) the respective discount factors, which are rooted in the monetary conditions expected to emerge in each firm owner's *native* country;[9] and (2) the explicit introduction of the exchange rate, $\frac{1}{S}$, into the Foreign firm's calculation of expected revenues. One can observe that if the exchange rate were fixed and conditions in both countries were governed by a common monetary innovation or monetary authority, a Home and Foreign firm's expected discounted profits from sales in the Home-country market would be distinguishable only by their unique productivity levels.

Combining the ZCP conditions (d.12a) and (d.12b) with the semi-reduced form equations for expected profits in (d.13) and (d.14), one attains expressions relating the cutoff productivity levels for both Home- and Foreign-owned firms to the average per-unit production cost in the Home economy:

$$\hat{\varphi}_H = \left(\frac{fE[\frac{1}{M}]}{E[(1-\alpha\left(\frac{1}{E[M]}\right)M)]}\right)^{\frac{1}{\mu-1}} \bar{\varphi} \tag{d.15}$$

$$\hat{\varphi}_F = \left(\frac{f_{MNE}E[\frac{1}{M^*}]}{E[\left(\frac{M}{M^*}\right)\left(\frac{1}{S}\right)(1-\alpha\left(\frac{\kappa E\left[\left(\frac{M}{M^*}\right)\frac{1}{S}\right]}{\alpha\varphi_F E\left[\left(\frac{M}{M^*}\right)\left(\frac{M}{S}\right)\right]}\right)M)]}\right)^{\frac{1}{\mu-1}} \bar{\varphi} \tag{d.16}$$

It is assumed that the distribution of possible productivity levels for firm i, $g(\varphi)$ is the same for all firms. It is also useful to generalize the notation for the cutoff productivity level: $\hat{\varphi}_j = \varphi(\hat{\imath}_j)$. A Home firm which draws $\varphi < \hat{\varphi}_H$, for instance, will immediately exit– before initiating production. The distribution of successful firms' productivity levels, $\eta_j(\varphi)$, is therefore the probability of drawing a particular φ, given that $\varphi \geq \hat{\varphi}_j$. Let $G(\varphi)$ be the cumulative distribution of the probability density $g(\varphi)$. Then, as in Melitz (2002), the equilibrium distribution for Home firms is therefore defined by

$$\eta_H(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\hat{\varphi}_H)} \text{ if } \varphi \geq \hat{\varphi}_H \\ 0 \text{ if } \varphi < \hat{\varphi}_H. \end{cases} \tag{d.17}$$

[9] Explicitly, the discount factor for residents of the Home country is $E[U_c] = E[C^{-1}] = E[M^{-1}]$, whereas the discount factor for residents of the Foreign country is $E[U_{c^*}] = E[C^{*-1}] = E[M^{*-1}]$.

There is a similar distribution for Foreign firms operating in the Home market,

$$\eta_F(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\hat{\varphi}_F)} \text{ if } \varphi \geq \hat{\varphi}_F \\ 0 \text{ if } \varphi < \hat{\varphi}_F^*. \end{cases} \tag{d.18}$$

From (d.17) and (d.18), it is clear that the average per-unit labor cost, equation (d.10), is actually a function of the cutoff productivity levels:

$$\bar{\varphi} = \bar{\varphi}(\hat{\varphi}_H, \hat{\varphi}_F) = \left[\frac{n_H}{N}\bar{\varphi}_H^{\mu-1} + \frac{n_F}{N}\bar{\varphi}_F^{\mu-1}\right]^{\frac{1}{\mu-1}}, \tag{d.19}$$

where

$$\bar{\varphi}_H = \bar{\varphi}(\hat{\varphi}_H) = \left[\frac{1}{1-G(\hat{\varphi}_H)}\int_{\hat{\varphi}_H}^{\infty} \varphi^{\mu-1} g(\varphi) d\varphi\right]^{\frac{1}{\mu-1}} \tag{d.20a}$$

$$\bar{\varphi}_F = \bar{\varphi}(\hat{\varphi}_F) = \left[\frac{1}{1-G(\hat{\varphi}_F)}\int_{\hat{\varphi}_F}^{\infty} \varphi^{\mu-1} g(\varphi) d\varphi\right]^{\frac{1}{\mu-1}} \tag{d.20b}$$

Thus, (d.15) and (d.16) provide two equations with three unknows: the cutoff productivity levels for the Home market, $\hat{\varphi}_H$ and $\hat{\varphi}_F$, and the nominal exchange rate, S. Dividing (d.16) by (d.15) and rearranging, one finds $\hat{\varphi}_F$ as a function of $\hat{\varphi}_H$ and S:

$$\hat{\varphi}_F = \left(\frac{\frac{f E[\frac{1}{M}]}{E[(1-\alpha\left(\frac{1}{E[M]}\right)M)]}}{\frac{f_{MNE} E[\frac{1}{M^*}]}{E[\left(\frac{M}{M^*}\right)\left(\frac{1}{S}\right)(1-\alpha\left(\frac{\kappa E\left[\left(\frac{M}{M^*}\right)\frac{1}{S}\right]}{E\left[\left(\frac{M}{M^*}\right)\left(\frac{M}{S}\right)\right]}\right)M)]}} \right)^{\frac{1}{\mu-1}} \hat{\varphi}_H \tag{d.21}$$

Finding a reduced form for the nominal exchange rate will allow the system to be solved and indicate how volatility in the monetary variables underlying the exchange rate impact the willingness of Foreign-owned firms to invest in the Home-country market.

D.4.2 The Foreign Exchange Market

Thus, to solve the model, it is necessary to obtain a solution for the nominal exchange rate. In the case of incomplete markets, this must be done by introducing a market-clearing condition in the market for foreign exchange.[10] First, it is noted that by assumption, all goods consumed by Home-country residents must be produced and purchased within the Home country, in Home-country currency. An analogous rule applies for goods consumed by residents of the Foreign country. Therefore, all revenues earned by Home-country firms from sales in the Foreign market must be expatriated to the Home country, and revenues earned by Foreign-owned firms in the Home market must also be expatriated to the Foreign country. The exchange of revenues from overseas plants is made at an exchange rate, S, which is the equilibrating engine in the market for foreign currency.

The market-clearing condition[11] for foreign currency can therefore be expressed as

$$n_F\left(p_F(\bar{\varphi}_F)c_F(\bar{\varphi}_F) - w l_F(\bar{\varphi}_F)\right) = S n_H^*\left(p_H^*(\bar{\varphi}_H^*)c_H^*(\bar{\varphi}_H^*) - w^* l_H^*(\bar{\varphi}_H^*)\right), \tag{d.22}$$

[10] In the intertemporal complete-markets case with money in the utility function introduced in Section 2, the nominal exchange rate is fully specified from the first-order conditions for money and bonds. The for-ex market in that case clears entirely due to consumer's ability to share risk, without a separate market-clearing condition.

[11] The condition here is specified in the spirit of Bacchetta and van Wincoop's (2000) foreign exchange market in their model of exchange rate volatility and trade.

As defined in (d.20a), $\bar{\varphi}_F$ is the average productivity level of Foreign firms in the Home country and $\bar{\varphi}_H^*$ is the same index for Home firms operating abroad. If both countries are identical, with identical processes for monetary shocks, the average level of productivity of overseas plants will be the same for Home- and Foreign-owned firms ($\bar{\varphi}_F = \bar{\varphi}_H^*$), the proportion of prospective Home- and Foreign-owned entrants that choose to invest abroad will be equal ($n_F = n_H^*$), as will the average price multinationals charge for their good in overseas markets. Suppose that

$$S = \frac{M}{M^*}.$$

Then we have a symmetric equilibrium and, if M and M^* are lognormally distributed with a mean-preserving spread, similar to the growth rates in the main text, equation (d.21) becomes

$$\frac{\hat{\varphi}_F}{\hat{\varphi}_H} = \left(\frac{f E[\frac{1}{M}]}{f_{MNE} E[\frac{1}{M^*}]} \right)^{\frac{1}{\mu-1}}, \tag{d.24}$$

which relates the threshold productivity level of Foreign firms operating in the Home economy to that of the Home firms in terms of the fundamental variables, M and M^*; the fixed costs, f and f_{MNE}; the elasticity of substitution, μ (recall $\alpha = \frac{\mu-1}{\mu}$); and the coefficient of relative risk aversion, ρ.

D.5 The Effect of Exchange-Rate Volatility on Firm Entry

The relationship between the threshold productivity levels of Home and Foreign firms operating in the Home market reveals the effect of exchange-rate volatility on the willingness of Foreign investors to engage in ventures overseas. If $\hat{\varphi}_H$ is greater than $\hat{\varphi}_F$, that means that the least productive Foreign firms will have to be more productive than the least productive Home firms producing in the Home country[12] Thus, one can take the derivative of $\frac{\hat{\varphi}_H}{\hat{\varphi}_F}$ with respect to the underlying parameters to determine how they impact the relative ability of Foreign firms to enter the Home market without expecting to go bankrupt. In addition to permitting equation (d.15) to be solved for $\hat{\varphi}_H$, equation (d.24) provides the expression for the ratio of the productivity level of the least productive Foreign and Home firm which allows one to conduct comparative statics.

To conduct the analysis, it is assumed that the Home and Foreign money supplies are both lognormally distributed. The growth process is defined by

$$M = (1 + \psi)e^{v},$$

where m is log of the Home money supply and ν is randomly distributed with mean $-\frac{1}{2}\sigma_m^2$ and variance σ_m^2. Let the ratio $\frac{\hat{\varphi}_F}{\hat{\varphi}_H}$ in the setting of incomplete asset markets be called γ_{IM}. Under this money supply process assumed, γ_{IM} is equal to the expression in equation (18) in the main text:

$$\gamma_{IM} = \frac{\hat{\varphi}_F}{\hat{\varphi}_H} = \left(\frac{(1+\psi) f_{MNE} e^{\sigma_{m^*}^2 - \sigma_m^2}}{(1+\psi^*) f} \right)^{\frac{1}{\mu-1}} = \gamma. \tag{d.25}$$

[12]Since the cumulative distribution of productivity draws, $G(\varphi)$, is monotonically increasing in φ, the probability that a Foreign firm successfully entering the Home market, $p_{ME} = \frac{1-G(\hat{\varphi}_F)}{1-G(\hat{\varphi}_F^*)}$, will fall as $\hat{\varphi}_F$ increases.