

Lucas's Econometric Policy Evaluation - Investment Tax Credit

The paper presents a model of investment demand with the purpose of analyzing the effects of an investment tax credit. As Lucas notes, typical statistical estimates implicitly assume that the change in the investment tax credit is permanent - but, by nature, the tax credit is constructed to be temporary. (Why is this so?) This model shows that the econometric estimates will severely underestimate the investment response.

Like any market, there are two sides: demand and supply. Supply is the more interesting component and reflects investment decisions made by firms to maximize expected profits. Demand is simply modeled as a linear demand curve with a random intercept.

0.0.1 Supply

We assume a perfectly competitive industry that takes prices as given. Output, y_t , is a linear function of beginning of period capital, k_t .

$$y_t^s = \lambda k_t \tag{1}$$

0.0.2 Demand

Demand is represented by the demand function:

$$y_t^d = a_t - bp_t \tag{2}$$

where a_t is a random variable. This shifts the intercept of the demand curve.

We want to solve for the industry investment demand function. This will be determined by two conditions:

1. Firms are maximizing present discounted value of expected profit(3)
2. Supply is equal to demand.

0.0.3 Firm's Investment decisions

Firms are taxed on profit, i.e. revenue minus costs, at the constant rate, θ . In this model, costs are determined entirely by the depreciation of capital. That is, capital is the only input, so the capital lost in producing output - this is what is meant by depreciation - is the cost. Also, for every unit of investment undertaken, firms receive a rebate of ψ_t which is random. Also, note that

investment at time t does not affect that period's revenue since output depends on beginning of period capital. Investment affects next period's capital and, therefore, output through the law of motion for capital:

$$k_{t+1} = k_t (1 - \delta) + i_t \quad (4)$$

Consider the firm's investment decision: It receives the revenue from sales and then must pay taxes on profits. From the after tax profits, the firm chooses investment; for each unit of investment, the firm receives a tax rebate. Let's look at these terms:

$$\begin{aligned} \text{Revenue} & : p_t y_t \\ \text{Taxes} & : [\theta (p_t y_t - \delta k_t) - \psi_t i_t] \\ \text{Investment} & : i_t \end{aligned}$$

Note that the tax rebate implies that the price of investment in period t is $(1 - \psi_t)$ (This implies that the price of output is measured in terms of capital goods. That is why the price of investment in the absence of the rebate is just 1.) As a consequence, the value of the firm's capital after production is given by:

$$\text{Value of post-production capital: } (1 - \psi_t) k_t (1 - \delta)$$

Let's put these pieces together by looking at the revenue, costs, taxes and investment expenditures for periods t and $t + 1$:

$$\begin{aligned} & p_t y_t - \theta (p_t y_t - \delta k_t) - (1 - \psi_t) i_t + (1 - \psi_t) k_t (1 - \delta) + \quad (5) \\ & \frac{1}{1+r} \left[\begin{array}{l} p_{t+1} y_{t+1} - \theta (p_{t+1} y_{t+1} - \delta k_{t+1}) - \\ (1 - \psi_{t+1}) i_{t+1} + (1 - \psi_{t+1}) k_{t+1} (1 - \delta) \end{array} \right] \end{aligned}$$

Note that choosing investment at time t reduces revenues by the price $(1 - \psi_t)$. But this will increase revenues next period because of the greater capital stock which produces more output. These effects are discounted by the current interest rate. Now expand this over the infinite horizon. The firm's present discounted value of net revenues is

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t [p_t y_t (1 - \theta) + \theta \delta k_t - (1 - \psi_t) i_t + (1 - \psi_t) k_t (1 - \delta)]$$

We are almost ready to find the necessary condition associated with the optimal choice of i_t . First, we want to substitute out for k_{t+1} and y_{t+1} using eqs. (4) and (1). Second, the firm does not know with certainty what future net revenues will be since both the price and the tax rebate are random. Hence, the firm will maximize the *expected value* (remember what this is from your Stats course) of net revenue. Making these two changes yields the following maximization problem:

$$\max_{\{i_t\}_{t=1}^{\infty}} E \left\{ \begin{array}{l} \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t \{ [p_t \lambda (k_{t-1} (1 - \delta) + i_{t-1})] (1 - \theta) + \\ \theta \delta (k_{t-1} (1 - \delta) + i_{t-1}) + (1 - \psi_t) (k_{t-1} (1 - \delta) + i_{t-1}) (1 - \delta) - \\ (1 - \psi_t) i_t \} \end{array} \right\}$$

Taking the derivative with respect to i_t yields the following necessary condition:

$$(1 - \psi_t) = \left(\frac{1}{1+r} \right) \left\{ \underbrace{\lambda(1-\theta) E_t(p_{t+1})}_{\text{revenue}} + \underbrace{\theta\delta}_{\text{depreciation}} + \underbrace{(1-\delta)[1 - E_t(\psi_{t+1})]}_{\text{capital gains or losses}} \right\} \quad (6)$$

The above expression represents the marginal cost = marginal benefit condition associated with optimal investment. The LHS represents the cost at time t of investment. As shown by the expression in brackets, greater investment has three effects: it increases after-tax revenue, it lowers costs through depreciation, and the remaining capital can be sold resulting in capital gains or losses. Since neither next period's price or the tax credit are known at time t , the expected value of these random variables is the relevant term. Remember that the solution of this problem would involve two conditions (see eq. (3)), eq. (6) is the first of these two conditions.

Now we use the other condition, $y_t^s = y_t^d$, to solve for investment demand as a function of the expected tax credit. This involves several steps. 1. Use the market clearing condition to get an expression for p_t . 2. Since this holds every period, agents can use this condition to form the expected value of next period's price, $E_t(p_{t+1})$. 3. We then use eq. (6) to eliminate $E_t(p_{t+1})$. Here we go:

Step 1. Supply equals demand.

$$\lambda k_{t+1} = \lambda(k_t(1-\delta) + i_t) = a_{t+1} - bp_{t+1} \quad (7)$$

Step 2: Forecast of next period's price.

Since eq.(7) holds every period, firm's can forecast next period's price using the equation (this is the assumption of rational expectations). That is, at time t we can take expectations of both sides

$$E_t[\lambda(k_t(1-\delta) + i_t)] = E_t[a_{t+1} - bp_{t+1}]$$

But the left-hand side is known at time t , so this becomes:

$$\lambda(k_t(1-\delta) + i_t) = E_t(a_{t+1}) - bE_t(p_{t+1})$$

or:

$$k_t(1-\delta) + i_t = \frac{1}{\lambda}E_t(a_{t+1}) - \frac{b}{\lambda}E_t(p_{t+1}) \quad (8)$$

Step 3: Eliminate $E_t(p_{t+1})$

The first sub-step is to isolate $E_t(p_{t+1})$ using eq.(6). This involves some algebra. First multiplying by $(1+r)$ yields:

$$(1 - \psi_t)(1+r) = \lambda(1-\theta) E_t(p_{t+1}) + \theta\delta + 1 - E_t(\psi_{t+1}) - \delta + \delta E_t(\psi_{t+1})$$

Isolating the term with the expected price yields:

$$\lambda(1-\theta)E_t(p_{t+1}) = [r + \delta(1-\theta)] - [\psi_t(1+r) - E_t(\psi_{t+1})(1-\delta)]$$

Or

$$E_t(p_{t+1}) = \frac{1}{\lambda} \left[\frac{r}{1-\theta} + \delta \right] - \frac{1}{\lambda(1-\theta)} [\psi_t(1+r) - E_t(\psi_{t+1})(1-\delta)] \quad (9)$$

Now we can substitute this into eq. (8)

$$\begin{aligned} & k_t(1-\delta) + i_t & (10) \\ = & \underbrace{\frac{1}{\lambda}E_t(a_{t+1})}_{(1)} \\ & - \underbrace{\frac{b}{\lambda^2} \left[\frac{r}{1-\theta} + \delta \right]}_{(2)} + \underbrace{\frac{b}{\lambda^2} \left[\frac{\psi_t(1+r) - E_t(\psi_{t+1})(1-\delta)}{(1-\theta)} \right]}_{(3)} \end{aligned}$$

This is the industry demand function for the desired capital stock - that is, the industry investment demand function. First note that the term $\frac{b}{\lambda^2}$ is the slope of the demand curve for capital (it is a combination of how the factors (2) and (3) (to be explained) affect the expected price (see eq. (10)) and how this affects desired capital (see eq. (8)).

The noted terms have the following interpretation:

(1) This is the pure demand effect. If firms expect greater demand tomorrow (a larger intercept term), they want to produce more so they require more capital.

(2) This term represents a “price” effect due to interest rates, depreciation and taxes. In a more complicated model, possibly all three of these terms would be random. Notice that an increase in the real interest rate decreases the demand for investment just as we model in the typical IS curve. Also, greater taxes, θ , would reduce investment demand.

(3) The final term represents expected capital gains or losses (adjusted for taxes) due to expected changes in the investment tax credit. This term is at the heart of the policy analysis. Note that if the investment tax credit is expected to fall, this will increase the price of capital next period. Hence the expected capital gains increases the demand for investment.

0.0.4 Policy Analysis

If you were an economist working for the Council of Economic Adviser’s circa 1974, your task would be estimate the investment demand function given in eq. (10). Given data on (k_t, i_t, ψ_t, a_t) (and taxes and interest rates if these were random), one could use econometric methods to estimate the parameters, (b, λ) .

Lucas then asks the question, “How would one use these estimates to evaluate the consequences of a particular investment tax credit policy?”

The state-of-the art methodology at the time was to assume that the tax credit policy represented a once-and-for-all change in ψ_t . That is, suppose that there was no tax credit in place and then the tax credit goes into effect at time t . The economist models this as $\psi_t = E_t(\psi_{t+1}) = \bar{\psi}$. Substitute this into eq. (10) to yield

$$\Delta k_{t+1} = \frac{b}{\lambda^2} \frac{r + \delta}{1 - \theta} \bar{\psi}$$

If the tax credit truly is permanent, then this estimate is accurate. But the whole purpose of a tax credit is to stimulate investment today rather than tomorrow, so by nature it needs to be temporary. Lucas models this in the following way: Suppose the tax credit can take on two values:

$$\psi_t = \begin{cases} 0 \\ \bar{\psi} \end{cases}$$

The associated conditional transition probabilities are given by

$$\Pr(\psi_{t+1} = \bar{\psi} | \psi_t = 0) = q$$

$$\Pr(\psi_{t+1} = 0 | \psi_t = 0) = 1 - q$$

$$\Pr(\psi_{t+1} = \bar{\psi} | \psi_t = \bar{\psi}) = p$$

$$\Pr(\psi_{t+1} = 0 | \psi_t = \bar{\psi}) = 1 - p$$

With this, we can calculate the conditional expectations (the expectations conditional on the investment tax credit in place at time t)

$$E_0(\psi_{t+1}) = q\bar{\psi} + (1 - q)0 = q\bar{\psi} \quad (11)$$

$$E_{\bar{\psi}}(\psi_{t+1}) = p\bar{\psi} + (1 - p)0 = p\bar{\psi} \quad (12)$$

Now use these to evaluate the term (3) in eq. (10)

$$\text{If } \psi_t = 0 \implies \frac{b}{\lambda^2} \left[\frac{0(1+r) - q\bar{\psi}(1-\delta)}{1-\theta} \right] = \frac{b\bar{\psi}}{\lambda^2(1-\theta)} (-q(1-\delta)) \quad (13)$$

$$\text{If } \psi_t = \bar{\psi} \implies \frac{b}{\lambda^2} \left[\frac{\bar{\psi}(1+r) - p\bar{\psi}(1-\delta)}{1-\theta} \right] = \frac{b\bar{\psi}}{\lambda^2(1-\theta)} (1+r - p(1-\delta)) \quad (14)$$

Now use this to predict the change in the capital stock (the change in investment) if the policy goes from 0 to $\bar{\psi}$

$$\Delta k(0, \bar{\psi}) = k(\bar{\psi}) - k(0) = \frac{b\bar{\psi}}{\lambda^2(1-\theta)} [1+r + (q-p)(1-\delta)] \quad (15)$$

Now consider two extremes cases:

Scenario 1. The policy is hardly ever expected to be offered ($q \approx 0$), but once offered, it is almost permanent ($p \approx 1$). This is the case implicitly assumed by the econometric studies of the day. Then, we have

$$\Delta k_1(0, \bar{\psi}) = \frac{b\bar{\psi}}{\lambda^2(1-\theta)} [r + \delta] \quad (16)$$

Scenario 2. The policy is used frequently ($q \approx 1$), but is temporary ($p \approx 0$). Then we have

$$\Delta k_2(0, \bar{\psi}) = \frac{b\bar{\psi}}{\lambda^2(1-\theta)} (2 + r - \delta)$$

The ratio of the two effects is

$$\frac{\Delta k_2(0, \bar{\psi})}{\Delta k_1(0, \bar{\psi})} = \frac{2 + r - \delta}{r + \delta} \quad (17)$$

Lucas assumes that $r = 0.14, \delta = 0.15$. This implies that

$$\frac{\Delta k_2(0, \bar{\psi})}{\Delta k_1(0, \bar{\psi})} \approx 7$$

Hence, this is a big difference.

Lucas notes that to be able to forecast the effects of policy, we need rules.