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Decision Rules for Selecting between Logistic and Exponential STAR Models*

Abstract

A new LM specification procedure to choose between Logistic and Exponential Smooth Transition Autoregressive (STAR) models is introduced. This procedure has better consistency and power properties than that previously available in the literature. Monte-Carlo simulations and empirical evidence are provided in support of our claims.

- **JEL Codes:** *C12, C22.*
- **Keywords:** *Lagrange multiplier test, smooth transition autoregressive model, non-linear model.*

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1 Introduction

This paper[1] introduces an alternative specification procedure to that proposed in Teräsvirta (1994) for the specification of Smooth Transition Autoregressive (STAR) models. STAR models are a general class of state-dependent non-linear time-series models in which the transition between states is endogenously generated.[2] Together with Hamilton's (1989) regime-switching model, where the transition between states is exogenously determined by a Markov Chain, state-dependent models are reduced-form models that allow for different dynamic responses that depend on the "state." Consequently, these models are particularly well suited to accommodate the asymmetric behavior of economic fluctuations recently documented in a variety of studies.[3]

The two main results of this paper rely on a Taylor series approximation of the transition function (between states) around the scale parameter.[4] The first result is the introduction of a new specification strategy to choose between logistic and exponential STAR models. This alternative strategy is simpler and significantly more successful in selecting the correct model while avoiding the pitfalls of the procedure proposed in Teräsvirta (1994). The second result concerns the test of the null hypothesis of linearity against STAR-type nonlinearity.[5] We propose a simple modification of this test that improves its power against alternatives containing an exponential STAR model as a special case. Our claims are supported by Monte Carlo evidence and an empirical example based on Teräsvirta and Anderson (1992).

The paper is organized as follows: Section 2 briefly reviews STAR models and nonlinearity

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[2] In addition, STAR models encompass other popular families of non-linear time-series models such as the Threshold Autoregressive (TAR) and the Exponential Autoregressive (EAR). See Haggan & Ozaki (1981), Tong (1983), Tsay (1989) and Granger and Teräsvirta(1993).

[3] See Neftçi (1984), Rothman (1991), and Teräsvirta and Anderson (1992) for example.

[4] Luukkonen, Saikkonen and Teräsvirta (1988a), based on Davies (1977), introduced this solution for STAR models.

[5] Note: this alternative is an assumption imposed by the practitioner. This allows one to narrow down general nonlinearity into a workable family of non-linear models, namely STAR models.

testing. Section 3 discusses the decision rule proposed by Teräsvirta (1994) and introduces the alternative procedure advanced in this paper. Section 4 reports on the small sample properties of the alternative test procedures using Monte-Carlo techniques. Section 5 presents an empirical application, and Section 6 concludes.

2 STAR Models and Nonlinearity Testing

2.1 Overview

Consider the following STAR model:

$$y_t = \boldsymbol{\pi}'\mathbf{x}_t + F\left(z_{t-d}, \gamma, c\right)\boldsymbol{\Theta}'\mathbf{x}_t + u_t \tag{1}$$

where y_t is a scalar; $\mathbf{x}_t = (1, y_{t-1}, ..., y_{t-p})' = (1, \widetilde{\mathbf{x}}_t')'; \boldsymbol{\pi}' = (\pi_0, \pi_1, ..., \pi_p) = \left(\pi_0, \widetilde{\boldsymbol{\pi}}'\right); \boldsymbol{\Theta}' = (\Theta_0, \Theta_1, ..., \Theta_p) = \left(\Theta_0, \widetilde{\boldsymbol{\Theta}}'\right)$ and $1 \leqslant d \leqslant p$. z_{t-d} is usually chosen to be y_{t-d}, (although it could be any other predetermined or exogenous variable), u_t is a martingale difference sequence with constant variance.[6] The process in (1) is assumed to be stationary and ergodic. The function $F\left(z_{t-d}, \gamma, c\right)$ is at least fourth order continuously differentiable with respect to the scale parameter γ.

The exponential STAR model (ESTAR)[7] has a transition function F, defined by

$$F\left(z_{t-d}, \gamma, c\right) = \left[1 - \exp\left\{-\gamma\left(z_{t-d} - c\right)^2\right\}\right]. \tag{2}$$

The logistic STAR model (LSTAR)[8] has as transition function:

$$F\left(z_{t-d}, \gamma, c\right) = \left[\left\{1 + \exp\left(-\gamma\left(z_{t-d} - c\right)\right)\right\}^{-1} - \frac{1}{2}\right]. \tag{3}$$

[6] This assumption is usually introduced to simplify the derivation of the asymptotic distribution of the LM test, see White (1984).

[7] The exponential autoregressive (EAR) model of Haggan and Ozaki (1981) is a particular case of ESTAR when $\theta_0 = c = 0$ and $z_{t-d} = y_{t-d}$.

[8] The term $\frac{1}{2}$ is added here merely for convenience and does not affect the results.

Testing linearity against STAR-type nonlinearity implies testing the null hypothesis, $H_0 : \boldsymbol{\Theta}' = \mathbf{0}$ in (1). However, under the null, the parameters γ and c are not identified. Alternatively, we could choose $H_0^* : \gamma = 0$ as our null hypothesis in which case neither c nor $\boldsymbol{\Theta}'$ would be identified. Davies (1977) first showed that conventional maximum likelihood theory is not directly applicable to this problem. A solution proposed in Luukkonen et al. (1988a) and adopted in Teräsvirta (1994) is to replace $F(z_{t-d}, \gamma, c)$ with a suitable Taylor series approximation. Under the null of linearity, the LM test is shown to possess the usual χ^2 distribution asymptotically.[9]

In practice, the test is performed by constructing the following auxiliary regression:

$$y_t = \boldsymbol{\pi}'\mathbf{x}_t + F\gamma\left(z_{t-d}, \gamma = 0, c\right)\boldsymbol{\Theta}'\mathbf{x}_t\gamma + v_{1t} \tag{4}$$

where $F\gamma(.)$ indicates the first derivative of $F(z_{t-d}, \gamma = 0, c)$ with respect to γ. Substituting the expression for $F\gamma(.)$ into (4) gives:

$$y_t = \delta_0 + \boldsymbol{\delta}_1'\widetilde{\mathbf{x}}_t + \boldsymbol{\beta}_1'\widetilde{\mathbf{x}}_t z_{t-d} + \boldsymbol{\beta}_2'\widetilde{\mathbf{x}}_t z_{t-d}^2 + v_{1t} \tag{5}$$

where the null hypothesis of linearity becomes $H_0' : \boldsymbol{\beta}_1' = \boldsymbol{\beta}_2' = \mathbf{0}$. Call this test NL2. In the special case where $F(.)$ is the logistic function $\boldsymbol{\beta}_2' = \mathbf{0}$. The process in (5) can be explosive and generally it is not a meaningful time series model (see Granger and Andersen (1978)).[10] Luukkonen et al. (1988a) realized that this test could have low power against alternatives where $\widetilde{\boldsymbol{\Theta}}'$ is "small" and Θ_0 is "large" in absolute value if the model is LSTAR. To overcome this difficulty, they proposed to include up to third order powers. The final version of their test therefore becomes:

$$y_t = \delta_0 + \boldsymbol{\delta}_1'\widetilde{\mathbf{x}}_t + \boldsymbol{\beta}_1'\widetilde{\mathbf{x}}_t z_{t-d} + \boldsymbol{\beta}_2'\widetilde{\mathbf{x}}_t z_{t-d}^2 + \boldsymbol{\beta}_3'\widetilde{\mathbf{x}}_t z_{t-d}^3 + v_{3t} \tag{6}$$

where the null hypothesis is $H_0'' : \boldsymbol{\beta}_1' = \boldsymbol{\beta}_2' = \boldsymbol{\beta}_3' = \mathbf{0}$. The LM test based on the auxiliary

[9] The delay parameter d is usually unknown. Based on Tsay (1989), Teräsvirta (1994) proposes choosing d that minimizes the p-value of the nonlinearity test.

[10] Also note that the alternative hypothesis will include models other than the STAR.

regression (6) is the test adopted by Saikkonen and Luukkonen (1988), Teräsvirta et al. (1994), and Teräsvirta (1994). Their test-statistic is computed by the following procedure: First, estimate (6) under the null hypothesis H_0'' by OLS and calculate the sum of squared residuals, SSR_0. Second, using the residuals from the previous step, estimate a model that contains the regressors of (6) to compute the sum of squared residuals SSR_1. Then, the statistic $T(SSR_0\text{-}SSR_1)/SSR_0$ will have an asymptotic χ^2 distribution with degrees of freedom given by the number of parameter restrictions under H_0''. We refer to this test-statistic as NL3 and make it the focus of our analysis. In practice, the use of the approximation given by the F-distribution is recommended because of the better size and power properties it has in small samples. An alternative approach is to use the Wald test of Hansen (1996). This procedure approximates the unknown limiting distribution by generating p-values based on simulation methods.[11]

2.2 Modifying the Nonlinearity Test

The transition function of a STAR model exhibits two important features. First, the logistic function in (3) has a single inflection point, while the exponential function in (2) has two inflection points. Second, the even powers of the Taylor series expansion of a logistic function around $\gamma = 0$ are zero. Conversely, the odd powers of the Taylor series expansion of an exponential function around $\gamma = 0$ are zero. In this section we use the first feature to improve the power of NL3. The second feature is exploited in the next section to derive the new selection procedure (decision rule) introduced in this paper.

The power of the nonlinearity test NL3 depends crucially on the quality of the Taylor series approximation to the transition function. Consequently, the difference in shape between the logistic and the exponential functions suggests that at least a *second order* Taylor expansion is necessary to capture the two inflection points of the exponential function. Therefore, fourth order terms should be included in regression (6). We have:

[11] See Pesaran and Potter (1997) for an interesting application of this technique.

$$F(z_{t-d},\gamma,c) \simeq F_{\gamma}(z_{t-d},\gamma=0,c)\gamma + \frac{1}{2}F_{\gamma\gamma}(z_{t-d},\gamma=0,c)\gamma^2 \tag{7}$$

which for the exponential becomes:

$$F(z_{t-d},\gamma,c) \simeq (z_{t-d}-c)^2\gamma - \frac{1}{2}(z_{t-d}-c)^4\gamma^2. \tag{8}$$

The second term of a Taylor series expansion for the logistic function is simply zero. Based on the previous discussion, we recommend augmenting the auxiliary regression (6) as follows:

$$y_t = \delta_0 + \boldsymbol{\delta}_1'\widetilde{\mathbf{x}}_t + \boldsymbol{\beta}_1'\widetilde{\mathbf{x}}_t z_{t-d} + \boldsymbol{\beta}_2'\widetilde{\mathbf{x}}_t z_{t-d}^2 + \boldsymbol{\beta}_3'\widetilde{\mathbf{x}}_t z_{t-d}^3 + \boldsymbol{\beta}_4'\widetilde{\mathbf{x}}_t z_{t-d}^4 + v_{4t} \tag{9}$$

where the null hypothesis of linearity now becomes $H_0''' : \boldsymbol{\beta}_1' = \boldsymbol{\beta}_2' = \boldsymbol{\beta}_3' = \boldsymbol{\beta}_4' = \mathbf{0}$. The LM test-statistic of this joint hypothesis is called NL4.[12] Compared to NL3, NL4 requires p extra regressors in the auxiliary regression. Testing nonlinearity in practice involves several important steps such as choice of lag length of the AR model and choice of delay parameter d. These are well documented in Teräsvirta (1994).

In situations where lack of parsimony is of concern, we recommend a simplified version of NL4 based on the results in Luukkonen et al. (1988a). Parallel to their *augmented first order procedure,* the auxiliary regression in the STAR case becomes:

$$y_t = \delta_0 + \boldsymbol{\delta}_1'\widetilde{\mathbf{x}}_t + \boldsymbol{\beta}_1'\widetilde{\mathbf{x}}_t z_{t-d} + \beta_2^* z_{t-d}^3 + \beta_3^* z_{t-d}^4 + \beta_4^* z_{t-d}^5 + v_{4t}^*. \tag{10}$$

The corresponding null hypothesis of linearity is $H_0^{IV} : \boldsymbol{\beta}_1' = \mathbf{0}$; $\beta_2^* = \beta_3^* = \beta_4^* = 0$.

3 Choosing between LSTAR or ESTAR

This section exploits an attractive feature of Taylor series approximations to the transition function of a STAR model. We begin by presenting Teräsvirta's (1994) specification procedure and detailing

[12] When $z_{t-d} = y_{t-d}$, this test is similar in spirit to a high order RESET test, see Ramsey (1969).

its deficiencies. In response to these concerns, we propose an alternative procedure. Upon rejecting the null hypothesis of linearity one might consider using a STAR model as a useful non-linear alternative. Teräsvirta (1994) suggests a model selection procedure (which we will denominate **TP** for short) based on Equation (6). Teräsvirta motivates his procedure by observing the following sequence of nested F-tests:

1. Test the null: $H_{03} : \boldsymbol{\beta}_3' = \mathbf{0}$ with an F-test (F_3). Teräsvirta notes that in principle, rejection of this null would imply rejection of the ESTAR specification since cubic powers of z_{t-d} in a first order approximation of $F(z_{t-d}, \gamma, c)$ are 0.

2. Test the null: $H_{02} : \boldsymbol{\beta}_2' = \mathbf{0} | \boldsymbol{\beta}_3' = \mathbf{0}$ with an F-test (F_2). Teräsvirta's reasoning is that the z_{t-d}^2 terms of a first order Taylor series approximation to a logistic function are zero when $c = \Theta_0 = 0$, see (1). However, these terms will be nonzero in the ESTAR case (except in the unlikely case when $\widetilde{\boldsymbol{\Theta}}' = \mathbf{0}$). Failure to reject this null is taken as evidence in favor of a LSTAR model. Nevertheless, rejection of H_{02} is not very informative one way or the other.

3. Test the null: $H_{01} : \boldsymbol{\beta}_1' = \mathbf{0} \; | \boldsymbol{\beta}_2' = \boldsymbol{\beta}_3' = \mathbf{0}$ with an F-test (F_1). Following Teräsvirta, failing to reject H_{01} after rejecting H_{02} points to an ESTAR model. On the other hand, rejecting H_{01} after failing to reject H_{02} supports the choice of LSTAR.

Based on these tests, Teräsvirta's rule consists on noting which hypotheses are rejected and then comparing the relative strengths of the rejections. If the model is LSTAR, typically H_{01} and H_{03} are rejected more strongly than H_{02}. Therefore, Teräsvirta proposed to select an ESTAR specification if the p-value of F_2 is the smallest of F_1, F_2, F_3, otherwise the LSTAR alternative is preferred.

Teräsvirta recognizes that this procedure might cause problems. For example, even if the model is ESTAR, H_{03} might be rejected since $\boldsymbol{\beta}_3' = \mathbf{0}$ only if $c = \Theta_0 = 0$ in (1). These concerns and others are best illustrated by analyzing the terms of the Taylor series expansion for each non-linear state in (1):

$$\boldsymbol{\Theta}'\widetilde{\mathbf{x}}_t\left[\{1+\exp\left(-\gamma\left(z_{t-d}-c\right)\right)\}^{-1}-\frac{1}{2}\right]\simeq\boldsymbol{\Psi}_1'\widetilde{\mathbf{x}}_t(z_{t-d}-c)+\boldsymbol{\Psi}_3'\widetilde{\mathbf{x}}_t(z_{t-d}-c)^3 \qquad (11)$$

for the logistic third order expansion, and

$$\boldsymbol{\Theta}'\widetilde{\mathbf{x}}_t\left[1-\exp\left\{-\gamma\left(z_{t-d}-c\right)^2\right\}\right]\simeq\boldsymbol{\Psi}_2'\widetilde{\mathbf{x}}_t(z_{t-d}-c)^2+\boldsymbol{\Psi}_4'\widetilde{\mathbf{x}}_t(z_{t-d}-c)^4 \qquad (12)$$

for the exponential second order expansion. The shorthand notation $\boldsymbol{\Psi}_i'$ for $i = 1, 2, 3, 4$ collects the parameters associated with each of the terms that result from expanding $\widetilde{\mathbf{x}}_t(z_{t-d}-c)^i$, namely the $\boldsymbol{\beta}_i'$ parameters in (9).

Teräsvirta's concerns can now be easily understood by inspecting (12). Whenever c, π_0 and/or θ_0 are non-zero, expansion of $\widetilde{\mathbf{x}}_t(z_{t-d}-c)^4$ yields non-zero $\widetilde{\mathbf{x}}_t z_{t-d}^3$ terms. In addition, when the variance of the error term is "large" the distribution of the data into each state around the threshold c is asymmetric. As a result, the null hypothesis $H_{02} : \boldsymbol{\beta}_2' = \mathbf{0}|\boldsymbol{\beta}_3' = \mathbf{0}$ does not discriminate between a LSTAR with $c \neq 0$ and an ESTAR in general.

An additional source of complications lies in the design of the rule itself. For example, if the true model is LSTAR, it is unclear that by conditioning on the cubic terms to be zero (that is, restricting $\boldsymbol{\beta}_3' = \mathbf{0}$ in (1)), the joint significance of the square terms, $\widetilde{\mathbf{x}}_t z_{t-d}^2$ (from (11), these are non-zero since $c \neq 0$) will be also zero. These terms are now left to approximate the transition function – an approximation that the cubic terms presumably were successfully capturing.

Inspection of equations (11) and (12) provides clues on how to correct the deficiencies of TP. Consider the following example. Suppose $c = 0$. Based on (11), it is clear that if the model is LSTAR, the terms $\widetilde{\mathbf{x}}_t z_{t-d}^j$ for $j = 2, 4$, are zero (i.e. $\boldsymbol{\beta}_2' = \boldsymbol{\beta}_4' = \mathbf{0}$ in (9)). Alternatively, if the model is ESTAR, based on (12), the terms $\widetilde{\mathbf{x}}_t z_{t-d}^j$ for $j = 1, 3$, are zero (i.e. $\boldsymbol{\beta}_1' = \boldsymbol{\beta}_3' = \mathbf{0}$ in (9)). This behavior of the Taylor series approximation suggests a natural alternative selection procedure (which we will call **EJP** for short) based on (9). Conditional on rejecting linearity:

1. Test the null: $H_{0E} : \boldsymbol{\beta}_2' = \boldsymbol{\beta}_4' = \mathbf{0}$ with an F-test (F_E).

2. Test the null: $H_{0L} : \boldsymbol{\beta}_1' = \boldsymbol{\beta}_3' = \mathbf{0}$ with an F-test (F_L).

If the minimum p-value corresponds to F_L, select LSTAR, otherwise select ESTAR.

This selection procedure is clearly consistent when $c = 0$ following the motivation in our example. However, when $c \neq 0$, EJP is still effective since we are comparing the joint significance of linear and cubic terms relative to the joint significance of quadratic and fourth order terms, *without conditioning on other parameters being zero.*

Aside from its simplicity and effectiveness, EJP provides information about non-zero thresholds, c. Linear and cubic terms are exactly zero when $c = 0$ and the model is ESTAR. Quadratic and fourth order terms are exactly zero when $c = 0$ and the model is LSTAR. Therefore, rejecting H_{0L} and failing to reject H_{0E} suggests an LSTAR model with $c = 0$. Rejecting H_{0E} and failing to reject H_{0L} suggests a ESTAR model with $c = 0$. This feature is useful to set $c = 0$ as a good starting value in the estimation stage.

4 Monte Carlo Experiments

This section provides Monte Carlo evidence in support of our recommended decision procedure, EJP. Additional evidence is provided to compare the nonlinearity tests NL3 and NL4. The models simulated in this study are taken from Luukkonen et al. (1988a,b) and Teräsvirta (1994). Each experiment is replicated 1,000 times. The first 100 observations of each series are disregarded to avoid initialization problems.[13]

4.1 Selection Frequencies of the EJP Selection Procedure

Table 1 summarizes the simulation results when the true data generating process (DGP) is an ESTAR model. Table 2 presents similar evidence when the true DGP is a LSTAR model. Both tables compare the accuracy of EJP and TP in selecting the correct model. The correct selection

[13] Extensive Monte Carlo evidence and detailed description of the experiments is available in Escribano and Jorda (1997).

rate is reported as a percentage of those replications for which linearity was first rejected at the conventional 95% confidence level.

Two features are worth noting from this exercise. First, EJP significantly outperforms TP. The difference is most notable when the true DGP is an ESTAR model (see Table 1). While the margin by which EJP outperforms TP can be as wide as 76.5% to 3.9%, only in a few cases of the $LSTAR$ did TP outperform EJP and then by a small amount (the biggest difference was 80.2% versus 63%). The second feature is the lack of consistency of TP, argued in section 3 and clearly revealed in our simulations. For example, consider $\mu = 1$ in Table 1 TP's correct selection frequency is 12.9%, 9.5% and 3.9% for sample sizes of 50, 100 and 200 observations, respectively. The selection rates for EJP in the same example are 62.4%, 70.4% and 76.5%, respectively. Except for one case (Table 2, $\pi_1 = -0.5; \theta_1 = 0.5$; EJP = 73.6% for T = 50 versus EJP = 72.3% for T = 100), EJP's selection rates improved as the sample size increased. We view these results as strong support in favor of EJP.

4.2 Power Properties of the NL4 Test

Table 3 compares the power of the nonlinearity tests NL3 and NL4. The key question is whether the gain in power from augmenting the auxiliary regression in (6) with the terms $\widetilde{\mathbf{x}}_t z_{t-d}^4$ outweigh the losses from including these additional regressors in (9). For $c = 0$, if the true DGP is a LSTAR model, we will lose power by including redundant regressors. However, if the true model is an ESTAR, NL4 should have higher power. If $c \neq 0$, the power of each test needs to be evaluated on a case by case basis.

The simulations indicate that with large sample sizes (in our study 300 observations), there is little difference between NL3 and NL4. Both tests detect non-linearity adequately, with the power approximating 1 in most cases. However, for smaller sample sizes, (in our study 100 observations), NL4 has higher power than NL3 when the true model is ESTAR (in particular when the variance of the error term is high and/or c and Θ_0 are nonzero). When the true model is LSTAR, NL4 does not significantly lose power with respect to NL3. In view of these results and with the disclaimer of their limited scope, we recommend NL4 when the sample size is small and parsimony is not an issue.

5 Teräsvirta and Anderson (1992) Revisited

Teräsvirta and Anderson (1992) analyze the dynamic properties of industrial production indices of thirteen OECD countries and an European aggregate using STAR models. The data is quarterly, seasonally unadjusted,[14] and spans from 1960:I to 1986:IV.[15] This section replicates nonlinearity testing and model selection with our alternatives, NL4 and EJP. Our goal is to compare the performance of the new procedures in practice. Table 4 reports p-values of nonlinearity tests, delay parameter choice, and STAR model selection for those countries in which nonlinearities were detected either by NL3 or NL4.[16]

NL3 and NL4 obtain their minimum p-values for the same choice of delay parameter, d, except in the case of the U.S.A. While the results of both tests are similar, NL4 fails to reject linearity at the usual 5% level for 3 countries.[17] With regard to EJP, the same models are selected as with TP except for Austria and Sweden. In the case of Japan, Teräsvirta and Anderson (1992) report that choosing between models (LSTAR or ESTAR) was hard with TP and hence estimated both specifications. The preferred model was an ESTAR — a choice that EJP selects unequivocally.

We estimated both LSTAR and ESTAR specifications for Austria and Sweden to determine which specification fitted best. The basic statistics of the preferred models for each specification are reported in Table 5 The estimates for Austria are harder to compare since the final models have a different number of parameters — Schwarz's information criterion (SIC) favors the ESTAR specification while Akaike's (AIC) favors the LSTAR specification. However, in the case of Sweden,

[14] They make the series approximately stationary by fourth lag differencing (x_t-x_{t-4}).

[15] Source: *OECD Main Economic Indicators.*

[16] French and Italian indices were adjusted for strikes and other anomalies and are therefore not considered here.

[17] A sample of 104 observations and the extra regressors required by NL4 probably justify this result.

the final models have the same number of parameters. The preferred specification is the ESTAR (which was selected by EJP but not by TP) with a better fit overall than its LSTAR counterpart. Of course, the true model is unknown. The value of this exercise was to check what specification seemed to work best and what specification test led us to it.

6 Conclusion

This paper provides a new selection procedure to choose between a logistic and an exponential specification when the alternative to linearity considered is a STAR model. This new decision rule, EJP, is simpler, more intuitive, and has better power and consistency properties than TP. Along the way, we have also provided practical guidelines regarding nonlinearity testing. In support of our claims, we conducted Monte-Carlo simulations and applied our procedures to the industrial production indices of thirteen OECD countries and to a European aggregate. All the tests developed here can be easily generalized for use in Smooth Transition Regression and multivariate models.[18]

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[18] See Escribano and Jorda (1998) and Teräsvirta(1997).

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Table 1

Relative frequencies of correct specification of STAR model (as a proportion of replications for which linearity is rejected at a 95% confidence level). Table 4, pg. 172, Luukkonen et al. (1988b). 1 = 100% accuracy selecting the correct model, 0 = 0% accuracy. DGP:

$$y_t - 0.3y_{t-1} - \{\exp(-y_{t-1}^2) - 1\}0.9y_{t-1} = m + e_t \qquad e_t \sim N(0\ ,0.36)$$

Sample Size	m	TSP	EJSP	Power NL4
50	0	0.632	0.792	0.106
	0.3	0.472	0.736	0.125
	1	0.129	0.624	0.210
100	0	0.805	0.898	0.256
	0.3	0.552	0.830	0.317
	1	0.095	0.704	0.493
200	0	0.899	0.963	0.616
	0.3	0.659	0.923	0.692
	1	0.039	0.765	0.881

Table 2

Relative frequencies of correct specification of STAR model (as a proportion of replications for which linearity is rejected at a 95% confidence level). Fig. 2, pg. 496, Luukkonen et al. (1988a). 1 = 100% accuracy selecting the correct model, 0 = 0% accuracy. DGP:

$$y_t - p_1 y_{t-1} + (q_1 y_{t-1})(1 + \exp\{-0.5y_{t-1}\})^{-1} = e_t \qquad e_t \sim N(0\ ,25)$$

	p_1 = -0.5				p_1 = 0.5			
Sample Size	q_1	**TSP**	**EJSP**	**Power NL4**	q_1	**TSP**	**EJSP**	**Power NL4**
50	-0.4	0.500	0.594	0.064	-1.4	0.568	0.947	0.322
	0	.	.	0.039	-1	0.872	0.872	0.203
	0.5	0.736	0.736	0.072	-0.5	0.802	0.630	0.081
	1	0.853	0.871	0.170	0	.	.	0.047
	1.5	0.904	0.936	0.467	0.5	0.841	0.690	0.113
100	-0.4	0.459	0.811	0.122	-1.4	0.594	0.978	0.744
	0	.	.	0.043	-1	0.941	0.935	0.491
	0.5	0.811	0.724	0.127	-0.5	0.898	0.814	0.118
	1	0.952	0.936	0.498	0	.	.	0.037
	1.5	0.963	0.978	0.883	0.5	0.919	0.860	0.272

Table 3

Power simulations. Data generated from models 4.1-4.2 and 4.6, pg. 210-211, in Teräsvirta (1994).
DGP:

$$y_t = 1.8 y_{t-1} - 1.06 y_{t-2} + (\pi_{20} - 0.9 y_{t-1} + 0.795 y_{t-2}) F(y_{t-1}) + u_t ; \quad u_t \sim N(0\ ,0.04)$$

$$F(y_{t-1}) = (1 + \exp\{-\gamma(y_{t-1} - c)\})^{-1}, \quad LSTAR; \quad F(y_{t-1}) = 1 - \exp\left\{-1000(y_{t-1} - c)^2\right\}, \quad ESTAR$$

	ESTAR		LSTAR		ESTAR		LSTAR	
Model	**NL3**	**NL4**	**NL3**	**NL4**	**NL3**	**NL4**	**NL3**	**NL4**
π_{20} = c = 0	0.612	0.722	0.962	0.951	0.825	0.835	1.000	1.000
π_{20} = 0.02; c = 0	0.983	0.997	0.691	0.656	1.000	1.000	0.993	0.993
π_{20} = 0.04; c = 0.02	0.611	0.623	0.157	0.139	0.984	0.992	0.378	0.373
	Sample Size = 100				Sample Size = 300			

Table 4

Linearity testing, determining the delay parameter and selecting between LSTAR and ESTAR models

Country	Max. Lag (AIC)	P - value NL3	P - value NL4	Delay Parameter	TSP Choice	EJSP Choice
Austria	5	0.010	0.033	1	LSTAR	ESTAR
Belgium	5	0.050	0.259	1	LSTAR	LSTAR
Japan	5	0.000	0.000	1	?	ESTAR
Norway	8	0.031	0.200	5	LSTAR	LSTAR
Sweden	5	0.015	0.040	3	LSTAR	ESTAR
U.K.	8	0.047	0.192	4	ESTAR	ESTAR
U.S.A.	6	0.006	0.054/0.016*	3/5*	LSTAR	LSTAR
EUR	9	0.015	0.043	3	ESTAR	ESTAR

Note: For U.S.A. NL4 minimum p-value was for d = 5.

Table 5

Summary Statistics for STAR model estimation: Austria and Sweden.

	AUSTRIA			SWEDEN	
Summary Statistics	**LSTAR**	**ESTAR**	**Summary Statistics**	**LSTAR**	**ESTAR**
R-Squared	0.7079	0.6778	*R-Squared*	0.7251	0.7311
Adj. R^2	0.6819	0.6607	*Adj.* R^2	0.7039	0.7104
SSR	0.0503	0.0554	*SSR*	0.0538	0.0526
AIC	-7.4036	-7.3770	*AIC*	-7.3560	-7.3781
SIC	-7.1677	-7.2207	*SIC*	-7.1463	-7.1684
Durbin-Watson	2.3350	2.0845	*Durbin-Watson*	1.8476	2.0132
No. of params.	9	6	*No. of params.*	8	8