Inflation Targeting in a Simple New Keynesian Model

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Components of the Model

- Expectations augmented Phillips Curve (PC curve)
- Monetary Policy Rule (MPR) derived from policymaker's preferences

Equilibrium

- Short Run: intersection of PC and MPR curves
- Long Run: $\pi^e = \pi^T$ That is: Expected inflation = inflation target

Results

- "New Policy Tradeoff" - volatility tradeoff in the economy
- Characterization of optimal policy in terms of an interest rate rule.

The expectations augmented Phillips Curve

The Phillips curve is expressed as:

$$\pi = \pi^e + ax + e$$

- π^e = expected inflation
- $x = \frac{y - y^n}{y^n}$ where y = GDP and y^n = full employment GDP. Therefore x = % deviation from full employment.
- a = the slope of the Phillips curve
- e = random shock to inflation (say due to oil prices).

Note that the location of the Phillips curve is determined by inflationary expectations and the shock e

Policymaker's preferences

We assume that the policymaker cares about inflation and output volatility:

$$\min \frac{1}{2}\left[k\left(\pi - \pi^{T}\right)^{2} + \lambda x^{2}\right]$$

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- We know from the PC that $\pi=\pi^{e}+ax+e$. We will ignore uncertainty so set $e=0$. Use this to eliminate x.

$$\min_{\pi} \frac{1}{2}\left[k\left(\pi-\pi^{T}\right)^{2}+\lambda\left(\frac{\pi-\pi^{e}}{a}\right)^{2}\right]$$

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$$\min_{\pi} \frac{1}{2}\left[k\left(\pi-\pi^{T}\right)^{2}+\lambda\left(\frac{\pi-\pi^{e}}{a}\right)^{2}\right]$$

- Take the derivative with respect to π and set $=0$.

Monetary Policy Rule

The first-order condition is:

$$k\left(\pi - \pi^T\right) + \lambda\left(\frac{\pi - \pi^e}{a}\right)\frac{1}{a} = 0$$

or

$$k\left(\pi - \pi^T\right) + \frac{\lambda}{a}\left(\underbrace{\frac{\pi - \pi^e}{a}}_{x}\right) = 0$$

or

$$ak\left(\pi - \pi^T\right) = -\lambda x$$

This has standard $MC = MB$ interpretation

Interpreting MC = MB condition

Recall preferences are given by:

$$\min_{\pi} \frac{1}{2}\left[k\left(\pi - \pi^{T}\right)^{2} + \lambda x^{2}\right]$$

Suppose that $x < 0$ (below full employment). Then policymakers need to increase output $\Delta x > 0$.

- Then $MB = -\lambda x \Delta x$ (use minus sign so that $MB > 0$).

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- But along the Phillips curve $\Delta\pi = a\Delta x$ so $MC = k\left(\pi - \pi^{T}\right)a\Delta x$
- Setting $MC = MB$ we have $k\left(\pi - \pi^{T}\right)a\Delta x = -\lambda x \Delta x$

Monetary Policy Rule

Solve this expression for x : $ak\left(\pi - \pi^T\right) = -\lambda x$

$$x = -\left(\frac{k}{\lambda}\right) a \left(\pi - \pi^T\right)$$

- If $\pi > \pi^T$ then Fed needs to create a recession $(x < 0)$: *Leaning against the Wind*

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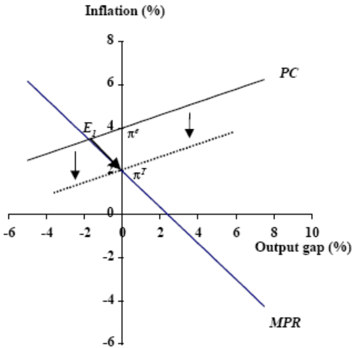
- We add shocks to the economy that affect output: $x = -\left(\frac{k}{\lambda}\right) a \left(\pi - \pi^T\right) + u$
- Solve for π

$$\underbrace{\pi = \pi^T - \alpha\left(x - u\right)}_{Monetary\ \ Policy\ \ Rule\ \ (MPR)}$$

where $\alpha = \left(\frac{\lambda}{k}\right)\frac{1}{a}$. The slope of *MPR* is determined by the relative importance of output/inflation fluctuations to policymaker

Equilibrium Analysis

FIGURE 1
Output and Inflation with Inflation Targeting



Suppose inflation is currently above the inflation target - at the point E_1 in the figure.

- Short run equilibrium is determined by the intersection of $MPR = PC$
- Long run equilibrium determined by $\pi^e = \pi^T = \pi$.

Equilibrium Analysis

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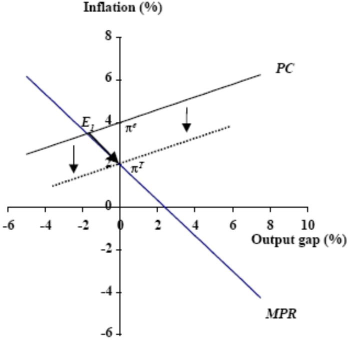
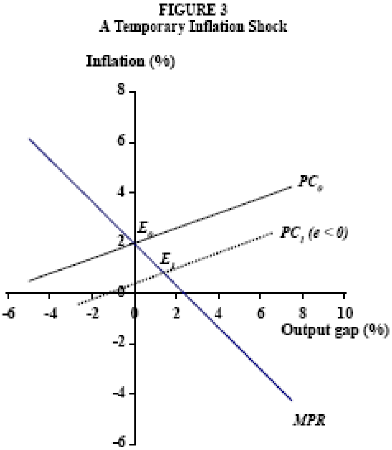


- With $\pi > \pi^T$, the Fed runs a recession $(x < 0)$.
- Note that $\pi < \pi^e$ so over time π^e falls and this causes the PC curve to shift down to restore equilibrium

Negative Inflation Shock (Fall in Oil Prices)

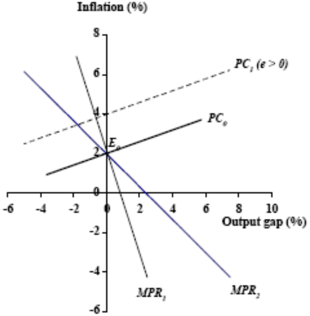


Fed stimulates the economy to offset the fall inflation ($\pi^T = 2\%$ by assumption)

New Policy Tradeoffs

Recall the slope of the *MPR* curve is $-(\lambda/k)\frac{1}{a}$. Consider two economies with $\lambda_1 > \lambda_2$ (same k)

FIGURE 5
The Role of Policy Preferences



With positive inflation shock $(e > 0)$, Economy 2 experiences greater fall in output but smaller inflation increase.

Lars Svensson of Princeton: Central Banks should tell us their λ!

New Policy Tradeoffs

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- Recall that in the bad old days (before rational expectations) - it was thought the policy tradeoff was between the level of inflation and the level of output.

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- Recall that in the bad old days (before rational expectations) - it was thought the policy tradeoff was between the level of inflation and the level of output.
- Now, in the new policy models, the policy tradeoffs are in terms of the implied changes in inflation and output - determined by the slope of the Monetary Policy Rule

Deriving an interest rate rule

Now monetary policy is characterized in terms on an interest rate rule. We can derive this in this setup.

- Start with the IS curve:

$$y = y_0 - br + u \tag{1}$$

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- Start with the IS curve:

$$y = y_0 - br + u \tag{1}$$

- Scale this by full employment and use the Fisher relationship:

$$\frac{y}{y^n} = \frac{y_0}{y^n} - b\left(i - \pi^e\right) + u \tag{2}$$

Deriving an interest rate rule

Define the long run real interest rate when $y = y^n$. and $u = 0$. Use eq. (2)

$$\frac{y}{y^n} = \frac{y_0}{y^n} - b\left(i - \pi^e\right) + u$$

$$\left(\frac{y^n - y_0}{y^n}\right) = -br^* \Longrightarrow r^* = \frac{1}{b}\left(\frac{y_0 - y^n}{y^n}\right) = \frac{1}{b}x_0 \tag{3}$$

Now subtract 1 from both sides of eq. (2) and rearrange terms

$$\frac{y - y^n}{y^n} = \frac{y_0 - y^n}{y^n} - b\left(i - \pi^e\right) + u = x_0 - b\left(i - \pi^e\right) + u$$

$$x = br^* - b\left(i - \pi^e\right) + u = -b\left(\underbrace{i - \pi^e}_{r} - r^*\right) + u \tag{4}$$

$$\text{If } r > r^* \Longrightarrow x < 0$$

Deriving an interest rate rule

Use our two previous key relationships to derive the reduced form model for x

$$\pi = \pi^e + ax + e \ \ (PC)$$

$$\pi = \pi^T - \alpha (x - u) \ \ (MPR)$$

Setting the two expressions equal and solving for x yields (also, once again, we will ignore shocks - set $e = 0$):

$$x = \left(\frac{1}{a+\alpha}\right)\left(\pi^T - \pi^e\right) + \left(\frac{\alpha}{a+\alpha}\right) u$$

We will assume that the Fed can not respond to u (unknown) - so drop from the equation to get the reduced form:

$$x = \left(\frac{1}{a+\alpha}\right)\left(\pi^T - \pi^e\right)$$

Deriving an interest rate rule

Recall the IS curve: $x = -b\left(i - \pi^e - r^*\right)$. Since we are seeking the optimal setting for the nominal interest rate, solve for i

$$i = -\frac{1}{b}x + \pi^e + r^*$$

Since $x = \left(\frac{1}{a+\alpha}\right)\left(\pi^T - \pi^e\right)$ this becomes

$$i = -\frac{1}{b}\left[\left(\frac{1}{a+\alpha}\right)\left(\pi^T - \pi^e\right)\right] + \pi^e + r^*$$

or

$$i = r^* + \pi^e + \frac{1}{b}\left(\frac{1}{a+\alpha}\right)\left(\pi^e - \pi^T\right)$$

Deriving an interest rate rule....Almost Done!!!!

We have

$$i = r^* + \pi^e + \frac{1}{b}\left(\frac{1}{a+\alpha}\right)\left(\pi^e - \pi^T\right)$$

Define the nominal interest rate target as $i^T = r^* + \pi^T$. Introduce this into the above expression by adding and subtracting π^T to the RHS

$$i = \left(r^* + \pi^T\right) + \left(\pi^e - \pi^T\right) + \frac{1}{b}\left(\frac{1}{a+\alpha}\right)\left(\pi^e - \pi^T\right)$$

Or, using the definition of the nominal interest rate target:

$$i = i^T + \underbrace{\left[1 + \frac{1}{b}\left(\frac{1}{a+\alpha}\right)\right]}_{>1}\left(\pi^e - \pi^T\right) + \frac{e}{b\left(a+\alpha\right)}$$

The critical factor is that the nominal interest rate should move more than one-for-one with changes in expected inflation.

The Taylor rule

We have:

$$i = i^T + \underbrace{\left[1 + \frac{1}{b}\left(\frac{1}{a+\alpha}\right)\right]}_{>1}\left(\pi^e - \pi^T\right) + \frac{e}{b\left(a+\alpha\right)}$$

If expected inflation is above the target inflation rate, then the nominal interest rate must increase by a greater amount to raise the real interest rate. This intuition is reflected in the famous Taylor rule that is used to characterize monetary policy.