

Discussion of Poole's paper: Choice of the optimal monetary policy

I describe below the steps used in Poole's analysis presented in Section III of his paper. There are three steps:

1. Within a deterministic IS-LM model, solve for equilibrium output, Y , as a function of the instrument - either M (money supply) or r (interest rate). Then determine the optimal choice of r and M given a full employment level of output.

2. Introduce stochastic shocks to both the goods market and the money market. Because of these shocks, output will now be random. We assume that the monetary authorities choose M or r to minimize the variance of output fluctuations. With this objective function, we solve for the optimal values of the instruments. It turns out that these values are identical to those obtained under Step 1. This illustrates *certainty equivalence*.

3. Using these optimal values for a **given** policy, we compare the loss implied by the two alternative instruments. This comparison allows us to pick the **optimal policy**. We demonstrate that if monetary market shocks are small relative to goods market shocks, then M is the optimal instrument; if monetary shocks are relatively large, then r is the optimal instrument.

On to Poole's analysis.

A static, deterministic IS-LM model

We start with a simple IS-LM model

$$Y = a_0 + a_1 r, \quad a_1 < 0 \quad (\text{goods market}) \quad (1)$$

$$M = b_0 + b_1 Y + b_2 r, \quad b_1 > 0, b_2 < 0 \quad (\text{money market}) \quad (2)$$

The first step is to get this structural model into reduced form. That is, express the endogenous variables as a function of the exogenous variables. The reduced form will, therefore, depend on the choice of instrument. If r is the instrument, (Y, M) are endogenous; if M is the instrument (Y, r) are endogenous. Since the focus of this analysis is output, we will derive only the reduced form for Y .

Interest rate as instrument

If the interest rate is the instrument, then (1) is in reduced form already.

$$Y = a_0 + a_1 r \quad (3)$$

We assume that there is a full employment level of output that is the target of the monetary authorities, Y_f . Then the interest rate should be set as:

$$r^* = \frac{Y_f - a_0}{a_1} \quad (4)$$

Money stock instrument

Solve (1) for r and substitute the expression in (2). Solving the resulting expression for Y produces the reduced form:

$$Y = \frac{a_0 b_2 - a_1 b_0}{b_2 + a_1 b_1} + \left(\frac{a_1}{b_2 + a_1 b_1} \right) M \quad (5)$$

Given the target of Y_f , eq. (5) implies that the money stock should be set to:

$$M^* = \frac{Y_f (a_1 b_1 + b_2) - a_0 b_2 + a_1 b_0}{a_1} \quad (6)$$

This completes Step I.

A stochastic IS-LM model

We add shocks to both the goods market and money market:

$$Y = a_0 + a_1 r + u \quad (7)$$

$$M = b_0 + b_1 Y + b_2 r + v \quad (8)$$

where $E(u) = 0$, $E(v) = 0$, $E(u^2) = \sigma_u^2$, $E(v^2) = \sigma_v^2$, $E(uv) = \sigma_{uv}$. Output will now be random. We assume that policy makers want to stabilize output around full employment output, denoted Y_f . This assumption is reflected in the quadratic *loss function*:

$$L = E \left[(Y - Y_f)^2 \right] \quad (9)$$

The goal is to find the optimal setting for r under an interest rate instrument and M under a money stock instrument.

Optimal r^*

Again, eq. (7) already expresses output in reduced form. Hence, substitute this expression in the loss function, eq (9). Then choose r to minimize this function:

$$\min_r E \left\{ [(a_0 + a_1 r + u) - Y_f]^2 \right\} \quad (10)$$

Setting the derivative equal to zero yields:

$$E [2a_1 (a_0 + a_1 r + u - Y_f)] = 0 \quad (11)$$

Dividing through by $2a_1$ and taking expectations of the resulting expression yields:

$$r^* = \frac{Y_f - a_0}{a_1} \quad (12)$$

Note that this is identical to eq. (4) - this illustrates certainty equivalence: because of the quadratic loss function, the value of r^* is the same in both the deterministic and stochastic settings.

Optimal M^*

We do the same steps to find the optimal setting for the money stock. This involves first getting the reduced form for Y using eqs.(7) and (8).

$$Y = \frac{a_0 b_2 + b_2 u - a_1 v + a_1 (M - b_0)}{a_1 b_1 + b_2} \quad (13)$$

Next, use the reduced form to eliminate Y in the loss function. Then the monetary authorities face the following minimization problem:

$$\min_M E \left\{ \left[\left(\frac{a_0 b_2 + b_2 u - a_1 v + a_1 (M - b_0)}{a_1 b_1 + b_2} - Y_f \right) \right]^2 \right\} \quad (14)$$

where the term in parentheses is the reduced form for Y . Again, taking the derivative and setting it equal to zero, we obtain:

$$E \left[\frac{2a_1}{a_1 b_1 + b_2} \left(\frac{a_0 b_2 + b_2 u - a_1 v + a_1 (M - b_0)}{a_1 b_1 + b_2} - Y_f \right) \right] \quad (15)$$

Taking expectations and solving for M yields:

$$M^* = \frac{Y_f (a_1 b_1 + b_2) - a_0 b_2 + a_1 b_0}{a_1} \quad (16)$$

Again, we have certainty equivalence.

The optimal instrument

It is critical to note the distinction between the deterministic and stochastic models. The best *value* of the instrument, whether it is r or M , is unaffected by introducing uncertainty – this is due to certainty equivalence. But Y will still be a random variable in the stochastic model - hence even though the values for the interest rate and money stock in eqs. (12) and (16) respectively are the best under a given instrument policy, we have not yet determined what the optimal policy is. This is done by comparing the loss implied by the two instruments. To do this, we first determine what output will be under the two policies, and then compare the losses.

That is, under an interest rate target, r^* is given by eq. (12). Substitute this into the reduced form for output - eq.(7). This yields:

$$Y_r = Y_f + u \quad (17)$$

Under a money stock target, M^* is given by eq. (16); substitute this into the reduced form for output under this policy - eq.(13). This yields:

$$Y_M = Y_f + \frac{b_2 u - a_1 v}{a_1 b_1 + b_2} \quad (18)$$

We now compare the loss (i.e. the squared deviations from Y_f) implied by the two policies. To do this, substitute eqs. (17) and (18) into the loss function (eq. (9)).

The loss from using an interest rate instrument is:

$$L_r = E \left[(Y_r - Y_f)^2 \right] = E \left[u^2 \right] = \sigma_u^2 \quad (19)$$

The loss from using a money stock instrument is:

$$\begin{aligned} L_M &= E \left[(Y_r - Y_f)^2 \right] = E \left[\left(\frac{b_2 u - a_1 v}{a_1 b_1 + b_2} \right)^2 \right] \\ &= (a_1 b_1 + b_2)^{-2} E \left[b_2^2 u^2 + a_1^2 v^2 - 2a_1 b_2 u v \right] \\ &= (a_1 b_1 + b_2)^{-2} \left[b_2^2 \sigma_u^2 + a_1^2 \sigma_v^2 - 2a_1 b_2 \sigma_{uv} \right] \end{aligned} \quad (20)$$

We can compare the losses under the two policies by taking the ratio of L_M to L_r :

$$\frac{L_M}{L_r} = (a_1 b_1 + b_2)^{-2} \left[b_2^2 + a_1^2 \frac{\sigma_v^2}{\sigma_u^2} - 2a_1 b_2 \frac{\sigma_{uv}}{\sigma_u^2} \right] \quad (21)$$

If the ratio is greater than 1, then an interest rate policy is superior; if the ratio is less than 1, then a money stock instrument is best. In general, it is difficult to determine the magnitude. However, under the assumption that the shocks are independent, $\sigma_{uv} = 0$ and that the income elasticity of money demand is unity, $b_1 = 1$, things get a bit easier. The ratio of losses now becomes:

$$\frac{L_M}{L_r} = (a_1 + b_2)^{-2} \left[b_2^2 + a_1^2 \frac{\sigma_v^2}{\sigma_u^2} \right]$$

$$\frac{L_M}{L_r} < 1$$

If the variance of *LM* shocks is small relative to the variance of *IS* shocks (i.e. $\left(\frac{\sigma_v^2}{\sigma_u^2} \approx 0\right)$, the ratio will be less than one implying a money stock instrument is best. In contrast, if money shocks are dominant, then the ratio will be greater than one implying that using the interest rate as the instrument of monetary policy is optimal.

In reviewing this paper and analysis, it is important to understand the **steps** that were used and the distinction between the stochastic and deterministic models.