

MIDTERM ANSWER KEY

Economics 137 - Fall 2007

Prof. Kevin D. Salyer
UC Davis

October 29, 2007

PROBLEM 1: (10 Points)

- federal budget deficit: $\approx 2\%$ of GDP
- defense expenditures: $\approx 5\%$ of GDP
- social security and medicare: $\approx 10\%$ of GDP

PROBLEM 2: (10 Points)

The two functions are

1. government's optimal choice of the tax rate as a function of the capital stock and
2. households' optimal choice of investment (and hence the capital stock) as a function of their expectation of the tax rate.

The intersection of these is a time consistent solution, because it is a situation where agents' expectations of the tax rate exactly match the optimal tax rate from the government's perspective. Likewise households' optimal investment corresponds exactly to the capital stock that induces government to set agents' expected tax rate. Hence, nobody has an incentive to deviate from their choices if allowed to re-optimize.

PROBLEM 3: (10 Points)

- (a) Since the household has no income in the first period the lifetime budget constraint reduces to

$$0 + \left(\frac{1}{1+r}\right)y_1 = c_0 + \left(\frac{1}{1+r}\right)c_1. \quad (1)$$

Hence, the household has to solve the maximization problem

$$\max_{\{c_0, c_1\}} \ln(c_0) + \beta \ln(c_1) + \lambda \left[\frac{y_1 - c_1}{1+r} - c_0 \right], \quad (2)$$

with associated first order conditions:

$$\begin{aligned} \frac{1}{c_0} &= \lambda \\ \frac{1}{c_1} &= \left(\frac{1}{1+r}\right)\lambda. \end{aligned}$$

Elimination of λ yields the familiar *marginal rate of substitution=relative price* representation

$$\frac{\beta c_0}{c_1} = \frac{1}{1+r}. \quad (3)$$

Equation (3) says that the rate at which the household is willing to shift consumption from period 1 to period 0 must equal the relative price at which it can *trade* future consumption for current consumption. Here, the relevant price is the interest rate.

- (b) To find the demand functions we first solve for c_0 as a function of c_1 ,

$$c_0 = \beta^{-1}(1+r)^{-1}c_1 = \frac{1+\rho}{1+r}c_1, \quad (4)$$

where we made use of the definition $\beta \equiv \frac{1}{1+\rho}$, which expresses the time preference *factor*, β , in terms of the subjective time preference *rate*, ρ . The next step is to plug (4) into the budget constraint and solve for c_1 :

$$\begin{aligned} \frac{y_1}{1+r} &= \frac{1+\rho}{1+r}c_1 + \frac{1}{1+r}c_1 \\ y_1 &= (2+\rho)c_1 \\ c_1^* &= \frac{1}{2+\rho}y_1. \end{aligned} \quad (5)$$

Plugging c_1^* back into (4) we can solve for

$$c_0^* = \left(\frac{1}{1+r}\right) \left(\frac{1+\rho}{2+\rho}\right) y_1. \quad (6)$$

The optimal choice of consumption in period 1, c_1^* , does *not at all* depend on the interest rate, r . This makes perfect sense. Suppose the interest rate was zero. Then the household would split the endowment y_1 according to the time preference rate ρ and consume $\tilde{c}_0^* = \left(\frac{1+\rho}{2+\rho}\right) y_1$ in period 0 and $\tilde{c}_1^* = \frac{1}{2+\rho}y_1$ in period 1. If the interest rate was positive, the household could not afford such an allocation and has to reduce either period 0 or period 1 consumption by the cost of borrowing first period consumption. Since period 0 consumption is more expensive (due to the cost of borrowing) the household will reduce period 0 consumption and eventually consume $c_0^* < \tilde{c}_0^*$ in period 0 whereas consumption in period 1 remains unchanged, i.e. $c_1^* = \tilde{c}_1^*$.

- (c) We derive the indirect utility function by plugging the optimal values, c_0^* and c_1^* , into the utility function, which yields

$$\begin{aligned} V(y_1, r) &= \ln \left[\left(\frac{1}{1+r}\right) \left(\frac{1+\rho}{2+\rho}\right) y_1 \right] + \beta \ln \left[\frac{1}{2+\rho} y_1 \right] \\ &= \left[\ln \left(\frac{1+\rho}{2+\rho}\right) - \ln(2+\rho) \right] - \ln(1+r) + (1+\beta) \ln(y_1) \\ &= \gamma - \ln(1+r) + (1+\beta) \ln(y_1), \end{aligned} \quad (7)$$

where $\gamma \equiv \left[\ln \left(\frac{1+\rho}{2+\rho}\right) - \ln(2+\rho) \right]$. Hence, as we would expect, utility (or welfare) increases in the endowment, i.e. $\frac{\partial V(y_1, r)}{\partial y_1} = \frac{1+\beta}{y_1} > 0$, and decreases in r , i.e. $\frac{\partial V(y_1, r)}{\partial r} = -\frac{1}{1+r} < 0$.

PROBLEM 4: (10 Points) The Keynesian consumption function is usually written in the following way

$$C_t(Y_t, \tau_t) = a + b(Y_t - T_t), \quad (8)$$

where C_t denotes consumption in period t , a is autonomous consumption, b is the marginal propensity of consumption, Y_t is income and τ_t denotes (excise) taxes at time t . This function says that an individual's consumption decision *only* depends on the disposable income ($Y_t - T_t$) at time t and is independent of income in other periods of life. This has the effect, that a tax cut at time t , will increase period t consumption no matter whether the tax cut is transitory or permanent.

In contrast, the *permanent income hypothesis* (PIH) assumes that current consumption depends on the *present value* of disposable income. This implies that an infinitely lived individual only cares about permanent changes in the tax rate, because transitory tax cuts (or increases) don't significantly influence *lifetime income*.

The policy implication of the PIH is that transitory changes of the tax rate have virtually no effect on peoples' consumption choices.

PROBLEM 5: (10 Points) There are four crucial assumptions underlying the Ricardian Equivalence result:

1. *Infinitely lived agents*: Only if agents live forever, they will not react to changes in the tax rate, because they know *for sure* that they have to deal with the reverse tax change at some time in the infinite future. If this was not the case, an individual who knew that *the end was near* would definitely eat all the benefits of an unexpected tax cut and *not* save for the future.
2. *perfect capital markets*: This assumption assures that agents can save at the same rate as the government and that they don't face any financial constraints. Only if this is the case households are able to invest the benefits of a tax cut received today in order to fully finance the expected tax increase in the infinite future.
3. *lump sum taxes*: Lump sum taxes are the only form of taxes that do not distort people's behavior, because they are independent of people's choices. While this concept is convenient for many economic theories, we hardly ever observe it in reality. This is one of the reasons why it is very hard to empirically test the RE result.
4. *constant government spending*: Like lump-sum taxes, this assumption is very convenient for theoretic purposes but it is hard to find an economy whose government consumption doesn't change over time. This is another reason why it is very hard to empirically test the RE result.