Professor Salyer, Economics 135, Spring 2008

Final Exam - Answer Key

*Directions: Answer all questions; the questions are weighted equally. For full credit, you must provide **complete** explanations for your answers.*

1. Analyze the following statement, "The formula for the duration of a coupon bond is motivated by the fact that a coupon bond can be thought of as portfolio of pure discount bonds." Be precise in your analysis.

   ANSWER: First, I note that, for a pure discount bond (i.e. zero coupon bond), maturity is equal to duration. I denote this as $D_i^d = i$. Also, the bond pricing formula for a coupon bond is $P_b = \sum_{t=1}^{N} \frac{C_t}{(1+i)^t}$ where C_i denotes cash payment received in period i (and assuming annual payments). But this can be re-written as: $P_b = \sum_{t=1}^{N} P_i^d$ where $P_i^d = \frac{C_t}{(1+i)^t}$ is the price of a pure discount bond. This establishes that the coupon bond is indeed equal to the sum of the underlying pure discount bonds. Then the duration of the coupon bond is given by

   $$D_b = \sum_{t=1}^{N} w_i D_i^d$$

   where $w_i = \frac{P_i^d}{P_b}$ - this is the duration formula for the coupon bond.

2. Use the expectations hypothesis to answer the following questions:

   (a) Suppose that the current two and three year bond rates are 5% and 7% respectively. What is the one year rate expected to be 2 years from now?

   ANSWER: From the expectations hypothesis: $E_t\,(i1_{t+2}) = 3\,(i3_t) - 2\,(i2_t) = 21\% - 10\% = 11\%$.

   (b) Suppose that the current 2 year rate is 8% while the current one year rate is 6%. If you are planning to borrow next year to purchase a new car, what position in the future market should you take to hedge your interest rate risk exposure?

   ANSWER: From the expectations hypothesis, the one year interest rate is expected to jump to 10%. This will imply a fall in bond prices. The correct position is to take a long position in the futures market (buy at the current low price if interest rates do indeed rise).

3. Agents have wealth of W but face the prospect of a loss, x, with probability p. Actuarially fair insurance is available with premium h. Let c denote the amount of insurance purchased (i.e. $c = x$ implies full insurance). Let W_1 denote wealth if the loss does not occur (i.e. $W_1 = W - h$) and W_2 denotes wealth if the loss does occur ($W_2 = W - h - x + c$). Agents choose c in order to maximize $E\,[U\,(W)]$. Show that risk averse agents will purchase full insurance.

   ANSWER: Actuarially fair insurance implies that $h = pc$ since this implies the expected payout (pc) is equal to the average premium (h). Using this in the equations for wealth in the two states: $W_1 = W - pc$; $W_2 = W - x + (1-p)\,c$. Note that this implies the slope of the budget line is given by: $(dW_2/dW_1) = (-(1-p)/p)$: agents ability to transfer wealth across states is exactly equal to the ratio of the probabilities of each state. Substitituting the expression for wealth into the expected utility:

   $$\max\left[(1-p)\,U\,(W - pc) + pU\,(W - x + (1-p)\,c)\right]$$

   and taking the derivative with respect to c yields: $U'\,(W_1) = U'\,(W_2)$ which implies $W_1 = W_2$ which implies $c = y$.

4. In the model of banks as providers of liquidity insurance, we examined four cases. Autarky, a market economy with borrowing and lending, the Pareto optimum and how banks can achieve the Pareto optimum. Discuss (1) Why the equilibrium in the market economy was preferred to autarky, (2) Why the market economy was NOT a Pareto optimum and (3) What feature of the model permitted the economy with banks to achieve the Pareto optimum.

   ANSWER: (1) In the autarky equilibrium, agents will have to interrupt the high return project if they are a Type 1 person (early consumer). This results in a poor return. In the market economy, agents can borrow against the future income from the project so this yields a higher level of consumption. Or if they are type 2 agents, they can convert their storage (no net return) into bonds once they find out that they are a Type 2 person (late consumer). Since the productive asset is never interrupted consumption in the economy is higher. (2) But the market interest rate is determined by the rate of return on the productive asset. It is NOT determined by the probabilities of being a type 1 or type 2 agents. As shown in Q3, optimal insurance implies that the ability to transfer consumption across states (early or late consumer) must reflect the probabilities of each type. This is why the market economy is not a Pareto optimum. (3) Banks can achieve the Pareto optimum because of the Law of Large Numbers. The fraction of Type 1 and Type 2 agents is perfectly predictable given the infinite number of agents in the economy. Hence, banks can place exactly the right amount of initial wealth into the high productive asset and keep the remaining for the Type 1 agents.

5. In discussing the characteristics of their model of lending under uncertainty, Gertler and Hubbard state: "A second prediction is that investment fluctuations may exhibit asymmetries. Investment downswings in recessions may be sharper than upswings during booms." Explain the features of the model that lead to this prediction. Be precise in your explanation; use graphs when relevant.

   ANSWER: Because of the incentive compatibility constraint, the amount that banks will lend (when the constraint is binding) is determined by firms net worth. In recessions, firms net worth falls so this will cause less investment to take place. In expansions, as net worth grows, it is possible that the incentive compatibility constraint will not become binding. Then, ceteris paribus, there is a unique level of investment for the economy. Further increases in net worth will not change this level of investment.

6. The Gertler and Hubbard model of lending under uncertainty included an additional constraint: the incentive compatibility constraint. This was given by:

$$\left(\pi^g + \pi^b \alpha\right) f(K) - \left(\pi^g P^g + \pi^b P^b\right) \geq \left(\alpha f(K) - P^b\right) + r\nu K$$

   (In class, I used the notation $\eta = \nu$.) Explain in detail each term in this expression.

   ANSWER: $\left(\pi^g + \pi^b \alpha\right) f(K)$ = this is the expected return from investing in both hard and soft capital. $\left(\pi^g P^g + \pi^b P^b\right)$ = this is the expected payout to lenders. So the LHS is the expected net return to firms if they invest in hard and soft capital.$\left(\alpha f(K) - P^b\right)$ = This is the net return to firms if they only invest in hard capital. The first term is the return from production while the second is the payout to the bank since the bad state is certain to occur with no soft capital. $r\nu K$ = the return from putting the soft capital money that was borrowed into a Swiss bank account. Hence, the RHS is the total return to the firm if they borrow money for hard and soft capital but only use hard capital for productive purposes.

7. A critical component of the modern IS curve is the consumption Euler equation. Explain, in detail, what this is and where it comes from. What features of this equation help to distinguish consumption behavior in the modern macroeconomics policy model from that in the traditional IS curve?

   ANSWER: The consumption Euler equation is

$$U'(c_t) = \beta E_t \left[U'(c_{t+1})\right] rr_t$$

   where rr_t is the real return on savings. This condition is the necessary condition associated with choosing an optimal consumption path - each period the agent must determine how much to consume and how much to save. The LHS is the loss of utility (at the margin) from choosing more savings while the RHS is expected marginal utility gain from the extra consumption that savings will finance next period. At an optimum this loss (i.e. marginal cost) must equal the marginal benefit. The main difference between this behavior and consumption behavior in the traditional model is that a consumption PATH is being chosen...not consumption at a point in time. Therefore, optimal consumption at time t is determined by the current AND future short term interest rates (real) in the economy. This can be seen by log-linearizing the above expression around full employment consumption yields:

$$\tilde{c}_t = E\left[\tilde{c}_{t+1}\right] - \frac{1}{\gamma}\widetilde{rr}_t$$

where γ = agents relative risk aversion and the tilde denotes percentage deviation from full employment. Solving this forward (replacing future consumption) yields

$$\tilde{c}_t = \frac{1}{\gamma}\sum_{i=0}^{\infty}\widetilde{rr}_{t+i} = \frac{1}{\gamma}\widetilde{rr}_t^l$$

where $\widetilde{rr}_t^l$ is the long term interest rate at time t.