
This example is from Stanley Fischer's article, "Dynamic Inconsistency and the Benevolent Dissembling Government," (*Journal of Economic Dynamics and Control* (1980) 93-107.)

```
In[1] := Off[General::"spell1", General::"spell"]
```

- **This model presents a good illustration of dynamic inconsistency within the context of optimal taxation. We examine three cases: a model with no distortionary taxes, a model with distortionary taxes and precommitment and a model with distortionary taxes and no precommitment.**
- **First the model with no distortionary taxes. This introduces the baseline model: The economy lasts for two periods. In the first period agents have an endowment and face a simple consumption/savings decision. In the second period, agents receive income through the returns on capital (i.e. savings) and work effort. This is used to consume and pay taxes. The taxes are used to finance government expenditures. The details are:**

The utility function is:

$$U(c_1, c_2, 1 - n, g) = \ln c_1 + \delta(\ln c_2 + \alpha \ln(1 - n) + \beta \ln g)$$

where

c_1 is first period consumption, c_2 is second period consumption, n is labor supply (the household has one unit of time so $(1-n)$ is leisure) and g is government expenditures.

The parameter δ represents agents' time preference while α and β represent the importance of utility from leisure and government expenditures respectively.

We first solve the problem as a social planner problem - the government knows the preferences of the individuals and chooses the level of consumption, labor and government expenditures to maximize utility.

The first period resource constraint is: (1) $c_1 + k = y$

where k is capital and y is the endowment in the first period.

The second period resource constraint is: (2)

$$c_2 + g = an + Rk$$

where R is the rate of return on capital (this is exogenous) and a is the productivity of labor (the production function is linear). Note that taxes are ignored since in this example they are lump sum and do not influence agents' decisions.

The intertemporal budget constraint is obtained by solving (1) for k and substituting into (2). The maximization problem (written as a Lagrangian is):

$$\begin{aligned} \text{In}[2] := \mathbf{L} = & \\ & \mathbf{Log}[c_1] + \delta (\mathbf{Log}[c_2] + \alpha \mathbf{Log}[1 - n] + \beta \mathbf{Log}[g]) + \\ & \lambda (\mathbf{a} n + \mathbf{R} y - \mathbf{R} c_1 - c_2 - g) ; \end{aligned}$$

Next we obtain the first-order conditions

$$\begin{aligned} \text{In}[3] := \mathbf{focs} = \{ & \mathbf{D}[\mathbf{L}, c_1] == 0, \mathbf{D}[\mathbf{L}, c_2] == 0, \\ & \mathbf{D}[\mathbf{L}, n] == 0, \mathbf{D}[\mathbf{L}, g] == 0, \mathbf{D}[\mathbf{L}, \lambda] == 0 \} \\ \text{Out}[3] = \{ & -R\lambda + \frac{1}{c_1} == 0, -\lambda + \frac{\delta}{c_2} == 0, -\frac{\alpha\delta}{1-n} + a\lambda == 0, \frac{\beta\delta}{g} - \lambda == 0, \\ & -g + an + Ry - Rc_1 - c_2 == 0 \} \end{aligned}$$

Then we solve for the optimal choices: (c_1, c_2, n, g)

$$\begin{aligned} \text{In}[4] := \mathbf{sol} = \mathbf{Flatten} [& \\ & \mathbf{Simplify}[\mathbf{Solve}[\mathbf{focs}, \{c_1, c_2, n, g\}, \lambda]]] ; \end{aligned}$$

$$\text{In}[5] := \{c1^*, c2^*, n1^*, g1^*\} = \{c_1, c_2, n, g\} /. \mathbf{sol}$$

$$\text{Out}[5] = \left\{ \frac{a + Ry}{R(1 + (1 + \alpha + \beta)\delta)}, \frac{(a + Ry)\delta}{1 + (1 + \alpha + \beta)\delta}, \frac{-Ry\alpha\delta + a(1 + \delta + \beta\delta)}{a(1 + (1 + \alpha + \beta)\delta)}, \frac{(a + Ry)\beta\delta}{1 + (1 + \alpha + \beta)\delta} \right\}$$

Let's check the solution. Note eqs. (5)-(7) in Fischer's article - we check to see if these hold.

```
In[9] := {c12* == δ R c11*,  
          Simplify[n1* == 1 -  $\frac{\alpha}{a}$  δ R c11*], g1* == β c12*}
```

```
Out[9]= {True, True, True}
```

- **Now we introduce distortionary taxes. Since taxes are applied on labor and capital income in period 2, we need to acknowledge through our notation that the tax rate that prevails in the second time period may not be the same as announced in the first period.**
- **First we examine the optimal solution when there is precommitment, that is when some mechanism exists so that announced (in period 1) and actual tax rates are the same. Solving this is done in two steps.**
 1. **The individual makes choices based on the expected tax rates - this produces demand functions for (c_1, c_2, n) that are functions of the announced tax rates. Expected government expenditures do not affect household decisions since utility is separable - hence we will ignore g in period 1.**
 2. **The government then maximizes household indirect utility by choosing the tax rates subject to the government budget constraint.**

To simplify the notation, define the net after-tax returns from capital and labor as:

$Re_2 = (1 - \tau_{2k}^e) R$ and $ae_2 = (1 - \tau_{2l}^e) a$ where the notation 'e' denotes expected level and the "2" is used to denote Economy 2.

First we express the household's problem as the following Lagrangian (convince yourself that this is correct). Note the differences - the returns on labor and capital are affected by the expected tax rates and government expenditures are no longer a choice variable).

$$L_2 = \text{Log}[c_1] + \delta (\text{Log}[c_2] + \alpha \text{Log}[1 - n]) + \lambda (ae_2 n + Re_2 y - Re_2 c_1 - c_2) ;$$

Next we do the same steps as above to find agents demand functions:

```
In[12]:= focs2 = {D[L2, c1] == 0,
                 D[L2, c2] == 0, D[L2, n] == 0, D[L2, λ] == 0}
```

```
Out[12]= {1/c1 - λ Re2 == 0, -λ + δ/c2 == 0, -αδ/(1-n) + λ ae2 == 0, n ae2 - c2 + Y Re2 - c1 Re2 == 0}
```

```
In[13]:= sol2 = Flatten [
           Simplify[ Solve[focs2, {c1, c2, n}, λ] ] ] ;
```

This yields the optimal consumption and labor choices. I also use these to define optimal capital.

```
In[14]:= {c21*, c22*, n2*, k2*} = {c1, c2, n, Y - c1} /. sol2
```

```
Out[14]= { (ae2 + Y Re2) / ((1 + δ + α δ) Re2), δ (ae2 + Y Re2) / (1 + δ + α δ), (1 + δ) ae2 - Y α δ Re2 / ((1 + δ + α δ) ae2), Y - (ae2 + Y Re2) / ((1 + δ + α δ) Re2) }
```

Now use the households indirect utility (we now incorporate g since the government will need this when maximizing households' utility). The indirect utility function is obtained by substituting the demand functions for the choice variables in agents' (direct) utility function. Then utility is a function of tax rates rather than consumption and labor

```
In[15]:= IU2 = Log[c21*] + δ (Log[c22*] +
                       α Log[1 - n2*] + β Log[g2]) /. sol2 ;
```

Now express the government's Lagrangian and (try to) do the same steps. (Note that $R - Re_2 = \tau^e R$ and similarly for the tax on labor.)

```
In[16]:= GL2 = IU2 + γ ((R - Re2) k2* + (a - ae2) n2* - g2) ;
```

This will get messy fast...

In[24] := **gfocs2 = {D[GL2, Re₂] == 0, D[GL2, ae₂] == 0,
D[GL2, g2] == 0, D[GL2, γ] == 0}**

$$\begin{aligned} \text{Out [24]} = & \left\{ \frac{(1 + \delta + \alpha \delta) \text{Re}_2 \left(\frac{\gamma}{(1 + \delta + \alpha \delta) \text{Re}_2} - \frac{\text{ae}_2 + \gamma \text{Re}_2}{(1 + \delta + \alpha \delta) \text{Re}_2^2} \right)}{\text{ae}_2 + \gamma \text{Re}_2} + \gamma \right. \\ & \left. \left(-\gamma - \frac{\gamma \alpha \delta (a - \text{ae}_2)}{(1 + \delta + \alpha \delta) \text{ae}_2} + \frac{\text{ae}_2 + \gamma \text{Re}_2}{(1 + \delta + \alpha \delta) \text{Re}_2} + (R - \text{Re}_2) \left(-\frac{\gamma}{(1 + \delta + \alpha \delta) \text{Re}_2} + \frac{\text{ae}_2 + \gamma \text{Re}_2}{(1 + \delta + \alpha \delta) \text{Re}_2^2} \right) \right) \right\} + \\ & \delta \left(\frac{\gamma}{\text{ae}_2 + \gamma \text{Re}_2} + \frac{\gamma \alpha^2 \delta}{(1 + \delta + \alpha \delta) \text{ae}_2 \left(1 - \frac{(1 + \delta) \text{ae}_2 - \gamma \alpha \delta \text{Re}_2}{(1 + \delta + \alpha \delta) \text{ae}_2} \right)} \right) = 0, \\ & \frac{1}{\text{ae}_2 + \gamma \text{Re}_2} + \gamma \left(\frac{(1 + \delta) (a - \text{ae}_2)}{(1 + \delta + \alpha \delta) \text{ae}_2} - \frac{R - \text{Re}_2}{(1 + \delta + \alpha \delta) \text{Re}_2} - \frac{(a - \text{ae}_2) ((1 + \delta) \text{ae}_2 - \gamma \alpha \delta \text{Re}_2)}{(1 + \delta + \alpha \delta) \text{ae}_2^2} - \right. \\ & \left. \frac{(1 + \delta) \text{ae}_2 - \gamma \alpha \delta \text{Re}_2}{(1 + \delta + \alpha \delta) \text{ae}_2} \right) + \delta \left(\frac{1}{\text{ae}_2 + \gamma \text{Re}_2} + \frac{\alpha \left(-\frac{1 + \delta}{(1 + \delta + \alpha \delta) \text{ae}_2} + \frac{(1 + \delta) \text{ae}_2 - \gamma \alpha \delta \text{Re}_2}{(1 + \delta + \alpha \delta) \text{ae}_2^2} \right)}{1 - \frac{(1 + \delta) \text{ae}_2 - \gamma \alpha \delta \text{Re}_2}{(1 + \delta + \alpha \delta) \text{ae}_2}} \right) = 0, \\ & -\gamma + \frac{\beta \delta}{g2} = 0, -g2 + \frac{(a - \text{ae}_2) ((1 + \delta) \text{ae}_2 - \gamma \alpha \delta \text{Re}_2)}{(1 + \delta + \alpha \delta) \text{ae}_2} + (R - \text{Re}_2) \left(\gamma - \frac{\text{ae}_2 + \gamma \text{Re}_2}{(1 + \delta + \alpha \delta) \text{Re}_2} \right) = 0 \end{aligned}$$

This defines three equations in the three unknowns (Re₂, ae₂, g₂) - but it can not be solved analytically. So let's use some numbers.

In[25] := **parms = {R → 1.5, γ → 3,
a → 1, δ → 0.9, α → 0.25, β → 0.5}**

Out [25] = {R → 1.5, γ → 3, a → 1, δ → 0.9, α → 0.25, β → 0.5}

In[26] := **ngfocs2 = gfocs2 /. parms**

$$\begin{aligned} \text{Out [26]} = & \left\{ 0.9 \left(\frac{0.0794118}{\text{ae}_2 \left(1 - \frac{0.470588 (1.9 \text{ae}_2 - 0.675 \text{Re}_2)}{\text{ae}_2} \right)} + \frac{3}{\text{ae}_2 + 3 \text{Re}_2} \right) + \right. \\ & \frac{2.125 \text{Re}_2 \left(\frac{1.41176}{\text{Re}_2} - \frac{0.470588 (\text{ae}_2 + 3 \text{Re}_2)}{\text{Re}_2^2} \right)}{\text{ae}_2 + 3 \text{Re}_2} + \gamma \left(-3 - \frac{0.317647 (1 - \text{ae}_2)}{\text{ae}_2} + \right. \\ & \left. \frac{0.470588 (\text{ae}_2 + 3 \text{Re}_2)}{\text{Re}_2} + (1.5 - \text{Re}_2) \left(-\frac{1.41176}{\text{Re}_2} + \frac{0.470588 (\text{ae}_2 + 3 \text{Re}_2)}{\text{Re}_2^2} \right) \right) \right\} = 0, \\ & \gamma \left(\frac{0.894118 (1 - \text{ae}_2)}{\text{ae}_2} - \frac{0.470588 (1 - \text{ae}_2) (1.9 \text{ae}_2 - 0.675 \text{Re}_2)}{\text{ae}_2^2} - \right. \\ & \left. \frac{0.470588 (1.9 \text{ae}_2 - 0.675 \text{Re}_2)}{\text{ae}_2} - \frac{0.470588 (1.5 - \text{Re}_2)}{\text{Re}_2} \right) + \frac{1}{\text{ae}_2 + 3 \text{Re}_2} + \\ & 0.9 \left(\frac{0.25 \left(-\frac{0.894118}{\text{ae}_2} + \frac{0.470588 (1.9 \text{ae}_2 - 0.675 \text{Re}_2)}{\text{ae}_2^2} \right)}{1 - \frac{0.470588 (1.9 \text{ae}_2 - 0.675 \text{Re}_2)}{\text{ae}_2}} + \frac{1}{\text{ae}_2 + 3 \text{Re}_2} \right) = 0, \frac{0.45}{g2} - \gamma = 0, \\ & -g2 + \frac{0.470588 (1 - \text{ae}_2) (1.9 \text{ae}_2 - 0.675 \text{Re}_2)}{\text{ae}_2} + (1.5 - \text{Re}_2) \left(3 - \frac{0.470588 (\text{ae}_2 + 3 \text{Re}_2)}{\text{Re}_2} \right) = 0 \end{aligned}$$

```
In[32]:= tax2 = FindRoot[ngfocs2,
  {{{Re2, 1.0}, {ae2, 0.5}, {g2, 0.75}, {γ, 0.6}}}]
```

```
Out[32]= {Re2 -> 0.999441, ae2 -> 0.668364, g2 -> 0.776475, γ -> 0.579542}
```

That's better. Now compute the actual tax rates and the choice variables. After we have these, we can compute household utility.

```
In[33]:= taxrates2 = {R - Re2, a - ae2} /. parms /. tax2
```

```
Out[33]= {0.500559, 0.331636}
```

```
In[68]:= choices2 = {nc21*, nk2*, nc22*, nn2*, g2*} =
  {c21*, k2*, c22*, n2*, g2} /. parms /. tax2
```

```
Out[68]= {1.72646, 1.27354, 1.55295, 0.419122, 0.776475}
```

```
In[69]:= utility2 = IU2 /. parms /. tax2
```

```
Out[69]= 0.706147
```

■ For comparison, let's go back to the first case and get numerical values

```
In[28]:= choices1 = {nc11*, nk1*, nc12*, nn1*, ng1*} =
  {c11*, y - c11*, c12*, n1*, g1*} /. parms
```

```
Out[28]= {1.42395, 1.57605, 1.92233, 0.519417, 0.961165}
```

```
In[30]:= utility1 =
  Log[c11*] + δ (Log[c12*] + α Log[1 - n1*] +
  β Log[g1*]) /. parms
```

```
Out[30]= 0.758923
```

Note that utility is higher in the economy with no distortionary taxes...precisely what you expect.

■ Now we examine the case with no precommitment. This involves two economies: one which is a time-consistent

equilibrium and one which is a time inconsistent solution (but can not be a rational expectations equilibrium).

- **The time consistent example (Economy 3) - as in the previous example, we require in equilibrium that the expected and actual tax rates are the same. But here there is no precommitment, so to determine the path of taxes, we need to work backwards: First determine the optimal set of taxes in the last period given the capital stock inherited from period 1 and then households, in the first period, choose capital knowing that these are the taxes they will face.**

For notational purposes we define the net after tax return on capital as $R_3 = (1 - \tau_{3k}) R$ and $a_3 = (1 - \tau_{3l}) a$ where the "3" denotes that we are analyzing Economy 3.

The government maximizes second period utility taking as given k . First, we need to analyze the optimal consumer behavior to obtain the demand functions for c_2 and n . Then the government maximizes the household's indirect utility function subject to the government budget constraint. The household's problem is (note k_3 is exogenous since it was chosen in period 1 and again g is ignored by the household):

$$\text{In}[34] := L_3 = \text{Log}[c_{3_2}] + \alpha \text{Log}[1 - n_3] + \lambda (R_3 k_3 + a_3 n_3 - c_{3_2}) ;$$

$$\text{In}[35] := \text{focs3} = \{D[L_3, c_{3_2}] == 0, D[L_3, n_3] == 0, D[L_3, \lambda] == 0\} ;$$

$$\text{In}[36] := \text{sol3} = \text{Flatten} [\text{Simplify} [\text{Solve} [\text{focs3}, \{c_{3_2}, n_3\}, \lambda]]] ;$$

$$\text{In}[37] := \{c_{3_2}^*, n_3^*\} = \{c_{3_2}, n_3\} /. \text{sol3}$$

$$\text{Out}[37] = \left\{ \frac{a_3 + k_3 R_3}{1 + \alpha}, \frac{a_3 - k_3 R_3 \alpha}{a_3 + a_3 \alpha} \right\}$$

```
In[38]:= IU3 = Simplify[
      Log[c32] + α Log[1 - n3] + β Log[g3] /. sol3] ;
```

The government chooses α_3 and R_3 in order to maximize household utility subject to the government budget constraint. The Lagrangian is

```
In[39]:= GL3 = IU3 + γ ((R - R3) k3 + (a - a3) n3* - g3) ;
```

```
In[40]:= gfocs3 = {D[GL3, R3] == 0, D[GL3, a3] == 0,
      D[GL3, g3] == 0, D[GL3, γ] == 0} ;
```

```
In[42]:= gsol3 = Flatten [
      Simplify[ Solve[gfocs3, {R3, a3, g3}, γ] ] ]
```

```
Out[42]= {g3 →  $\frac{(a + k_3 R) \beta}{1 + \alpha + \beta}$ , R3 →  $\frac{k_3 R (1 + \alpha) - a \beta}{k_3 (1 + \alpha + \beta)}$ , a3 → a}
```

```
In[43]:= {g3*, R3*, a3*} = {g3, R3, a3} /. gsol3
```

```
Out[43]= { $\frac{(a + k_3 R) \beta}{1 + \alpha + \beta}$ ,  $\frac{k_3 R (1 + \alpha) - a \beta}{k_3 (1 + \alpha + \beta)}$ , a}
```

Note that the result $a_3 = a$ implies that the tax on labor is zero. That is, since capital is already in place, a capital tax is not distortionary so the optimal choice for the government is to not tax labor (given that this tax is distortionary) and tax capital only. Also note that the tax rate on capital is $(R - R_3) = g_3/k_3$ so that the tax rate (g_3/k_3) times the tax base (k_3) is equal to total expenditures (g_3).

We now turn to the individual's problem in the first period. In the first period, the household has two choice variables: (c_{31}, k_3) and, from the indirect utility function, it already knows the level of utility it can achieve for any choice of k_3 . Also, the agent makes these choices using its forecast of taxes given by the solution `gsol3` above. The individual's Lagrangian is (note again, we ignore government choices...the individual takes this as given). First define the indirect utility function that does not include government

```
In[44]:= IU33 = Log[c32] + α Log[1 - n3] /. sol3
```

```
Out[44]= Log[ $\frac{a_3 + k_3 R_3}{1 + \alpha}$ ] + α Log[ $1 - \frac{a_3 - k_3 R_3 \alpha}{a_3 + a_3 \alpha}$ ]
```

```
In[45]:= L33 = Log[c31] + δ IU33 + λ (y - c31 - k3) ;
```

```
In[46]:= focs33 =
```

```
      {D[L33, c31] == 0, D[L33, k3] == 0, D[L33, λ] == 0}
```

```
Out[46]= { $-\lambda + \frac{1}{c_{31}} = 0, \left( \frac{R_3}{a_3 + k_3 R_3} + \frac{R_3 \alpha^2}{(a_3 + a_3 \alpha) \left(1 - \frac{a_3 - k_3 R_3 \alpha}{a_3 + a_3 \alpha}\right)} \right) \delta - \lambda = 0, -k_3 + y - c_{31} = 0$ }
```

```
In[47]:= sol33 = Flatten [
```

```
      Simplify[Solve[focs33, {c31, k3}, λ] ] ] ;
```

```
In[48]:= {c31*, k3*} = {c31, k3} /. sol33
```

```
Out[48]= { $\frac{a_3 + R_3 y}{R_3 + R_3 \delta + R_3 \alpha \delta}, \frac{-a_3 + R_3 y (1 + \alpha) \delta}{R_3 (1 + \delta + \alpha \delta)}$ }
```

Note we have two equations in two unknowns. From the government's problem, we have that the optimal choice of taxes will imply the after-tax return on capital of: $R_3 = \frac{k_3 R (1 + \alpha) - a \beta}{k_3 (1 + \alpha + \beta)}$ - That is, the implied tax rate will be a function of the capital stock. Then, agents choose a capital stock based on their expectations of the tax rate: $k_3 = \frac{-\alpha_3 + R_3 y \delta (1 + \alpha)}{R_3 (1 + \delta + \alpha \delta)}$. In equilibrium, we require these to be consistent so use the second expression to eliminate k_3 in the first expression.

We can see this by graphing the two expressions. We can use *Mathematica* to plot this via the Implicit Plot function. First define the two functions:

```
In[49]:= aftertaxreturn = R3* - R3 /. parms
```

```
Out[49]=  $\frac{0.571429 (-0.5 + 1.875 k_3)}{k_3} - R_3$ 
```

```
In[50]:= cap = k3* - k3 /. parms /. a3 → 1
```

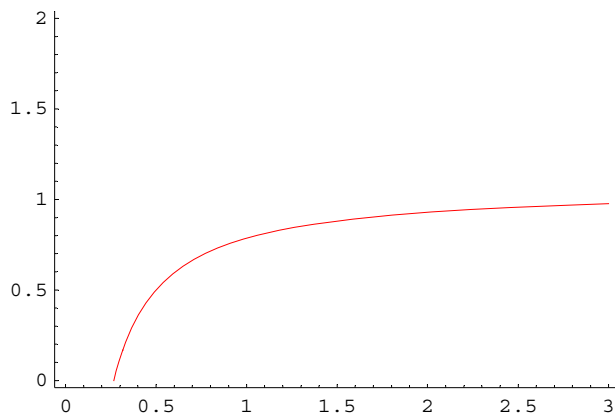
```
Out[50]=  $-k_3 + \frac{0.470588 (-1 + 3.375 R_3)}{R_3}$ 
```

Now import the required *Mathematica* function:

```
In[51]:= Needs["Graphics`ImplicitPlot`"]
```

First plot the tax function

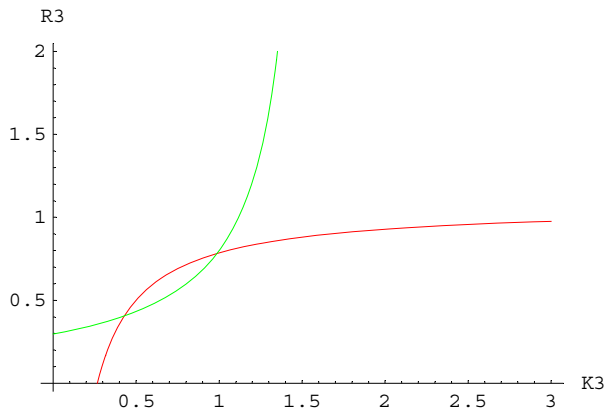
```
In[52]:= ImplicitPlot[aftertaxreturn == 0,  
  {k3, 0.0001, 3}, {R3, 0.0001, 2},  
  PlotStyle -> RGBColor[1, 0, 0]
```



```
Out[52]= - ContourGraphics -
```

Now Plot the two functions together

```
In[53]:= ImplicitPlot[{aftertaxreturn == 0, cap == 0},
  {k3, 0.0001, 3}, {R3, 0.0001, 2},
  PlotStyle ->
    {RGBColor[1, 0, 0], RGBColor[0, 1, 0] },
  AxesLabel -> {"K3", "R3" }]
```



```
Out[53]= - Graphics -
```

Back to the algebraic solution - replace capital on the RHS of the TAX equation using the CAP equation:

```
In[54]:= rhs = R3* /. sol33 /. a3 -> a
```

```
Out[54]= 
$$\frac{R3 (1 + \delta + \alpha \delta) \left( -a \beta + \frac{R (1 + \alpha) (-a + R3 \gamma (1 + \alpha) \delta)}{R3 (1 + \delta + \alpha \delta)} \right)}{(1 + \alpha + \beta) (-a + R3 \gamma (1 + \alpha) \delta)}$$

```

Then we require this expression to be equal to R3...this produces a quadratic expression which can be seen by the analytic solution for R3.

```
In[55]:= R3soln = Simplify[Solve[R3 == rhs, R3]]
```

```
Out[55]= 
$$\left\{ \left\{ R3 \rightarrow \frac{a + R \gamma \delta + R \gamma \alpha \delta - a \beta \delta - \sqrt{-4 a R \gamma (1 + \alpha + \beta) \delta + (a + R \gamma (1 + \alpha) \delta - a \beta \delta)^2}}{2 \gamma (1 + \alpha + \beta) \delta} \right\}, \right. \\ \left. \left\{ R3 \rightarrow \frac{a + R \gamma \delta + R \gamma \alpha \delta - a \beta \delta + \sqrt{-4 a R \gamma (1 + \alpha + \beta) \delta + (a + R \gamma (1 + \alpha) \delta - a \beta \delta)^2}}{2 \gamma (1 + \alpha + \beta) \delta} \right\} \right\}$$

```

To get a numerical answer, use the same parameter values we used earlier

```
In[56]:= nR3 = R3soln /. parms
```

```
Out[56]= {{R3 -> 0.406099}, {R3 -> 0.781732}}
```

Note that the first value implies a very high tax rate on capital since R3 represents the net after tax return. It is likely that this would imply a lower level of utility. If this conjecture is correct, then it can not be an equilibrium - let's check.

```
In[57]:= R31* = nR3 // Last
```

```
Out[57]= {R3 -> 0.781732}
```

```
In[58]:= R32* = nR3 // First
```

```
Out[58]= {R3 -> 0.406099}
```

```
In[59]:= tax3 =
```

```
{R - R3, a - a3} /. a3 -> a /. parms /. R31*
```

```
Out[59]= {0.718268, 0}
```

First we calculate the choice variables in the low tax state (denoted as "1")

```
In[60]:= {c31_1*, k31*, c31_2*, n31*, g31} =
```

$$\{c_{31}, k_3, c_{32}^*, n_3^*, g_3\} /. g_3 \rightarrow \frac{(a + k_3 R) \beta}{1 + \alpha + \beta} /. \\ \text{sol133} /. a_3 \rightarrow a /. \text{parms} /. R31^*$$

```
Out[60]= {2.01375, 0.986254, 1.41679, 0.645803, 0.708394}
```

Now we calculate their equivalents in the high tax state (denoted as "2")

```
In[61]:= {c32_1*, k32*, c32_2*, n32*, g32} =
```

$$\{c_{31}, k_3, c_{32}^*, n_3^*, g_3\} /. g_3 \rightarrow \frac{(a + k_3 R) \beta}{1 + \alpha + \beta} /. \\ \text{sol133} /. a_3 \rightarrow a /. \text{parms} /. R32^*$$

```
Out[61]= {2.57057, 0.429432, 0.939514, 0.765122, 0.469757}
```

Note that, as expected, consumption in period 1 is higher in the high tax state since savings (i.e. capital) is low. Also note that labor is higher and govt. consumption is lower in the high tax state...the following shows that this does indeed result in lower utility:

```
In[62]:= utility31 =
  Log[c31] + δ (Log[c32*] + α Log[1 - n3*] +
    β Log[g3]) /. g3 →  $\frac{(a + k_3 R) \beta}{1 + \alpha + \beta}$  /.
  sol33 /. a3 → a /. parms /. R31*
```

Out[62]= 0.624883

```
In[63]:= utility32 =
  Log[c31] + δ (Log[c32*] + α Log[1 - n3*] +
    β Log[g3]) /. g3 →  $\frac{(a + k_3 R) \beta}{1 + \alpha + \beta}$  /.
  sol33 /. a3 → a /. parms /. R32*
```

Out[63]= 0.222025

Utility in the high tax equilibrium is lower so we ignore this as an equilibrium.

- **Finally, we calculate the time inconsistent solution... note that this is NOT a permissible rational expectations equilibrium since agents' expectations of future taxes are wrong (predictably wrong...this is not a stochastic model).**

To solve the model, we assume that agents believe in the first period that the optimal tax policy under precommitment (Economy 2) will be followed in the second period even though there is no commitment mechanism. Hence, we know the path of taxes: For period 1, the agent solves the following problem as in Economy 2 so that all values are the same. We repeat these below:

```
In[64]:= choices4 =
          {c41*, k4*} = {c21*, k2*} /. parms /. tax2
Out[64]= {1.72646, 1.27354}
```

Then in period 2, the government uses the optimal plan described in Economy 3: set labor taxes to zero and tax only capital. This solution was described in the first part of the time consistent equilibrium without precommitment; that is, we already have the government's optimal choice of taxes for a given capital stock and the household's demand functions. First we solve for taxes and government expenditures consistent with the level of capital inherited from the first period:

```
In[70]:= {g4*, R4*, a4*} =
          {g3*, R3*, a3*} /. k3 → nk2* /. parms
Out[70]= {0.831515, 0.847081, 1}
```

```
In[71]:= tax4 = R - R4* /. parms
Out[71]= 0.652919
```

Then we use the tax rate on capital to determine the optimal amount of consumption and labor in the second period.

```
In[72]:= {c42*, n4*} = {c32*, n3*} /.
          {R3 → R4*, a3 → a4*, k3 → nk2*} /. parms
```

```
Out[72]= {1.66303, 0.584242}
```

Finally, we use these values to compute utility:

```
In[73]:= utility4 =
          Log[nc21*] + δ (Log[c42*] + α Log[1 - n4*] +
          β Log[g4*]) /. parms
```

```
Out[73]= 0.723354
```

Note that utility is indeed higher in this solution, hence Fischer's title: the Benevolent Dissembling Government.

The table below summarizes the results:

Solution	U^{**}	c_1	k	c_2	n	τ_k	τ_k^e	τ_n	τ_n^e
Social Planner	0.758	1.42	1.58	1.92	0.519	-	-	-	-
Precommitment	0.706	1.73	1.27	1.55	0.419	0.50	0.50	0.33	0.33
Time Consistent Eq.	0.625	2.01	0.986	1.42	0.646	0.718	0.718	0	0
Time Inconsistent Soln.	0.723	1.73	1.27	1.66	0.584	0.653	0.50	0	0.33

