Section: Evidence of Financial Globalization and Crisis

Title: Carry Trade

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1. Introduction

One of the most basic principles of finance states that a zero-cost investment should have zero expected return – in the alternative there would be an arbitrage opportunity. Frictions and compensation for risk may alter the specifics but not the core of this principle: you should not get something for nothing. The carry trade is an example of a zero cost investment where the speculator borrows in a low-yielding currency and invests the proceeds on a high-yielding currency so as to profit from the spread in the yields. If these short-long positions involve government securities, usually assumed to have zero risk of default, then the only source of exposure comes from an appreciation of the funding currency relative to the target currency.

Interest rates over the period in which the carry trade is held are generally fixed and known when the speculator enters the transaction. Thus, the amount by which the funding currency would have to appreciate to eliminate arbitrage is directly related to the yield spread – another manifestation of the well-known *uncovered interest rate parity* (UIP) condition. However, the speculator could simultaneously enter into a forward contract that would determine with certainty the exchange rate at which he could unwind the carry trade at the end of the holding period. Because a forward contract eliminates all exposure in a carry trade, it establishes a natural link between the forward rate and the spot rate that is expected to prevail at the end of the holding period – another manifestation of the familiar *covered interest rate parity* (CIP) condition.

These parity conditions however, are associated with the most enduring puzzles in international finance. Under rational expectations, UIP suggests that the yield spread should be an unbiased predictor of fluctuations in the spot rate. Similarly, CIP suggests that the forward rate should also be such an unbiased predictor. But empirically, it is often the case that the high yielding currency tends to appreciate rather than depreciate, a result that is difficult to explain even if one entertains that investors require a considerable premium as compensation for risk. This *forward premium puzzle* has therefore generated a rather extensive literature that includes the seminal work of Frankel (1980), Fama (1984), Froot and Thaler (1990), and Bekaert and Hodrick (1993), to cite a few. However, over periods of a decade or two it is quite difficult to reject ex-post UIP (see e.g. Fujii and Chinn, 2000; Alexius, 2001; and Sinclair, 2005), suggesting that yield spreads are arbitraged in the long-run and that possible profit opportunities are a matter of timing.

The trinity of parity conditions in international finance is completed with the purchasing power parity (PPP) condition – the proposition that, once converted to common currency, national price levels should be the same, which describes an equilibrium level toward which exchange rates would be expected to settle in the long-run. This proposition, perhaps first articulated by scholars of the Salamanca school in the sixteenth century (see Rogoff, 1996 and references therein), is yet another arbitrage condition that links the prices at which goods can be sold across borders, and the exchange rate. While PPP is hardly a satisfactory predictor of exchange rate fluctuations in the short-run, there is now some agreement about the robustness of the mechanism in the long-run (see the literature reviews in Rogoff, 1996; and Taylor and Taylor, 2004). The notion that a misalignment of the exchange rate from this equilibrium condition can endure only for so long, provides another empirically plausible self-correcting mechanism in the quest of divining where carry trade opportunities may lay, as we shall see.

All three parity conditions provide disarmingly simple mechanisms by which one could explain how exchange rates should vary over time. But Meese and Rogoff (1983) conclude that yield spreads, forward rates, price levels and other classic explanations in international economics are poor predictors of exchange rates in the short-run, thus generating a host of research that assesses the predictive ability of competing explanations and includes Engel and Hamilton (1990), two chapters in the Handbook of International Economics (1995), one by Frankel and Rose and the other by Froot and Rogoff, and Kilian and Taylor (2003), to mention a few. Although these puzzles are not individually fleshed out here, the carry trade offers a different perspective on these issues that is discussed below.

At US $4 trillion daily turnover in the global foreign exchange (FX) market (see the Bank of International Settlements, 2010) and with the ability to leverage[1] an investment up to 50-to-1, it is hard to overstate the importance of understanding the carry trade – FX market turnover dwarfs the combined daily turnover of the largest stock exchanges in the world combined (see Menkhoff and Taylor, 2007). Modern investment management must therefore include currency trading as an essential asset class in any portfolio. Naïve carry trade, a strategy solely based on

[1] The latest ruling by the U.S. Commodity Futures Trading Commission, which goes into effect October 18, 2010 will allow investors to place a minimum 2 percent security deposit in the case of major currencies and a 5 percent for all other currencies. For illustration, Bear Stearns had a leverage ratio of 35-to-1 when it collapsed in March 2008.

currency yield spreads where no attempt is made to predict exchange rates, would have yielded about 6% per annum (p.a.) from January 2003 to January 2008. Moreover, Binny (2005) and Berge, Jordà and Taylor (2010) find that currency trades have low correlation with the returns of conventional assets. At the same time, any gains made over this five year window completely evaporated by the end of 2008. This illustrates one of the essential features of the carry trade: prolonged periods with positive returns peppered with sudden crashes, a phenomenon often described "as going up the stairs and coming down the elevator "or also as "the peso problem" (see e.g. Burnside, Eichenbaum, Kleshchelski and Rebelo, 2008).

The carry trade is primarily a financial strategy and the first order of business is to identify and evaluate the returns characteristics of the most popular carry trade designs. This will require tools in which the properties of a given strategy are evaluated, not solely through a statistical lens, but through the lens of an investor tinted by returns, risk, and left-tail risk (as the risk of a crash is sometimes described). For this reason, some of the theoretical explanations for the apparent profitability of the carry trade dwell on financial frictions (e.g. Shleifer and Vishny, 1997; Jeanne and Rose, 2002; Froot and Ramadorai, 2005; Brunnermeier and Pedersen, 2008; Burnside, Eichenbaum, Kleshchelski and Rebelo, 2008; and Brunnermeier, Nagel and Pedersen, 2009) more than on full-blown conventional models of international finance (Baccheta and van Wincoop, 2006; Fisher, 2006; Lustig and Verdehlan, 2007; and Ilut, 2008).

Much of the discussion will focus on the period of unfettered arbitrage in the current era of financial globalization, that is, from the mid 1980s on for major currencies. Starting in the 1960s, the growth of the Eurodollar markets had permitted offshore currency arbitrage to develop. Given the increasingly porous nature of the Bretton Woods era capital controls and the tidal wave of financial flows building up, the dams started to leak, setting the stage for the crisis of the Bretton Woods regime in 1970-73. Floating would permit capital account liberalization but the process was fitful, and not until the 1990s was the transition complete in Europe (see Bakker and Chapple, 2002).

2. Designing Carry Trade Strategies

Fundamental models of consumption-based asset pricing are a standard way to describe how an investor's patience and tolerance for risk determine the value of a strategy's pay-offs in different states of the world. This adjustment of returns has a similar flavor to the probability weights assigned to random events. In fact, the *stochastic discount factor,* which summarizes the interaction between investor preferences and outcomes, can be thought of as the set of pseudo-probabilities implied by the investor's consumption choices.

Suppose x_{t+1} denotes the returns of a zero-cost investment strategy, then it is clear that for a risk-neutral investor (an investor that only cares about expected returns but not the risk associated with each outcome), absence of arbitrage would suggest that

$$E_t(x_{t+1}) = 0, \tag{1}$$

that is, given information available at time *t,* the investor should expect to obtain zero returns *on average*. In the more general case, let m_{t+1} denote the stochastic discount factor, then expression (1) becomes

$$E_t(m_{t+1}x_{t+1}) = 0. \tag{2}$$

In practice, m_{t+1} would be determined by the consumption outcomes associated with different states of the world and other sources of risk so that the value of the investment depends on its ability to produce returns for those states of the world in which consumption is low and no alternative investment offers an adequate hedge.

The carry trade is an example of zero-cost investment (abstracting momentarily from transactions costs and limits on leverage) and it will be useful to consider the trinity of parity conditions in international finance discussed earlier to design carry trade strategies. Specifically and for convenience, make the U.S. the *home country* so that the spot exchange rate ϵ_t is expressed in terms of foreign currency units per dollar. In a carry trade, you could for example borrow $1 at an interest rate i_t^* (* will be used to denote home country) by selling *short* a security and then by buying ϵ_t units of a foreign security with the same maturity (by going *long*) that yields i_t. At the end of the holding period the transaction is reversed. The foreign security will return $(1 + i_t)$ foreign currency units which can be transformed back into dollars at the spot exchange rate prevailing at maturity, ϵ_{t+1}. The proceeds are then used to repay principal plus interest $(1 + i_t^*)$ and any difference represents the carry trade profit (or loss if the proceeds are insufficient to repay the short-sell). Under risk-neutrality, we know that such a transaction should have, on average, zero returns, that is

$$E_t \epsilon_{t+1}(1 + i_t^*) = \epsilon_t(1 + i_t). \tag{3}$$

Taking natural logarithms, using the approximation $ln(1 + i_t) \approx i_t$ and denoting $ln(\epsilon_t) = e_t$ and $\Delta e_{t+1} = e_{t+1} - e_t$, then carry trade returns are simply

$$x_{t+1} = \Delta e_{t+1} + (i_t^* - i_t) \tag{4}$$

so that absence of arbitrage under the risk-neutral measure implies

$$E_t(x_{t+1}) = 0 \tag{5}$$

or that the expected rate of appreciation of the exchange rate equals the yield spread, that is

$$E_t(\Delta e_{t+1}) = i_t - i_t^*.$$

This last relation is one way to express UIP.

Going back to expression (3), if the investor instead purchases a forward contract F_t at time *t* on the value that ϵ_{t+1} will take, then (3) can be expressed instead as

$$F_t(1 + i_t^*) = \epsilon_t(1 + i_t) \tag{6}$$

so that, under risk-neutrality, $E_t(\epsilon_{t+1}) = F_t$, that is, the forward rate is an unbiased predictor of ϵ_{t+1}. Equation (6) provides one way to express CIP. In practice expression (6) will be of little use in constructing carry trade strategies as it is found in practice that forward contracts are priced to meet expression (6) so that no independent source of variation can be obtained by using them.

The final parity condition, PPP, can be incorporated into this discussion by expressing (3) in real rather than nominal terms. Specifically, let $r_t = i_t - \pi_{t+1}$ with $\pi_{t+1} = \Delta p_{t+1}$ and let $p_t = ln(P_t)$, with P_t the foreign country's price level. The same notation but with a * refers to the home country. Define the logarithm of the real exchange as $q_{t+1} = e_{t+1} + (p_{t+1} - p_{t+1}^*)$. Under the (weak) PPP condition $q_{t+1} = \bar{q} + \phi(p_{t+1} - p_{t+1}^*)$ with $\bar{q}$ the mean *fundamental equilibrium exchange rate* (FEER) to which q_{t+1} reverts to in the long-run so that it is a *stationary* variable. Notice then that the returns of the carry trade in expression (4) can be rewritten as

$$x_{t+1} = \Delta q_{t+1} + (r_t - r_t^*) \tag{7}$$

and absence of arbitrage now implies that the expected real exchange rate appreciation is equal to the spread in real interest rates, that is, $E_t(q_{t+1}) = E_t(r_t - r_t^*)$.

The dynamic interaction between nominal exchange rates, nominal interest rates, inflation and long-run PPP equilibrium are the constituent elements of a system than can be used to describe the stochastic behavior of nominal exchange rates and with which to form forecasts. Consider the stationary random vector Δy_{t+1} given by

$$\Delta y_{t+1} = \begin{bmatrix} \Delta e_{t+1} \\ \pi^*_{t+1} - \pi_{t+1} \\ i^*_{t+1} - i_{t+1} \end{bmatrix} \quad (8)$$

and where $q_t = \bar{q} + \phi(p_t - p^*_t)$ is a (unique) *cointegrating vector* that captures the essence of the (weak) PPP condition. If one assumes that the stochastic process for the system in (8) is linear, then a *vector error correction model* (VECM) is a natural model choice. For example, in a first order VECM, the first equation of the system in expression (8) becomes

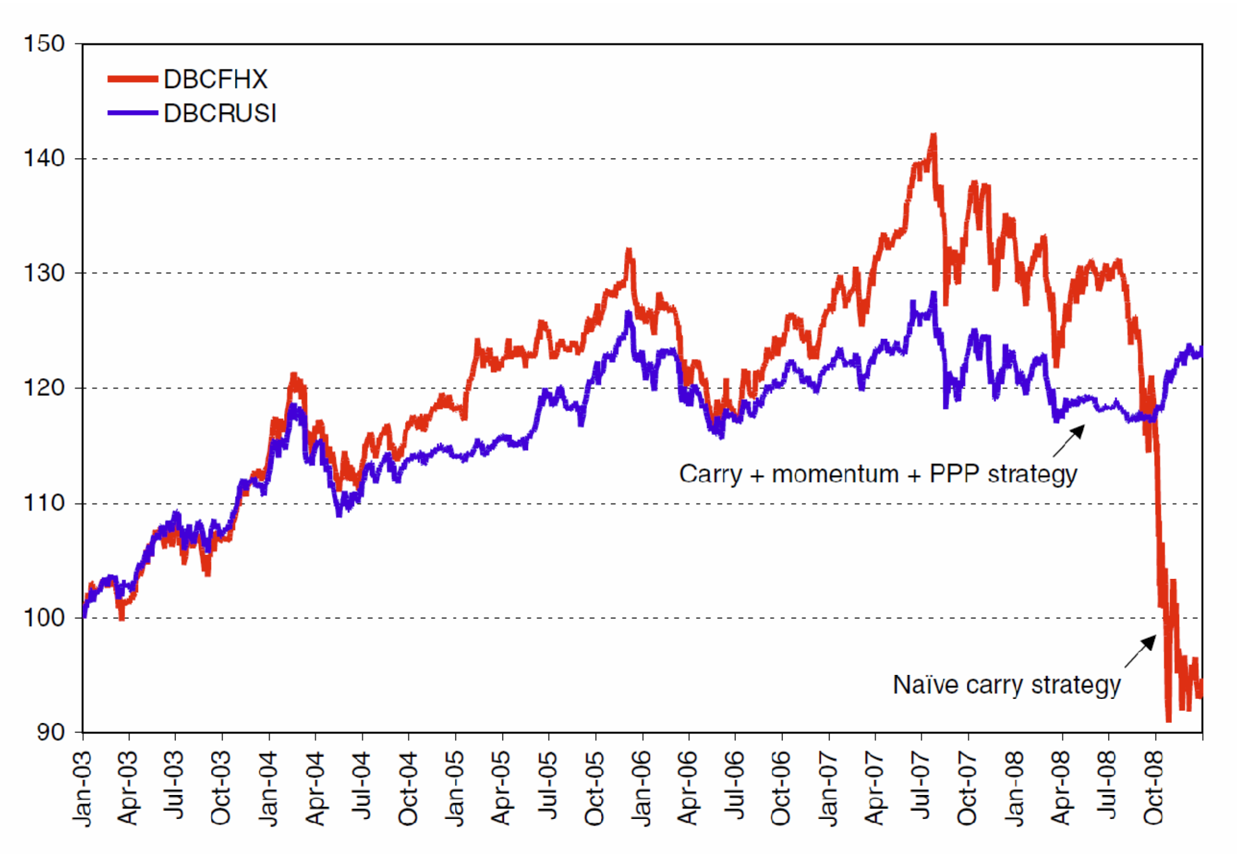
$$\Delta e_{t+1} = \beta_0 + \beta_e \Delta e_t + \beta_\pi(\pi^*_t - \pi_t) + \beta_i(i^*_t - i_t) + \gamma(q_t - \bar{q} - \phi(p_t - p^*_t)) + u_{t+1} \quad (9)$$

Expression (9) nests four popular approaches (to be described in detail below) to currency trading: *carry* (C), *value* (V) and *momentum* (M) signals used singly, and a composite based on a mix of all three CMV signals. For example, the CMV approach underlies each of three popular tradable ETFs[2] created by Deutsche Bank, where in each case a nine-currency portfolio are sorted into equal-weight long-neutral-short thirds based on the relative strength of each of the three signals, and regularly rebalanced. In addition, Deutsche Bank offers a composite rebalancing portfolio split one third between each of the CMV portfolios. Similar tradable indices and ETF products have since been launched by other financial institutions (e.g. Goldman Sachs' FX currents, and Barclays Capital's VECTOR). Figure 1 presents the

[2] ETF stands for exchange traded fund.

performance of two of these Deutsche Bank ETFs, one based on a naïve carry trade strategy and the other based on the composite strategy just described.

Figure 1. The performance of a naïve carry ETF versus a composite ETF



Notes: The ETFs are based on the Deutsche Bank G10 Currency Harvest USD Index (DBCFHX) and Deutsche Bank Currency Returns USD Index (DBCRUSI). The indices predate the ETF inception dates.

Summarizing, the four carry trade strategies, whose properties are examined here, can be succinctly described by how exchange rate appreciation is forecast:

- **Carry (C):** $\Delta\hat{e}_{t+1} = 0$; that is, the *carry* strategy solely focuses on $i_t^* - i_t$ to determine which currency to sell securities short with, and which to go long in.

- **Momentum (M):** $\Delta\hat{e}_{t+1} = \hat{\beta}_e \Delta e_t$**;** that is, if one simply goes with $\hat{\beta}_e = 0$ then the *momentum* strategy simply takes the current value of the exchange rate to be the best forecast of the exchange rate the next period.
- **Value (V):** $\Delta\hat{e}_{t+1} = \hat{\gamma}(q_t - \bar{q})$**;** where the PPP signal is used to forecast exchange rate appreciation.
- **VECM:** $\Delta\hat{e}_{t+1} = \hat{\beta}_0 + \hat{\beta}_e \Delta e_t + \hat{\beta}_\pi(\pi_t^* - \pi_t) + \hat{\beta}_i(i_t^* - i_t) + \hat{\gamma}(q_t - \bar{q} - \phi(p_t - p_t^*))$; that is, forecasts are based on a vector error correction representation for the system in expression (8), and specifically, using expression (9). This last specification is proposed in Berge, Jordà and Taylor (2010) as a way to encompass UIP and deviations from FEER.

At time *t* the investor uses one of these strategies to determine the direction of the carry trade he wishes to engage in depending on

$$\hat{d}_{t+1} = sign(\hat{x}_{t+1}) \in \{-1, 1\}; \qquad \hat{x}_{t+1} = \Delta\hat{e}_{t+1} + (i_t^* - i_t). \tag{10}$$

The ex-post returns realized by the trader are

$$\hat{\mu}_{t+1} = \hat{d}_{t+1} x_{t+1}. \tag{11}$$

In other words, the trader need not be particularly accurate in predicting Δe_{t+1} (which we know at least since Meese and Rogoff, 1983, to be a rather futile effort) as long as $\hat{d}_{t+1}$ correctly selects the direction of the carry trade. Recent work by Cheung, Chinn, and García Pascual (2005) and Jordà and Taylor (2010a) suggests that directional forecasts of exchange rate movements perform better than a coin-toss, suggesting that there may be an economic value to carry trade investment.

3. A Trading Laboratory for the Carry Trade

It is useful to examine the empirical properties of the strategies described in the previous section before commenting on formal methods of evaluating the investment potential of the carry trade. The data used throughout consists of a panel of nine countries (Australia, Canada, Germany, Japan, Norway, New Zealand, Sweden, Switzerland and the United Kingdom) relative to the United States, with the sample period being monthly observations between January 1986 and December 2008. The observed variables include the end-of-month nominal exchange rates expressed in foreign currency units per U.S. dollar; the one-month London interbank offered rates (LIBOR); and the consumer price indices. Exchange rate data and consumer process indices were obtained from the IFS[3] database, the LIBOR data are from the British Banker's Association.

Table 1, reproduced from Berge, Jordà and Taylor (2010), summarizes the panel based estimates of the four carry trade strategies described in the previous section: *carry, momentum, value* and *VECM*. These are fitted over the entire available sample. Generally speaking, it is clear that the model explains very little variation in the data. Nevertheless, the results justify some of the common carry trade strategies pursued. For instance, in the *momentum* strategy the coefficient on the lagged value of the change in exchange rates is positive and significant.

[3] International Financial Statistics, maintained by the International Monetary Fund.

Table 1. Four benchmark carry trade strategies

In-sample estimates, January 1986 – December 2008

Dep. V: Δe_{t+1}	**Carry Trade Strategy**			
	Carry	**Momentum**	**Value**	**VECM**
Δe_t	-	0.114*	-	0.128**
		(0.031)		(0.032)
$i_t^* - i_t$	-	-	-	0.864**
				(0.358)
$\pi_t^* - \pi_t$	-	-	-	0.230*
				(0.122)
$q_t - \bar{q}$	-	-	-0.014**	-0.023**
			(0.003)	(0.003)
R^2	-	0.013	0.004	0.025
Currencies	9	9	9	9
Periods	276	276	276	276
Total Obs.	2484	2466	2466	2455

Notes: Panel estimates with country fixed effects (not reported). Heteroskedasticity-robust standard errors reported in parenthesis. **/* indicates significance at the 95/90% confidence level. Slight differences in the total number of observations are due to differences in data lags.

Similarly, in the *value* strategy, there is some evidence that exchange rates do return to their fundamental value, albeit quite slowly, at a rate of about 1.4% per month. The more sophisticated *VECM* encapsulates both of these results, here with a long-run equilibrium speed of adjustment of about 2.3%.

Table 2, also reproduced from Berge, Jordà and Taylor (2010), reports some out-of-sample results that are more relevant for the speculator. They are based on a set of one period-ahead forecasts generated with rolling-window samples beginning with a forecast of January 2004 based on January 1986 to December 2003 data, and continuing on until December 2008. Thus, the out-of-sample period includes the turbulent fall of 2007, in which several "crash" episodes or "peso events" took place and therefore provides a realistic assessment of the type of returns that could have been made at the time.

Table 2. Four benchmark carry trade strategies.

Out-of-sample performance, January 2004 – December 2008

Realized Returns to an Equally-Weighted Portfolio	**Carry Trade Strategy**			
	Carry	**Momentum**	**Value**	**VECM**
Mean (monthly)	-0.0024	0.0027	-0.0022	0.0025
S.D.	0.018	0.018	0.015	0.018
Skewness	-2.97	1.59	-0.69	1.44
Avg. Ann. Ret (%)	-2.9	3.3	-2.6	3.0
Sharpe Ratio (ann.)	-0.47	0.51	-0.51	0.47

The results are based on an equally-weighted portfolio of the 9 currencies we consider against the U.S. dollar. A moment's reflection reveals the fickleness of currency trading. *Momentum* and *VECM* enjoy a low but positive rate of return of around 3% annually with a Sharpe ratio of about 0.5 and with a positive skew – almost the mirror image of the *carry* and *value* strategies. Thus, the results in Tables 1 and 2 seem to confirm popular wisdom: exchange rates are difficult to predict and therefore the carry trade is risky business. But, How risky? Are the negative profits *significantly* bad? And are the positive profits experienced by the *momentum* and *VECM* strategies *significantly* good? And how did they do relative to other investments, which also felt the fury of the financial crisis that began in the fall of 2007? To answer these questions, it is important to turn to the investor's problem in deciding among different strategies to come up with performance criteria that reflect, not a statistician's preference of one model over another based on the properties of the forecast errors, but an investor's returns and appetite for risk. This discussion is presented in the next section and along the way we will learn about a different way to evaluate UIP and PPP and the efficiency of currency markets.

4. The Trader's Decision Problem

Determining the profitable direction of a carry trade does not require an accurate model of exchange rate forecasting – just a model that accurately classifies when to go long or short with a particular currency pair. Specifically, recall that the returns realized ex-post are

$$\hat{\mu}_{t+1} = \hat{d}_{t+1} x_{t+1},$$

where using slightly more general notation, $\hat{d}_{t+1} = sign(\hat{\delta}_{t+1} - c)$, $\hat{\delta}_{t+1}$ is called a generic *scoring classifier*, for example, $\hat{\delta}_{t+1} = \hat{x}_{t+1}$; and c is a scalar that can take any value in the interval $c \in (-\infty, \infty)$ and only plays the role of a threshold. The more general notation $\hat{\delta}_{t+1}$ allows one to consider a more comprehensive list of scoring classifiers than not just conditional mean forecasts (this is explained in more detail in Jordà and Taylor, 2010b).

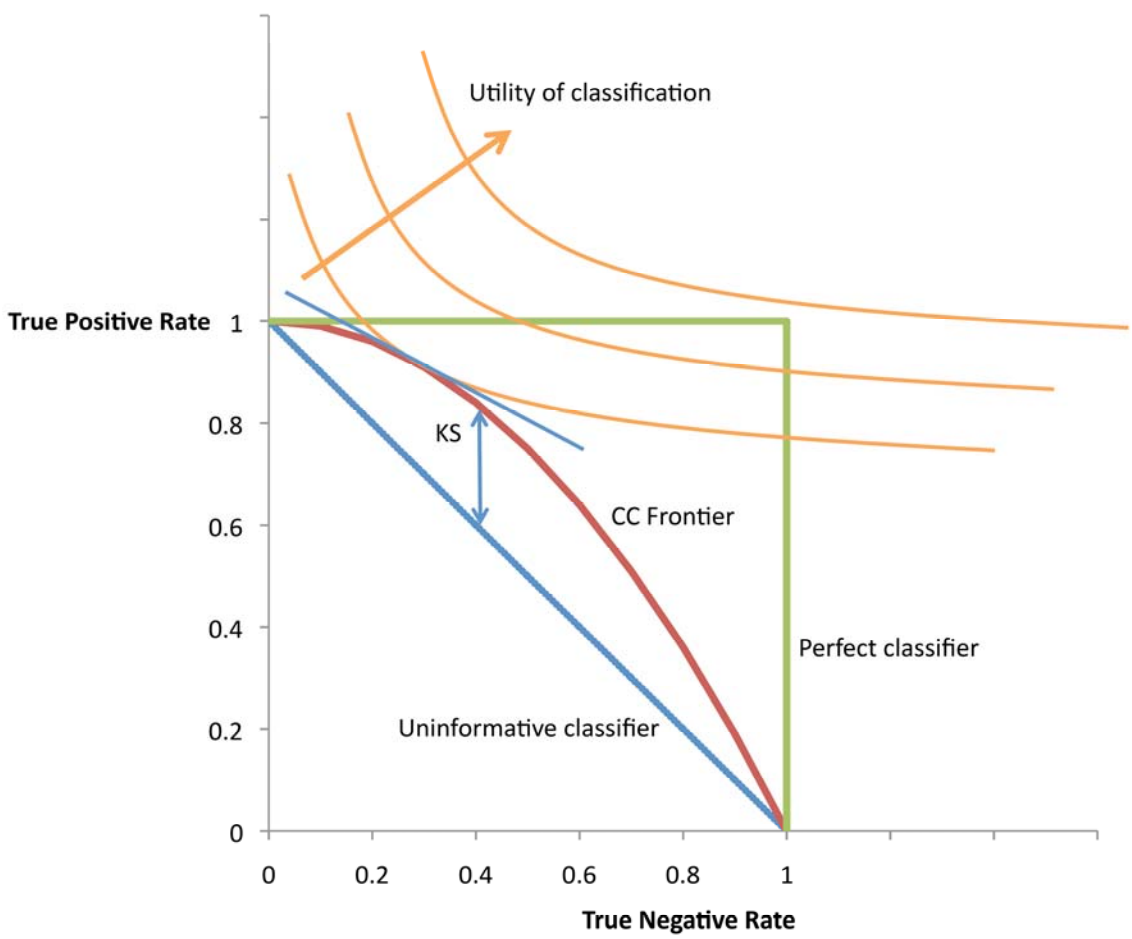
With these preliminaries, the trader's decision problem can be summarized by the following table:

Table 3. The trader's classification problem

		Prediction	
		Negative/Short	Positive/Long
Outcome	Negative/Short	$TN(c) = P(\hat{\delta}_t < c \mid d_t = -1)$	$FP(c) = P(\hat{\delta}_t > c \mid d_t = -1)$
	Positive/Long	$FN(c) = P(\hat{\delta}_t < c \mid d_t = 1)$	$TP(c) = P(\hat{\delta}_t > c \mid d_t = 1)$

Here *TN(c)* and *TP(c)* refer to the true classification rates of negatives and positives respectively; and *FN(c)* and *FP(c)* refer to the false classification rates of negatives and positives respectively. Clearly *TN(c)* + *FP(c)* = *1* and *FN(c)* + *TP(c)* = *1*. The space of combinations of *TP(c)* and *TN(c)* for all possible values of $c \in (-\infty, \infty)$ summarizes a sort of "production possibilities frontier" for the classifier $\hat{\delta}_t$, that is, the maximum *TP(c)* achievable for a given value of *TN(c)*. The curve that summarizes all possible combinations {*TP(c), TN(c)*} is called the *correct classification frontier* (or *CC* frontier) by Jordà and Taylor (2010b) and is related to other well-known curves in the statistics literature such as the *receiver operating characteristic (*ROC) curve, which displays combinations {*FP(c), TP(c)*}, and the *ordinal dominance curve*, which displays combinations {*FN(c), TN(c)*}. A stylized plot of a *CC* frontier is presented in Figure 2.

Figure 2. The correct classification frontier (CC frontier)



Notice that as $c \to -\infty$, then $TP(c) \to 1$ and $TN(c) \to 0$, and the limits are reversed as $c \to \infty$. For this reason it is easy to see that the *CC* frontier lives in the unit square [0,1] x [0,1]. A perfect classifier is one for which *TP(c) = 1* for any *TN(c)* and this corresponds to the north-east sides of the unit-square. An uninformative classifier on the other hand, is one where *TP(c) = FP(c) = 1 – TN(c)* for any value of *c* and this corresponds to the north-west/south-east "coin-toss" diagonal.

Just like equilibrium in the textbook two-goods market, equilibrium depends on the interaction between the production possibilities frontier (in our case, the *CC* frontier) and the utility that the investor extracts from each type of outcome. For a risk-neutral investor facing symmetric returns (and assuming no restrictions to trade), equilibrium will occur at that point where the marginal rate of substitution between profitable longs and profitable shorts is -1. It turns out that the vertical distance between the *CC* frontier and the coin-toss diagonal at this point is given by the Kolmogorov-Smirnoff (*KS*) statistic (displayed in figure 1). Briefly, the *KS* statistic compares the average correct classification ability of a classifier against a coin-tosser and has a formula that is relatively simple to compute and given by:

$$KS = max_c \left| 2 \left(\frac{TN(c) + TP(c)}{2} - \frac{1}{2} \right) \right|. \tag{12}$$

The empirical values for *TN(c)* and *TP(c)* are also easy to calculate. Given a candidate scoring classifier $\hat{\delta}_t$, denoting *S* the total number of shorts (negatives) in the sample and *L* the total number of longs (positives) such that *S* + *L* = *N* the number of observations, then

$$T\hat{N}(c) = \frac{\sum_{s=1}^{S} I(\hat{\delta}_s < c)}{S} \qquad T\hat{P}(c) = \frac{\sum_{l=1}^{L} I(\hat{\delta}_l > c)}{L} \tag{13}$$

where the indices *s(l)* are a convenient way reindexing the original observations into those for which $d_t = -1$ (for the shorts) and $d_t = 1$ (for the longs). Moreover, many software packages will calculate the *KS* statistic.

In practice, returns may not be symmetric, and even a risk-neutral investor may face transactions costs (and leverage limits) when short-selling that he does not when going long. Therefore, it is useful to explicitly cast the trader's utility of classification to account for all possible outcomes as

$$\begin{aligned} U(c) = U_{pP} TP(c)\pi + U_{nP}(1 - TP(c))\pi + \\ U_{pN}(1 - TN(c))(1 - \pi) + U_{nN} TN(c)(1 - \pi) \end{aligned} \tag{14}$$

where $\pi = P(d = 1)$, that is, the unconditional probability of a positive (and therefore $(1 - \pi)$ is the unconditional probability of a negative); and U_{aA} for $a \in \{n, p\}$ and $A \in \{N, P\}$ is the utility associated with each of the possible four states defined by the (classifier, outcome) pair.

The optimal operating point (where the investor's preferences are tangent to the *CC* frontier) can be easily found by taking the total differential in expression (14):

$$\frac{dTP}{dTN} = -\frac{(U_{nN} - U_{pN})}{(U_{pP} - U_{nP})} \frac{(1 - \pi)}{\pi}. \tag{15}$$

For example, if $U_{nN} = U_{pP} = 1$ so that correctly predicted outcomes are equally desirable and normalized to one; $U_{pN} = U_{nP} = -1$ so that incorrectly predicted outcomes are equally undesirable and are normalized to represent a loss symmetric to the gains of correct prediction; and $\pi = 0.5$, so that shorts and longs are equally likely, then the slope in expression (15) is -1 and coincides with the point at which the *KS* statistic is calculated as displayed in figure 2.

An alternative to the *KS* statistic that summarizes the properties of the *CC* frontier more broadly is the area under the *CC* frontier or *AUC*. From figure 2 it is clear that *AUC* = *0.5* for a coin-tosser, *AUC* = *1* for a perfect classifier, and with most cases falling somewhere in between. The *AUC* is helpful because it has a Gaussian large sample distribution with which to obtain inference conveniently and statistical packages such as STATA will calculate the *AUC*, its variance and basic statistics. A more detailed explanation of the *AUC* can be found in Jordà and Taylor (2010b).

5. Adjusting for Returns: *KS, *AUC** and Gain-Loss Ratio**

The *KS* and *AUC* statistics measure the success in correctly predicting the direction of trades. But one could predict 99 penny trades correctly, miss the dollar trade, and still lose money. In this section we adjust these statistics to account for returns and we use the modified statistics to relate these with other popular measures of performance in the finance literature. For a given sample of data, consider the maximally attainable profits, those of a trader with perfect foresight, and hence define

$$B_S = \sum_{d=-1} |x_t|, \qquad B_L = \sum_{d=1} x_t. \tag{16}$$

that is, the total returns from the shorts (B_S) and the total returns for the longs (B_L). These serve to construct the following weights for each *P* and *N* outcome:

$$\begin{aligned} w_s &= \frac{x_t}{B_S} \text{ if } \hat{\delta}_s < c \text{ and } d_s = -1 \text{ for } s = 1, ..., S \\ w_l &= \frac{x_t}{B_L} \text{ if } \hat{\delta}_l > c \text{ and } d_l = 1 \text{ for } l = 1, ..., L \end{aligned}$$

where, as before, the indices *s* and *l* each map *N* and *P* outcomes (respectively) to a unique observation *t*.

Using these weights, expression (13) can be easily modified to calculate the return-weighted statistics

$$\widehat{TN}^*(c) = \sum_{s=1}^{S} w_s I(\hat{\delta}_s < c), \qquad \widehat{TP}^*(c) = \sum_{l=1}^{L} w_l I(\hat{\delta}_l > c), \tag{17}$$

from which weighted versions of the *KS* and *AUC* statistics can be easily constructed so that *KS** and *AUC** evaluate returns-weighted directional performance.

Consider now the gains *G* and losses *L* an investor can make with a given classifier. These are given by:

$$\begin{aligned} G &= B_S TN^* + B_L TP^* \\ L &= B_S FP^* + B_L TN^*. \end{aligned}$$

In other words, net profit is simply *G* – *L*. However, this measure suffers from the problem that B_S and B_L are potentially unbounded as the sample size grows large. Therefore, a natural way to construct an investor's utility is by normalizing net profits by total potential profit, that is

$$U^* = \frac{G - L}{G + L} = \frac{B_S(2TN^* - 1) + B_L(2TP^* - 1)}{B_S + B_L}$$

using the results $TN^* + FP^* = 1$ and $TP^* + FN^* = 1$. It is now easy to see that this utility function can be expressed in terms of gain-loss ratios as

$$U^* = \frac{(G/L) - 1}{(G/L) + 1}.$$

Hence, maximizing the utility of the trading strategy is the same as maximizing the gain-loss ratio of the strategy, where the definition of this ratio matches the well-known Bernardo and Ledoit (2000) gain-loss ratio for the risk-neutral case, a measure widely used by finance practitioners and that can be used to provide tight bounds on an investor's tolerance for risk. Taking differentials, utility is maximized when

$$\frac{dTP}{dTN} = -\frac{B_S}{B_L}.$$

Thus, the optimal threshold is the point on the *CC** frontier with slope given by $-B_S/B_L$. When returns are symmetric, the slope is -1, which again coincides with the point at which the *KS** is calculated. For this special case, the gain-loss ratio can be directly calculated by realizing that

$$\frac{G}{L} = \frac{1 + KS^*}{1 - KS^*}$$

which makes it easy to compare with values of the gain-loss statistic of other investments reported in the literature. Moreover, Bernardo and Ledoit (2000) show that under risk neutrality and Gaussianity there is a one-to-one mapping between gain-loss and the Sharpe ratio, another popular method of summarizing the properties of an investment that consists in calculating the ratio of returns normalized by standard deviation.

Absence of arbitrage implies *G/L = 1* whereas an arbitrage opportunity implies that $G/L \rightarrow \infty$, most investment strategies falling somewhere in-between. In fact, one can easily illustrate the relationship between gain-loss and *TN** and *TP**. This is done in figure 3, which shows that the *CC** frontier for a coin-tosser coincides with a gain-loss ratio of 1, a perfect classifier coincides with a gain-loss ratio that approximates infinity, but the *CC** frontier for all other strategies fall somewhere in between with figure 3 providing a convenient way to map all these quantities.

Figure 3. Gain-Loss as a function of *TN and *TP***

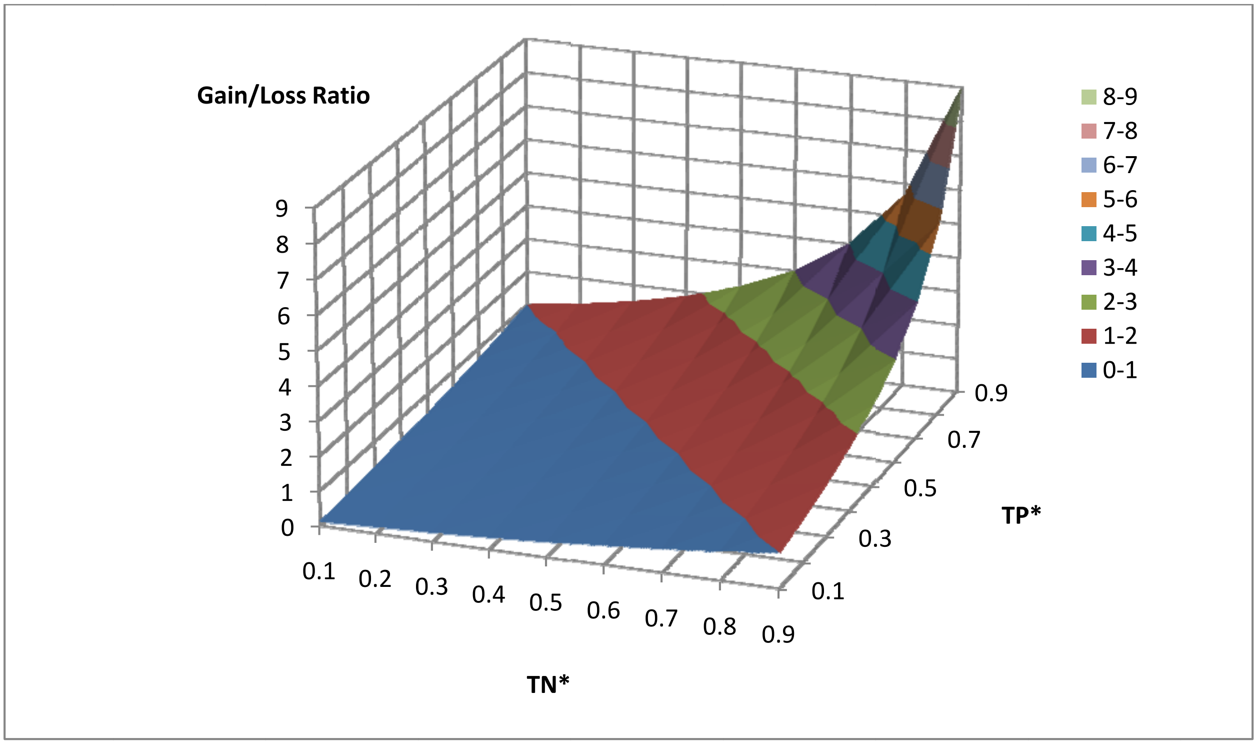


Table 4 illustrates how these techniques can be used to assess carry trade returns for the four strategies discussed in section 2 and in tables 1 and 2. Here the exercise consists in calculating the *KS* and *KS** statistics and hence the implied gain-loss ratio (and Sharpe ratio under the risk-neutral measure) as well as the *AUC* and *AUC** statistics discussed in sections 4 and 5. Table 4 reproduces results reported in Berge, Jordà and Taylor (2010) and focuses on the out-of-sample performance using the same rolling-window procedure described in table 2 for the period January 2004 to December 2008.

Table 4. Out-of-sample performance of four carry trade strategies. January 2004 – December 2008.

	Carry	Momentum	Value	VECM
KS	0.07	0.08	0.07	0.13**
	[0.52]	[0.35]	[0.55]	[0.02]
KS*	0.02	0.16***	0.02	0.17***
	[0.83]	[0.00]	[0.89]	[0.00]
AUC	0.52	0.53	0.51	0.57**
	(0.025)	(0.025)	(0.025)	(0.025)
AUC*	0.44**	0.60**	0.46	0.59**
	(0.025)	(0.024)	(0.025)	(0.024)
G/L ratio	1.04	1.38	1.04	1.41
Sharpe (ann.)	0.05	0.50	0.05	0.51
Obs.	540	540	540	540

Notes: *KS* and *KS** p-values reported in squared brackets. *AUC* and *AUC** standard errors reported in parenthesis. **/*** indicates significance at the 95/99% confidence level.

Several results deserve comment. Notice that for the *momentum* strategy, the *KS* and the *AUC* statistics would indicate poor ability in picking the correct direction of trades. However, once we weigh by returns, the strategy appears to be able to pick *big* winners more successfully than a simple coin-toss. The *VECM* strategy is more consistent and performs well by both metrics. Another interesting result is that both the *carry* and *value* strategies, which do not appear to do better than random chance in picking direction of trade, actually succeed slightly in picking the big *loosers* once one weighs by returns – that is, one would do better taking the opposite side of the bet! Gain-loss ratios are generally close to one (absence of arbitrage), but with an implied annualized Sharpe ratio for the winning strategies (*momentum* and *VECM*) that is similar to equities.

6. Are Carry Trade Returns Compensation for Risk?

Figure 1 succinctly illustrates that carry trades can be profitable over long periods of time, albeit subject to sudden crashes. Table 4, which includes the recent financial crisis, suggests that even a simple rule-of-thumb strategy such as *momentum* would produce positive average returns whose returns characteristics are on a par with equities. How could this be?

A natural answer is that carry trade returns are compensation for risk. For example, Brunnermeier, Nagel and Pedersen (2008) argue that during periods of asset market distress, investors tend to pull back from all risky asset classes, leading to short-run losses via order flow effects, or the price impact of trades. These authors provide support for this proposition using changes in the VIX[4] volatility index (used as a proxy for distress). An increase in VIX was found to correlate with lower carry trade returns at a weekly frequency, suggesting that liquidity may partially explain excess returns. However, lagged changes of VIX had little or no predictive power for next period's returns.

Burnside, Eichenbaum, Kleshchelski and Rebelo (2008) explore carry trades in conjunction with FX options in an effort to hedge against a tail event that takes the form of a collapse in the value of the high-yielding currency. Specifically, they pair a naïve strategy with the purchase fo at-the-money put options on the *long* currency (see also Bhansali, 2007). For the January 1987 to 2008 period, a naïve, unhedged, equally-weighted carry trade of major currencies had a return of 3.22% per annum with a Sharpe ratio of 0.54 and a skewness of -0.67. The U.S. stock market during the same period had a 6.59% return, with a 0.45 Sharpe ratio and a skewness of -1.16. Implementing the option's hedge cost about 0.71% per annum, bringing carry

[4] The Chicago Board Options Exchange produces the VIX and the VXO indices, which are meant to capture market volatility.

trade returns to 2.51% annually but with a Sharpe ratio of 0.71 and a positive skew of 0.75. Jordà and Taylor (2010a) achieve a similar level of protection against crash risk with a more sophisticated carry trade strategy that delivers higher returns and Sharpe ratio although in fairness to Burnside, Eichenbaum, Kleshchelski and Rebelo (2008), their objective is only to illustrate that carry trade returns are difficult to explain as compensation for risk alone.

At a more theoretical level, Lustig and Verdehlan (2007) claim that naïve carry trade returns could be explained in the context of a certain class of consumption-based asset pricing models although Burnside (2007) challenges the usefulness of this approach. Bacchetta and van Wincoop (2007) argue that agents make infrequent adjustments to their FX portfolio decisions because losses incurred are very small relative to observed FX management fees. However, their model relies on agents that process only partial information in that they adopt the rule-of-thumb that exchange rates behave like a random walk.

Rather than articulating frictions in financial markets or in the acquisition of information, or exotic investor preferences, we may ask directly whether carry trade returns are positively correlated with other risk factors: Are risk-adjusted carry trade returns significantly different from zero? Clarida and Taylor (1997) have shown, in a model with persistent short-run deviations from the risk-neutral efficient markets hypothesis, that expectational errors can induce a nonzero correlation between information in the forward yield curve and the future path of the exchange rate. Ang and Chen (2010) build on this idea and ask whether changes in interest rates at other maturities predict excess currency returns. They find that predictability from the yield curve persists up to 12 months and is robust to controlling for other common predictors of currency returns.

Berge, Jordà and Taylor (2010) explore these and other popular risk factors in an effort to determine whether adjusted for risk, carry trade returns are significant. Risk factors are explored from the perspective of the U.S. and include: the excess return to the value-weighted U.S. stock market, the size and the value premium; U.S. industrial production growth; the federal funds rate; the spread between the 10-year Treasury bond and the 3-month Treasury bill; the Pastor and Stambaugh (2003) two liquidity measures; and four measures of market volatility, specifically the Chicago Board Options Exchange volatility indexes VIS and VXO, as well as their differences.

Equal-weighted portfolio returns based on the sample of countries discussed above are regressed against each of the risk factors, one at a time, using the following specification

$$\hat{\mu}_{t+1} = \alpha_k + \beta_k f_{k,t+1} + u_{k,t+1} \tag{18}$$

where the index *k* denotes the *k-th* risk factor in our list of contenders. They consider *carry, momentum, value* and *VECM* strategies augmented with Nelson and Siegel (1987) yield curve factors and find that *momentum* and *VECM* strategies have non-significant betas so that out-of-sample, they are able to get a risk-adjusted annual return of 3% with a Sharpe ratio of 0.69 and with positive skewness (that is, low crash risk).

7. Conclusion

It does not take much sophistication for a speculator to generate risk-adjusted positive returns with the carry trade. A number of obvious improvements (such as optimally designed portfolios, strategies that permit the speculator to remain in a cash position when expected returns are small or uncertain, and others not considered here) would only improve the speculator's returns further. That this is so poses a challenge to conventional notions of market efficiency and long-standing puzzles in international finance. The carry trade is a risky investment but its positive returns are hard to justify on the basis of the investor's tolerance for exposure or how returns correlate with a wide range of alternative risk factors.

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