Choice Under Uncertainty

- So far, all decisions have taken place in a world of certainty
  - consumers know for sure the utility they will receive given a choice of goods
  - firms know for sure the profit they will receive from a chosen set of inputs
- This does not descirbe the real world
  - technological uncertainty
  - market uncertainty
- Many isues cannot be addressed without considering uncertainty eg. stock market, insurance, futures markets (investment and savings decisions)

- The next few lectures will proceed as follows
  - attitudes towards risk
  - uncertainty in the insurance market

Attitudes towards Risk

- Consider a representative agent model in which there is a single good (income)

- Secondly, assume that there are only two states of the world (state 1 and state 2)

  - Let y_1 denote income in state 1 and y_2 income in state 2

  - Let π denote the probability of state 1 so that $1 - \pi$ is the probability of state 2

- Given this, the expected value of income is

$$\overline{y} = \pi y_1 + (1 - \pi) y_2$$

  - the weighted average level of income received by the individual over the two states of the world

- Utility of the expected value is

$$u(\overline{y}) = u(\pi y_1 + (1 - \pi)y_2)$$

- ■ the utility received by the individual from the weighted average level of income $\overline{y}$

- Expected utility is given by

$$\overline{u} = \pi u(y_1) + (1 - \pi)u(y_2)$$

- ■ the utility received by the individual from fluctuating income levels across the two states of the world

Risk Aversion

- An individual is risk averse if

$$u(\overline{y}) > \overline{u}$$

- He/she prefers to have a constant amount of income rather than fluctuating amounts of a low income in state 1 of the world and a high income in state 2 of the world

  - averages are preferred to extremes

- An alternative definition uses the **certainty equivalent level of income** denoted by y_c

- This level of income must satisfy the following condition

$$u(y_c) = \overline{u}$$

  - y_c must be such that the utility received from getting y_c equals the utility from facing the gamble (expected utility)

- In other words, y_c is the level of income at which the individual is indifferent between getting that level of income for certain and facing fluctuating incomes levels

- This certainty equivalent level of income therefore gives us a notion of the value of the gamble to the individual. If the individual is risk averse

$$\overline{y} > y_c$$

  - that is, the individual values the gamble at less than the expected value

Risk Neutral

- An individual is risk neutral if he/she values the prospect at its' expected value

$$\overline{y} = y_c$$

or

$$u(\pi y_1 + (1-\pi)y_2) = \pi u(y_1) + (1-\pi)u(y_2)$$

Risk Loving

- An individual is loving if he/she values the prospect at more than its' expected value

$$y_c > \overline{y}$$

or

$$\pi u(y_1) + (1-\pi)u(y_2) > u(\pi y_1 + (1-\pi)y_2)$$

The Risk Premium

- Consider figure 1 and recall that y_c satisfies $u(y_c) = \overline{u}$

- In figure 1, the individual is willing to give up income *cd* rather than face the gamble with expected income $\overline{y}$

$$r = \overline{y} - y_c > 0$$

- If the individual is risk loving, the individual would pay to be able to face the gamble

$$r = \overline{y} - y_c > 0$$

Measures of Risk Aversion

- How risk averse is a particular individual ?
- The more concave the utility function, the more risk averse the individual

- Formally, risk aversion can be measured by

$$A(y) = -\frac{u''(y)}{u'(y)}$$

This is the Arrow-Pratt coefficient of absolute risk aversion

- If the individual is risk averse, $u''(y) < 0$ which implies $A(y) > 0$

- $A(y)$ may increase, decrease or remain constant as y increases
    - If the risk premium is decreasing (increasing) in wealth, the consumer has decreasing (increasing) risk aversion
    - If the risk premium is constant in wealth, the consumer has constant risk aversion

Insurance Under Uncertainty

- Consider a risk averse individual and suppose that there are two states of the world : state 1 and state 2
  - state 1 - initial income is y
  - state 2 - income is $y - L$
- The consumer can insure against the loss (cannot affect the loss or the probability of the loss ⇒ moral hazard issue)
- The insurance company sells insurance at a premium rate p $(0 < p < 1)$. Let q denote the amount of insurance cover purchased by the consumer

$$y_1 = y - pq$$

$$y_2 = y - L - pq + q$$

- Let π_1 denote the probability of state 1 and let π_2 denote the probability of state 2

$$\overline{y} = \pi_1(y - pq) + \pi_2(y - L - pq + q)$$

- Now suppose that the individual maximises expected utility

$$\max_q \pi_1 u(y_1) + \pi_2 u(y_2)$$

$$= \max_q \pi_1 u(y - pq) + \pi_2 u(y - L - pq + q)$$

- Rearranging the first order condition gives the following familiar condition

$$\frac{u'(y_2)\pi_2}{u'(y_1)\pi_1} = \frac{p}{1-p}$$

- An insurance company has an actuarily fair premium (p) if it does not alter the insured individual's expected income

$$\pi_1(y - pq) + \pi_2(y - L - pq + q)$$
$$= \pi_1 y + \pi_2(y - L)$$

- Hence, the fair premium is $p = \pi_2$

- In fact, perfect competition implies a fair premium

- Competition in the industry forces the expected profits of the firm to zero

$$\pi_1 pq + \pi_2(pq - q) = 0$$

$\Rightarrow$

$$q(p - \pi_2) = 0$$

Case 1

- Suppose $p = \pi_2$ (premium is fair). Then, from the foc, we can write

$$u'(y_1) = u'(y_2)$$

- Since $u''(y) < 0$, this implies $y_1 = y_2$

$$y - pq^* = y - L + (1 - p)q^*$$

$\Rightarrow$

$$L = q^*$$

  - the insurer completely insures against the loss