Take Home Final. Due: Tuesday, March 23, 1999. 11:00 pm. Office 1150 SSH.

Professor Oscar Jorda, 240 C Time Series Analysis

Instructions

You should attempt all problems in this exam. Please **do not collaborate** with your colleagues. You may use any class notes and references that you may deem appropriate. Some problems may not be analytically solvable. If such a situation arises, be as detailed as possible as to how would you try to come up with a solution. **Be very specific in the assumptions** and steps that you take. This will earn you partial credit if your answer is incorrect further down the line. However, you need not prove every single result as long as you **properly document the source.**

Since this is a take home exam, I will demand a **very high standard of precision and clarity**. I will not be as lenient as I was with the midterm (I want to get a nice curve!). Sloppy presentation will result in a poor score. Please invest time in presenting your work in an intelligible and **neat** manner. Unlike homework assignments, please do not stop by my office with questions. You may however, ask for clarification via e-mail. Please turn in each question separately. Good luck!

1

Suppose

$$y_t|F_{t-1} \sim N\left(y_{t-1} exp\left[\beta + \gamma y_{t-1}^2\right]; \sigma^2\right)$$

(a) Sketch the relation between y_t and y_{t-1} for $\gamma < 0$ (i.e. talk about dynamics and whatever characteristics of this model you consider worth mentioning).

(b) Test the hypothesis $\gamma = 0$ against the alternative $\gamma < 0$.

(c) How would you estimate the model? Be very specific. Report first order conditions, properly simplified. Give a detailed description of any numerical method you might have chosen and justify your choice.

(d) Test the final model against the alternative

$$y_t|F_{t-1} \sim N\left(y_{t-1} exp\left[\beta + \gamma y_{t-1}^2 + \delta y_{t-2}^2 + \alpha y_{t-2}\right]; \sigma^2\right)$$

2 Regression of a Random Walk on Deterministic Variables

Let the true D.G.P. be

$$y_t = y_{t-1} + u_t \tag{1}$$

where u_t is a MDS with variance σ_u^2 and also $sup_t E\left|u_t\right|^{2+\delta} < \infty$ for some $\delta > 0$. Also, assume that $y_0 = 0$. Consider the regression,

$$y_t = X_t\beta + \varepsilon_t \tag{2}$$

where $X_t = (1\ t)'$.

(a) Let $\widehat{\beta}$ denote the OLS estimator, show that,

$$\begin{bmatrix} T^{-1/2}(\widehat{\beta}_1 - \beta_1) \\ T^{-1/2}(\widehat{\beta}_2 - \beta_2) \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{bmatrix}^{-1} \begin{bmatrix} \int_0^1 W(\lambda)d\lambda \\ \int_0^1 \lambda W(\lambda)d\lambda \end{bmatrix} \sigma_u$$

(b) What are the probability limits of $\widehat{\beta}_1$ and $\widehat{\beta}_2$? Note: the problem is easier to attack by considering the following

$$T^{-1}D_T(\widehat{\beta} - \beta) = \left[D_T^{-1}\left(\sum X_t X_t'\right)D_T^{-1}\right]^{-1}\left[T^{-1}D_T^{-1}\sum X_t v_t\right]$$

where $v_t = \sum_{s=1}^t u_t$ and

$$D_T = \begin{bmatrix} T^{1/2} & 0 \\ 0 & T^{3/2} \end{bmatrix}$$

(c) Derive the limiting behavior of the t-statistic for the null $\widehat{\beta}_2 = 0$. What implications does this have for the usual procedure of comparing the t-statistic to ± 1.96 (for a 5% test)?

(d) Show that the R^2 from the regression in (2) has the random limit

$$R^2 \Longrightarrow 1 - \frac{\int W_T(\lambda)^2 d\lambda}{\int W\mu(\lambda)^2 d\lambda}$$

where $W\mu(\lambda) = W(\lambda) - \int W(s)ds$.

3 Money-Income Causality

Suppose that log money (m_t) and log output (y_t) are generated by a VAR(p) in first differences, i.e.

$$A(L)\begin{bmatrix} \Delta y_t \\ \Delta m_t \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

where $A(L)$ is a 2×2 lag polynomial matrix of order p with stable roots ($A(1)$ is non-singular) and $A_o = I_2$. Assume that the means in each regression are non-zero and that $\varepsilon_t = [\varepsilon_{1t}\varepsilon_{2t}]'$ is i.i.d. $(0, \sum)$, $\sum$ positive-definite with $E\left|\varepsilon_{it}^4\right| < \infty$, for $i = 1, 2$.

(a) Are m_t and y_t cointegrated? If they are, explain in detail why. If they are not, explain under what conditions they would be.

(b) You run the regression

$$y_t = \alpha_0 + \alpha_1 t + \beta(L) y_{t-1} + \phi(L) m_{t-1} + \varepsilon_{1t}$$

where $(p+1)$ lags are included (so that the regression is correctly specified) given the true model. You wish to show that only y_{t-1} enters in the regression, i.e. you wish to test $H_0 : \beta_i = 0,\ i = 2, ..., p+1$ versus $H_1 : \beta_i \neq 0$ for at least one i. Does the F-test of H_0 have the usual χ^2_p distribution?

(c) A common test is to examine that money does not Granger cause output in regressions such as the above. Consider the Granger-causality test of this hypothesis (i.e. excluding all lags of money from the regression). Does it have the usual χ^2_{p+1} distribution?

4

Consider the following D.G.P. for the cointegrated random variables z_t and y_t

$$(1-L)\begin{pmatrix} y_t \\ z_t \end{pmatrix} = (1-0.4L)^{-1}\begin{pmatrix} 1-0.8L & 0.8L \\ 0.1L & 1-0.6L \end{pmatrix}\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

where $\varepsilon_t = (\varepsilon_{1t}\ \varepsilon_{2t})' \sim N(0,\, I)$ with $z_0 = y_0 = 0$.

(a) Obtain the autoregressive representation of this D.G.P.

(b) Obtain the Error-Correction representation of this D.G.P.

(c) Deduce the long-run relation between z and y.

5

Consider the following model

$$\begin{aligned} x_t + a y_t &= u_t; \qquad & u_t &= \theta u_{t-1} + e_{1t} \\ y_t + 2a x_t &= v_t; \qquad & v_t &= \rho v_{t-1} + e_{2t} \qquad |\rho| < 1 \end{aligned}$$

e_{1t}, e_{2t} i.i.d. zero mean and independent of each other.

(a) If $\theta = 1$, are y_t and x_t cointegrated? If so find the Error Correction model.

(b) Does y_t cause x_{t+1}?

(c) Is y_t weakly exogenous for the estimator a in the first equation?

(d) If $0 < \theta < 1$ are y_t and x_t necessarily I(0)?

(e) If I want to do an impulse response function exercise, such as asking what will be the effect of a shock on y_t on x_{t+4}, can the model be used directly in its present form or should it be transformed in some way?