ARE/ECN 240C Time Series Analysis — Professor Òscar Jordà
Winter 2010 — Economics, U.C. Davis

PROBLEM SET 3 – DUE: JANUARY 31, 2006

Instructions

Please try to answer the questions rigorously by stating any implied assumptions and ensuring all the steps to your conclusion have been properly verified.

Part I – Analytical Questions

1. Suppose $\hat{\pi}_T \xrightarrow{p} \pi_0$ and let $g(\hat{\pi}_T;\theta) = \hat{\pi}_T - h(\theta)$, where h is a continuous function and where $V(\hat{\pi}_T) = \sigma^2$.

(a) Show that

$$\sqrt{T}\left(\hat{\theta}_T - \theta_0\right) \xrightarrow{d} N\left(0, (G'WG)^{-1} G'W\sigma^2 WG(G'WG)^{-1}\right)$$

where $G = \nabla_\theta g(\hat{\pi}_T;\theta)$ and find the specific version of this expression that applies to this problem with optimal weighting matrix $W = \left(\sigma^2\right)^{-1}$.

(b) Suppose $h(\theta) = 3\theta^2$. Derive the asymptotic distribution for this particular case.

(c) Instead, use the delta method to derive the asymptotic distribution of $\hat{\theta}_T$ in (b). Verify that you get the same results with both methods.

2. Suppose $y_t = \mu + \varepsilon_t + \theta\varepsilon_{t-1}$. Define $Y = \left(y_1, ..., y_T\right)'$; and $M = (\mu, ..., \mu)'$.

(a) Obtain $\Omega = E(Y - M)(Y - M)'$
(b) Hence derive the log-likelihood function for *Y*.
(c) Show that the log-likelihood for *Y* is equivalent to the log-likelihood based on $\frac{1}{T}\sum^{T} \ln f(y_t;\theta)$

3. The Hannan-Rissanen estimator for an MA(1) is a two step estimator in which the first step, a long autoregression is fitted to the data and in the second step, the residuals of the first step are used as regressors in the MA(1) specification. Suppose the D.G.P. is

$$y_t = \varepsilon_t + \theta\varepsilon_{t-1};\ |\theta| < 1$$

(a) The first step in the Hannan-Rissanen method consists of estimating

$$y_t = \rho_1 y_{t-1} + ... + \rho_p y_{t-p} + u_t$$

where p is fixed (for example, chosen by information criteria). Assuming $\hat{\rho}_j \xrightarrow{p} \theta^j$ for $j = 1,...,p$, derive the expression for $\hat{u}_t$ and show under what conditions $\hat{u}_t \xrightarrow{p} \varepsilon_t$.

(b) The second step consists in estimating θ with the regression

$$y_t = v_t + \theta \hat{u}_t$$

Show that this estimator is consistent for $\hat{\theta}$.

(c) Derive the asymptotic distribution of $\hat{\theta}$. Show that this estimator is less efficient than the maximum likelihood estimator.

4. Consider the following ARMA(2,1) model

$$y_t = 2 + 1.2 y_{t-1} - 0.5 y_{t-2} + \varepsilon_t + 0.25 \varepsilon_{t-1}$$

(a) Write this model in state-space form

(b) Using this form, verify whether the model is covariance stationary (you may use GAUSS).

(c) Compute the expression for $E(y_{t+2} \mid y_{t-1},...)$

(d) Derive the infinite MA representation for this model.

COMPUTER PROBLEM
Coming soon under separate cover.