

## Midterm Exam- Key

*Directions: Answer all questions; the questions are weighted equally. For full credit, you must provide **complete** explanations for your answers.*

1. Provide the dates (roughly) of three post-World War II recessions. Compare and contrast features of post-WWII and pre-WWI features of the business cycle as detailed in C. Romer's article. In your discussion, relate these features to M. Goodfriend's "Go-Stop" characterization of monetary policy.

*Answer: Any of the following dates can be used: 1948, 1953, 1957, 1960, 1969, 1973, 1980, 1981, 1990, 2000. Features of business cycles that are alike: average volatility, length of recessions, severity of recessions. Features that are different: length of expansions, distribution of recessions (more moderate in post-WWII). A good answer will give a description of "go-stop" policy. It appears that many business cycles, especially post 1960 were caused by expansionary monetary policy followed by contractionary policy as inflation became a problem.*

2. When studying policy without a commitment mechanism, Fischer solved for the indirect utility for both the government and households. Describe verbally the nature of the indirect utility function for the government and the problem that it is solving: that is, what are the choice variables for the government and what is taken as exogenous. Provide the same description for the household's maximization problem. Include in your analysis a discussion of the two functions that result from both optimization problems and how these were used to solve for equilibrium.

*Answer: The government makes decisions in the second period only when there is no commitment since tax policy announced in the first period is not credible. In the second period, the government takes as exogenous the capital household's choice in the first period. It also knows the household's demand functions for consumption and labor in the second period. Facing these demand functions and treating the capital stock as exogenous, the government chooses tax rates on capital and labor and the level of government spending. Note that all of these are functions of the beginning of period capital. In particular, the tax rate on capital  $\tau_k = f(k)$ . Household's optimize twice: in the last period, they take the capital stock as exogenous along with the tax rates and choose optimal consumption and labor. This optimization problem defines consumption and labor as functions of the capital stock and tax rates. These are used to define the indirect utility function for household's in the second period. In the first period, agents choose consumption and capital taking as exogenous the tax rates - the utility from capital is determined by the indirect utility function determined in the first optimization problem. Note this defines the choice of capital as a function of the*

tax rate on capital (the optimal tax rate on labor is zero.) That is  $k = g(\tau_k)$ . Equilibrium requires consistency which is a fix-point:  $\tau_k = f(g(\tau_k))$ .

3. Assume that a household lives for two periods and has preferences given by

$$U(c_0, c_1) = \ln c_0 + \beta \ln c_1$$

where  $c_0$  denotes first period consumption,  $c_1$  is second period consumption, and  $0 < \beta < 1$  represents the time discount factor. The household receives income  $Y_0$  in the first period and nothing when old. Consumption in the second period is financed out of savings - the interest rate on savings is constant so that the return on savings is  $(1 + r)$ .

- Set up the household's maximization problem as a Lagrangian and show that the intertemporal marginal rate of substitution is equal to  $(1 + r)$ . (Do **not** assume  $\beta(1 + r) = 1$ .) Interpret this result.
- Derive the household's savings function; i.e. the demand for savings. How responsive are savings to a change in  $r$ ? Interpret your result.
- Solve for the household's indirect utility function.

Answer: The first-order conditions are (after setting up the maximization problem as a Lagrangian - the multiplier is denoted  $\lambda$ ):

$$\frac{1}{c_0} = \lambda \tag{1}$$

$$\beta \frac{1}{c_1} = \lambda / (1 + r) \tag{2}$$

$$Y_0 = c_0 + \frac{c_1}{1 + r} \tag{3}$$

Using eqs. (1), (2) yields:  $c_1 = \beta(1 + r)c_0$ . Using this in the budget constraint yields  $c_0^* = \frac{Y_0}{1 + \beta}$ . Since savings =  $Y_0 - c_0$ , we have  $s = Y_0\beta / (1 + \beta)$ . Note that savings are not affected by the interest rate. This is because the income and substitution effects cancel out. Note that  $c_1^* = \beta(1 + r)Y_0 / (1 + \beta)$ . This implies the indirect utility function is:

$$\ln\left(\frac{Y_0}{1 + \beta}\right) + \beta \ln\left(\frac{\beta(1 + r)Y_0}{1 + \beta}\right)$$

4. The California government is faced with the Ramsey problem of choosing a sequence of excise taxes so that the utility of its citizens is maximized. It uses this revenue to pay for a new economics professor that the UC system plans to hire in the second period of the economy. The government knows that households' optimal consumption is given by:

$$c_t^* = \frac{\frac{r}{1+r} \left( \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^t} \right)}{1 + \tau_t} \tag{4}$$

$Y_t$  denotes income and it is assumed that  $Y_t = \$100,000$  in every period. The government plans to hire the professor by making a one-time payment of  $\$110,000$  in the second period and nothing in all other periods; that is,  $G_t = 0$  for  $t = 0; t \geq 2$ . The interest rate is 0.10. Many politicians are saying that taxes need to be raised next period in order to cover the entire cost of this new (and quite expensive) professor. Their argument is that it is unfair to burden future generations with this one-time expense. You have been invited to give your expert opinion on the matter and demonstrate that the optimal tax (note, this is not the tax rate) is simply  $\$10,000$  (NOTE: this is a mistake - it should have been  $\$9090.91$  - see below) every period. Present your proof and demonstrate that it is optimal for  $\tau^* = r$ . (Hint: Use the conditions that the tax smoothing hypothesis implies taxes are constant,  $\tau_t = \tau^*$ , and the government's intertemporal budget constraint implies that the present discounted value of tax revenues (given by  $c_t \tau_t$  in every period) equals the present discounted value of government expenditures. You do not have to solve for the indirect utility function.)

**Answer:** Given the numbers, optimal consumption is (using  $\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = \frac{1+r}{r}$ )

$$c^* = \frac{100,000}{1 + \tau^*}$$

Tax revenues are therefore

$$c^* \tau^* = 100,000 \frac{\tau}{1 + \tau}$$

The present discounted value of tax revenues must be equal to the present discounted value of government expenditures (again using  $\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = \frac{1+r}{r}$ )

$$100,000 \frac{\tau^*}{1 + \tau^*} \left( \frac{1.10}{0.10} \right) = \frac{110,000}{1.10}$$

which requires  $\tau^* = 0.10$ . Note that tax revenues are

$$c^* \tau^* = 100,000 \frac{\tau}{1 + \tau} = 100,000 \left( \frac{0.10}{1.10} \right) = 9090.91$$

5. What is Ricardian equivalence and what distinguishes this from the “tax-smoothing” hypothesis?

**Answer:** Ricardian equivalence is the proposition that the timing of a deficit financed tax cut does not affect economic activity. This result holds in economies with perfect capital markets, active bequest motives, and lump sum taxation. It is due to the fact that, with those assumption, the only thing that matters to households is the present discounted value of taxes (which is equal to the present discounted value of government expenditures). So the timing of individual taxes is irrelevant. The tax smoothing hypothesis is obtained in a world with active bequest motives and perfect capital markets BUT taxes are distortionary. Now the government wants to minimize the welfare losses associated with distortionary taxes. Given agents care about the path of consumption over their lifetimes, the most efficient tax scheme is one of constant tax rates.