ANSWER KEY – HOMEWORK 3

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Problem 1

a. Intertemporal budget constraint is

$$Y_0 = c_0 (1 + \tau_0) + \frac{c_1 (1 + \tau_1)}{1 + r}$$

b. The Lagrangian is:

$$L = \ln c_0 + \beta \ln c_1 + \lambda \left(Y_0 - c_0 (1 + \tau_0) - \frac{c_1 (1 + \tau_1)}{1 + r} \right)$$

The associated necessary conditions are (take derivative of L with respect to c_0, c_1, λ)

$$\begin{aligned} \frac{1}{c_0} - \lambda (1 + \tau_0) &= 0 \\ \beta \frac{1}{c_1} - \lambda \frac{(1 + \tau_1)}{1 + r} &= 0 \\ Y_0 - c_0 (1 + \tau_0) - \frac{c_1 (1 + \tau_1)}{1 + r} &= 0 \end{aligned}$$

c. Solving the above equations for c_0 and c_1 yields the demand functions:

$$\begin{aligned} c_0^* &= Y_0 \left(\frac{1 + r}{2 + r} \right) \frac{1}{1 + \tau_0} \qquad (1) \\ c_1^* &= Y_0 \left(\frac{1 + r}{2 + r} \right) \frac{1}{1 + \tau_1} \end{aligned}$$

d. The indirect utility function is obtained by replacing the arguments of the utility function with the demand functions:

$$\begin{aligned} V (Y_0, \tau_0, \tau_1) &= \ln c_0^* + \beta \ln c_1^* = \\ & - \ln (1 + \tau_0) - \beta \ln (1 + \tau_1) + (1 + \beta) \ln \left[Y_0 \left(\frac{1 + r}{2 + r} \right) \right] \end{aligned}$$

e. The government's budget constraint is:

$$c_0\tau_0 + \frac{c_1\tau_1}{1+r} = G_0$$

f. The Lagrangian associated with the Ramsey problem is (note we have used the demand functions in the government budget constraint):

$$\begin{aligned} L &= -\ln(1+\tau_0) - \beta\ln(1+\tau_1) + \ln\left[Y_0\left(\frac{1+r}{2+r}\right)(1+\beta)\right] + \\ &\quad \mu\left(Y_0\left(\frac{1+r}{2+r}\right)\frac{\tau_0}{1+\tau_0} + Y_0\left(\frac{1+r}{2+r}\right)\frac{\tau_1}{1+\tau_1}\frac{1}{1+r} - G_0\right) \end{aligned}$$

g. The associated necessary conditions are derived by taking the derivative of the government's Lagrangian with respect to taxes and the Lagrange multiplier, μ. Doing this yields (I do not write down the government's budget constraint, i.e. $\frac{\partial L}{\partial \mu}$. Also, I have used the assumption that $\beta = (1+r)^{-1}$)

$$\begin{aligned} -\frac{1}{1+\tau_0} + \mu Y_0\left(\frac{1+r}{2+r}\right)\left(\frac{1}{1+\tau_0} - \frac{\tau_0}{(1+\tau_0)^2}\right) &= 0 \\ -\frac{1}{1+\tau_1} + \mu Y_0\left(\frac{1+r}{2+r}\right)\left(\frac{1}{1+\tau_1} - \frac{\tau_1}{(1+\tau_1)^2}\right) &= 0 \end{aligned}$$

Rearranging

$$\begin{aligned} 1+\tau_0 &= \mu Y_0\left(\frac{1+r}{2+r}\right) \\ 1+\tau_1 &= \mu Y_0\left(\frac{1+r}{2+r}\right) \end{aligned}$$

Since the RHS in the expressions is constant, this implies $\tau_0 = \tau_1 = \tau^*$. This minimizes the deadweight loss due to taxes.

h. Replacing $\tau_0 = \tau_1 = \tau^*$ in the government budget constraint and rearranging terms yields:

$$\tau^* = \frac{G_0}{Y_0 - G_0}$$

Clearly, if government expenditures increase, taxes will have to increase as well. If income increases, this implies that greater consumption will occur in both periods - this increases tax revenues so taxes can be reduced.

Problem 2

In the Ramsey model studied in class (and developed in your reading), it was assumed that $\beta = (1+r)^{-1}$ and $U(c_t) = \ln c_t$. From this we first showed that optimal consumption is given by:

$$c_t^* = \frac{\mathbf{W}}{1+\tau_t}$$

Using this to form the indirect utility function, we then demonstrated that optimal taxes are constant - but this implies consumption is constant. We have from the government budget constraint:

$$\sum_{t=0}^{\infty} \frac{c_t \tau_t}{(1+r)^t} = \mathbf{G}$$

Since both taxes and consumption are constant, total tax revenue each period is given by:

$$c^* \tau^* = \frac{\mathbf{G}}{\sum_{t=0}^{\infty} (1+r)^{-t}} = \frac{r}{1+r} \mathbf{G}$$

Problem 3

You can solve for the demand functions by setting up the Lagrangian, but these are already given in the Mathematica program. Using those functions and the parameters given in the problem yields: $c_1 = 1.489, c_2 = 1.608, n = 0.196, g = 0.402$,utility$= 0.522$.

Problem 4

You are on your own here.